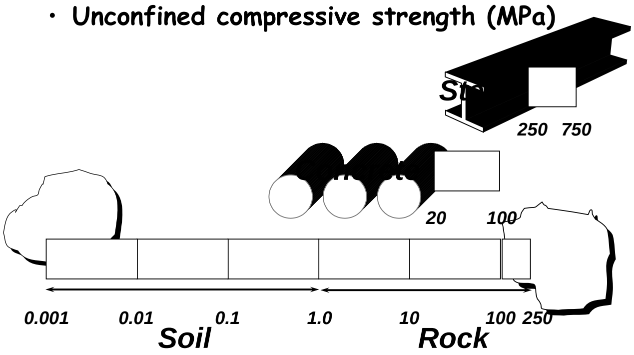
Chapter 8
Shear Strength

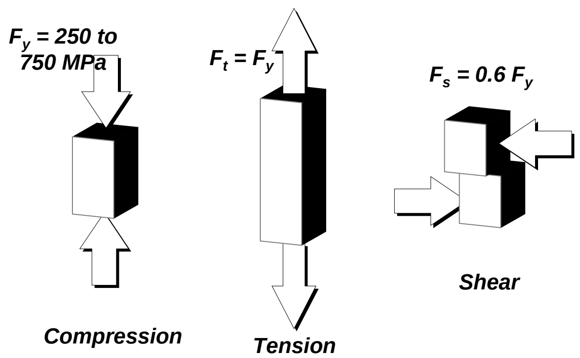
Dr. Talat Bader
May 2006

Soil and Rock Strength

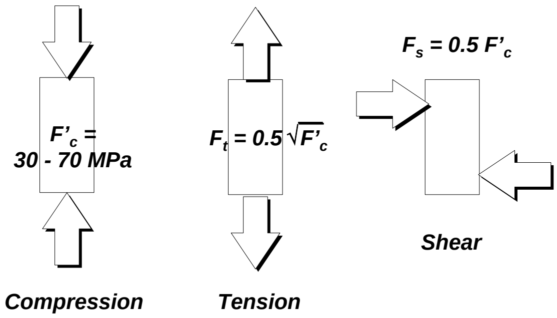
- Unconfined compressive strength (MPa)



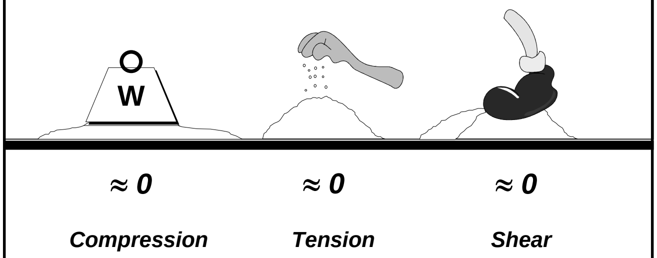
Steel Strength



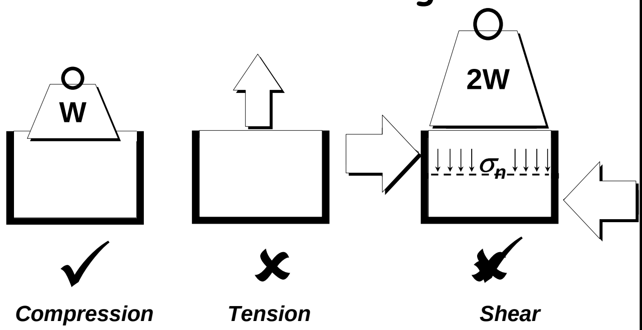
Concrete Strength



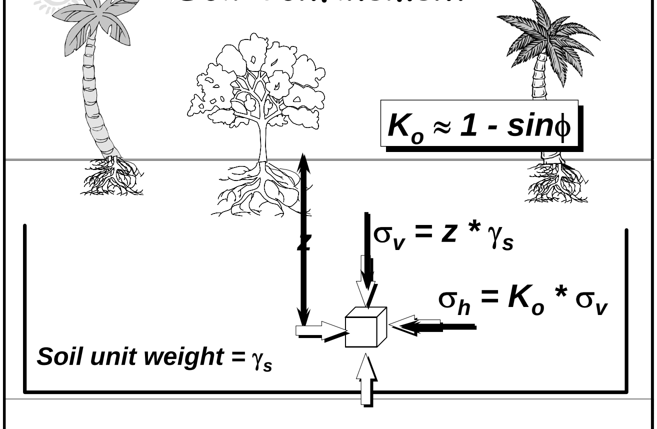
Sand Strength



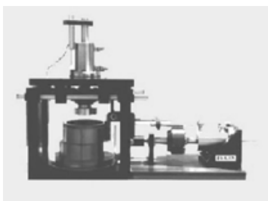
Confined strength

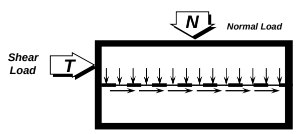


Soil Confinement



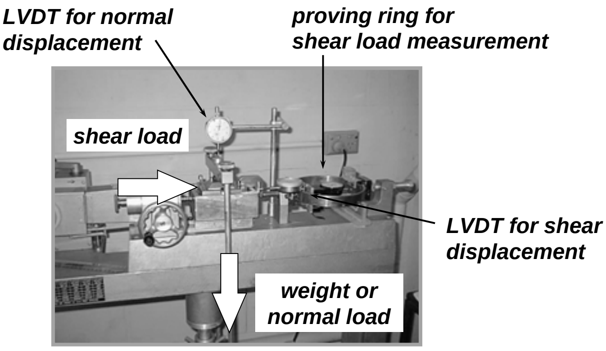
The Direct Shear Test



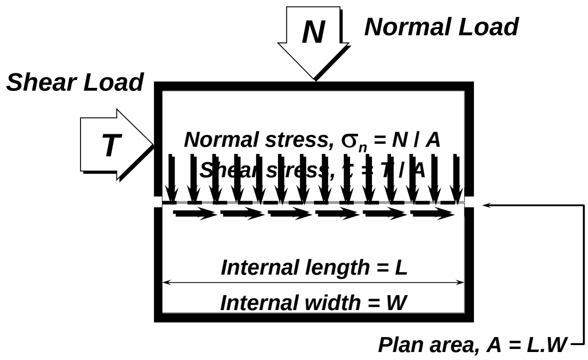


- Measuring directly the normal and shearing stresses on a failure plane.
- The standard apparatus consists of a box 60 mm square although a larger 150 mm square box is available for coarse grained soils where the behaviour of the soil mass is important.

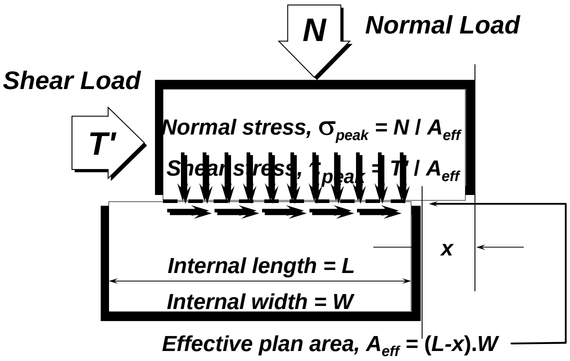
Direct Shear Test



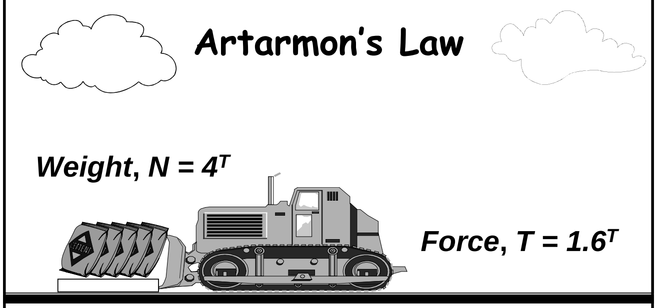
Shear Box Schematic



Peak strength



Artarmon's Law

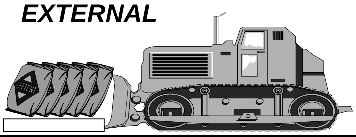


Coefficient of Friction, $\mu = 0.4$

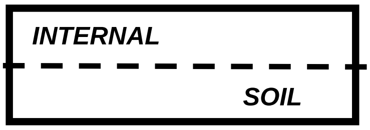
$$\boxed{T = N.\ \mu}$$

Friction

EXTERNAL



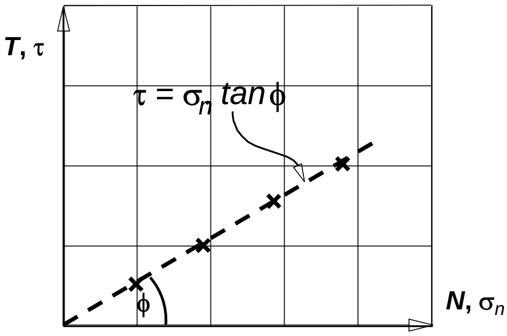
$T = N.\ \mu$



$T = N.\ tan\ (\phi)$

ϕ = *angle of internal friction*

Strength *vs* Stress



Advantages of The Shear Box

- Shear measured directly
- Cohesionless soils tested even quite coarse sizes.
- Measure volume change
- Shear plane forced at a particular point
- Large strain tests possible
- Relatively inexpensive

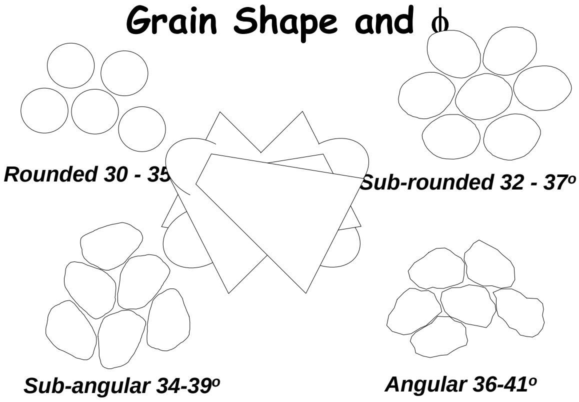
Disadvantages of The Shear Box

- No pore pressure measurements possible
- Normal load not variable
- Sample consistency difficult to achieve
- Non uniformity of normal stress across sample

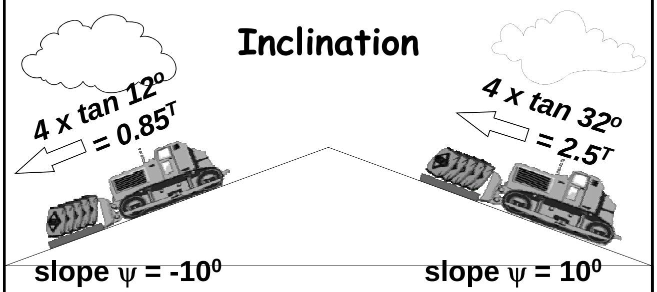
Mineral type and ϕ

Structural	Steel	15°
	Timber	20°
Sands	Quartz	30°
	Calcite	38°
Clays	Kaolinite	15°
	Illite	10°
	Smectite	5°

Grain Shape and ϕ



Inclination



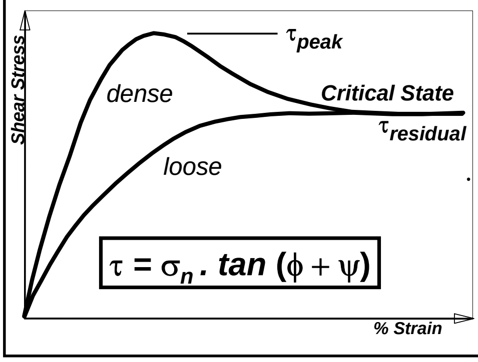
$N = 4^T$

$\mu = \tan\phi = 0.4$

$\Rightarrow \phi = 22^o$

$$\boxed{T = N.\ \tan(\phi + \psi)}$$

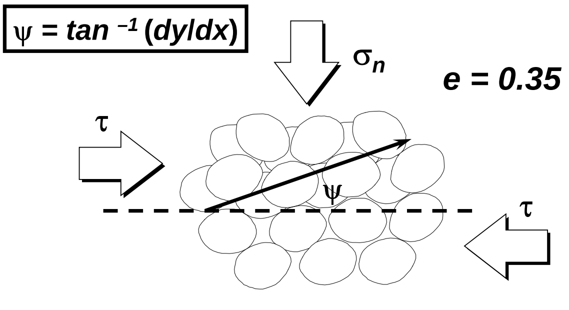
Shear-deformation response



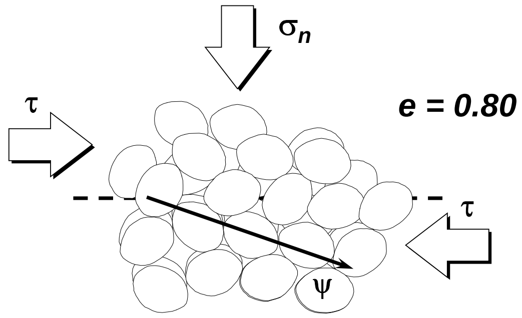
$$\boxed{\tau = \sigma_n.\ \tan(\phi + \psi)}$$

- The stress strain relationship for sands shows a slow rise to a steady value for a loose sand with slight densification creating a slight and apparent fall.
- A dense sand will reach a peak value and then fall to a level appropriate for a loose sand at large strains.

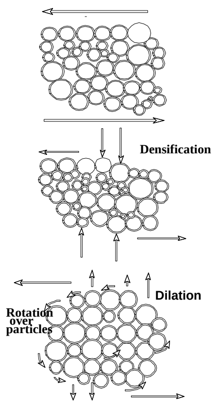
Shearing Dense Sand



Shearing Loose Sand

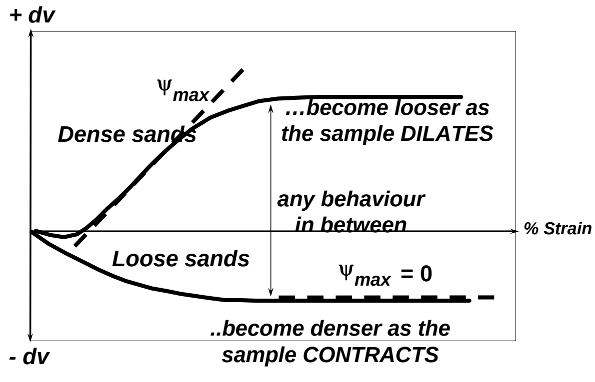


The Shear Behaviour of Sands and Clays



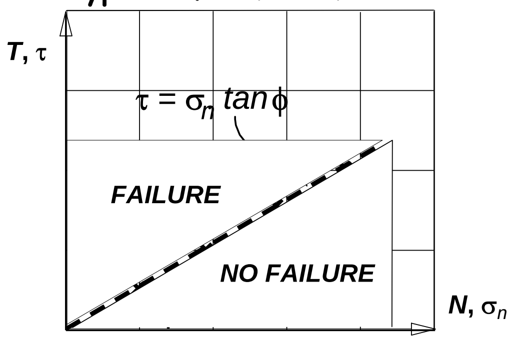
- This is due to the packing of sand to an optimal value followed by a rolling over of coarse grains at larger strains to loosen the packing.
- As the sample is sheared the particles (shown circular) move and adjust the packing
- Loosely packed sample densify packing closer together
- The densely packed samples when sheared may increase their densification slightly and then they start to rotate on individual particles and dilate. This densest point is called the critical state volume.
- The more single sized the sample the more marked the dilation.

Dilation / Contraction



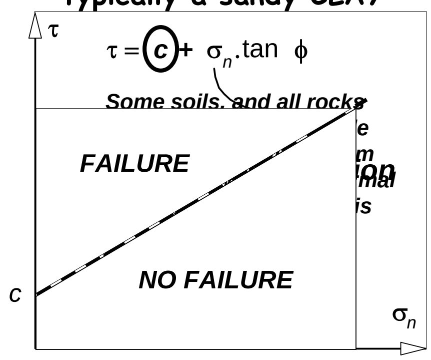
Cohesionless soil

typical of a medium SAND

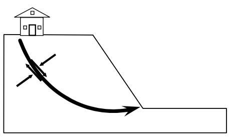


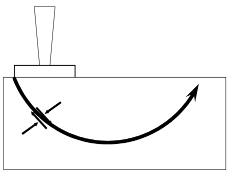
Cohesive Soils

typically a sandy CLAY



Shear Strength Failure Surface in Soils

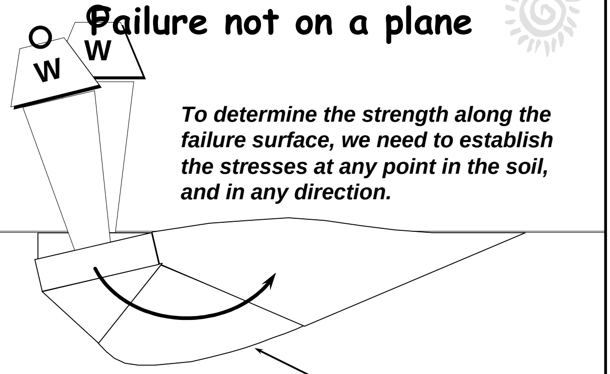




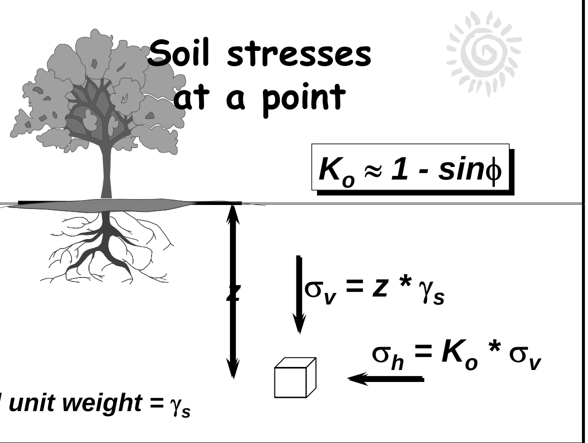
- Near any geotechnical construction (e.g. slopes, excavations, tunnels and foundations) there will be both mean and normal stresses and shear stresses.
- Failure will occur when the shear stress exceeds the limiting shear stress (strength).
- The mean or normal stresses cause volume change due to compression or consolidation.
- The shear stresses prevent collapse and help to support the geotechnical structure.
- Shear stress may cause volume change.

Failure not on a plane

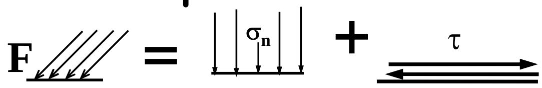
To determine the strength along the failure surface, we need to establish the stresses at any point in the soil, and in any direction.



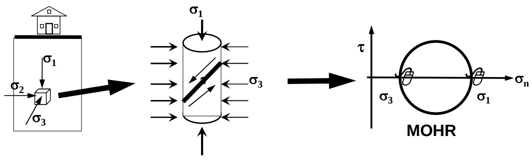
Soil stresses at a point



Principal Stresses

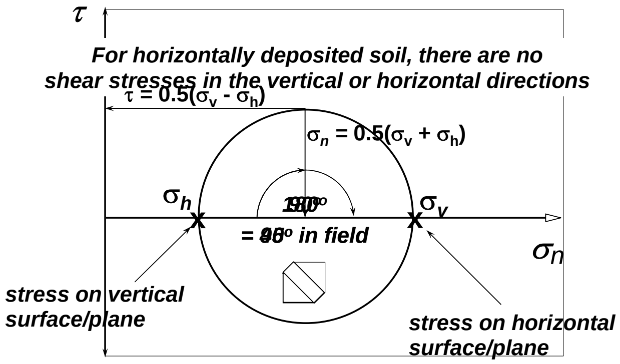


- In a tiny element in the soil mass will have a number of principal stresses acting on it:-

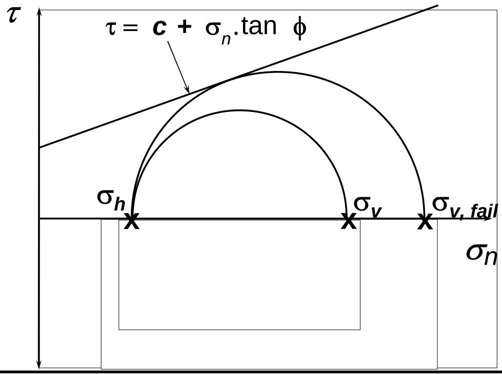


- The major principal stress is σ_1 and the minor principal stress is σ_3, the intermediate stress is σ_2 is ignored in the commonest form of analysis.

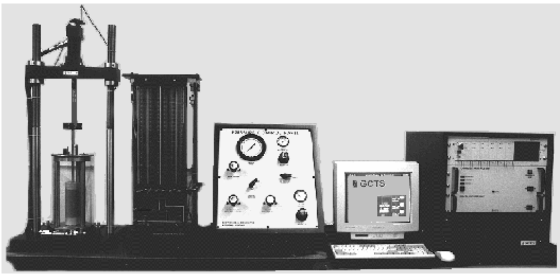
Mohr's circle plot



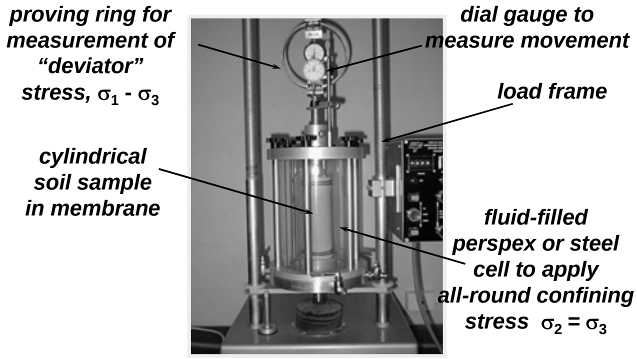
Failure Criterion



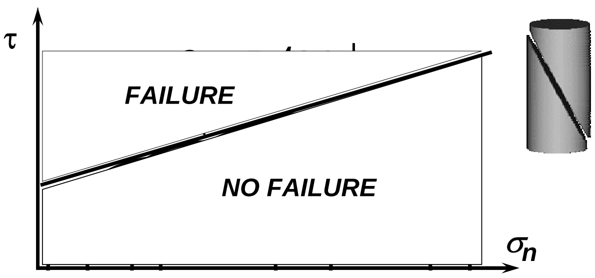
Triaxial Test Set-up



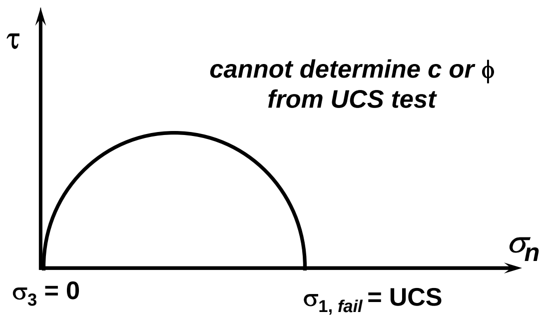
Soil Triaxial Test



Triaxial *c*-ϕ parameters

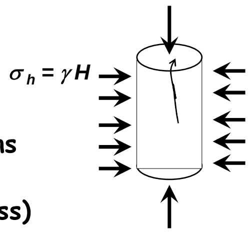


Unconfined Compressive Strength (UCS) Test



Shear Strength of Soils & Rocks

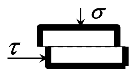
- Not a unique value (e.g. unlike mild steel)
- Depends on
  - geology
  - loading history
  - void ratio
  - water drainage conditions
  - rate of loading
  - depth (or confining stress)



Strength Testing

Direct shear

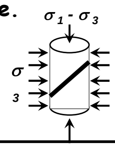
- All soil but best for sand
- short drainage path, relatively quick test
- 60 x 60 x 20 mm confined in box
- no control of drainage or pore press.



Triaxial

- best for clay
- long drainage path, relatively slow test
- 54 mm diameter x 108 mm long
- control drainage, pore press.

pore pressure & volume change measurements possible.



Types of Shear Test

- The way in which we shear soil, in particular the rate, will alter the type of shear strength parameters we obtain.
- DRAINED TEST: Alternatively if we shear the soil *relatively slowly* the pore pressures *will dissipate* and we refer to this test as a DRAINED TEST. The shear stresses are effective stresses and the shear strength parameters are:- c' , ϕ'
- UNDRAINED TEST: If we shear a soil *relatively rapidly* and the pore water pressures *cannot dissipate* or if we deliberately prevent dissipation by blocking drainage; the test is called an UNDRAINED TEST. The stresses will be total stresses and the shear strength parameters are referred to as:- c_u , ϕ_u

Triaxial Testing for Effective Strength

- Two main types of tests
  - Drained (D)
  - Consolidated Undrained with Pore Pressure measurement (CUPP)
- Both involve 3 stages :
  - saturation
  - consolidation
  - shearing

Saturation stage

- Place sample in cell
- Apply cell pressure and back pressure just less than cell pressure and leave
  - test level of saturation using "B test"

$$\Delta u = B\,\Delta\sigma$$

  - For S = 100%, B = 1, $\Delta u = \Delta\sigma$, $\Delta\sigma' = 0$
- if B > 0.95 then go to consolidation stage

Consolidation stage

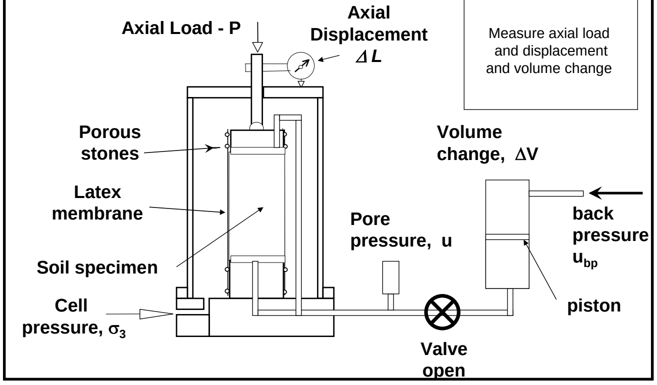
- Increase cell pressure by $\Delta\sigma_3$ to required effective confining stress $\sigma_3' = \sigma_3 - u_{BP}$
- pore pressure will increase by $\Delta\sigma_3$ (ie an excess pore pressure of $\Delta u = \Delta\sigma_3$ above back pressure)
- open drainage valves and allow excess pore pressures to dissipate - sample "consolidates"
- measure volume change of sample against time - coefficient of consolidation
- when volume stops changing $\longrightarrow$ shearing stage

Shearing Stage

D test	CUPP test
• leave back pressure lines open, i.e. drainage allowed	• close back pressure lines, i.e. no volume change
• apply vertical load at rate that allows complete drainage, i.e. no build-up in pore pressures	• apply vertical load at rate that allows equilibrium of pore pressures
• measure load, volume change, displacement	• measure load, pore pressure, displacement
• determine $\sigma_1' - \sigma_3'$ directly from axial load	• determine $\sigma_1 - \sigma_3$, u then σ_1', σ_3'
• determine E' and ν' (from ε_v and ε_a)	• determine E_u, $\nu_u = 0.5$

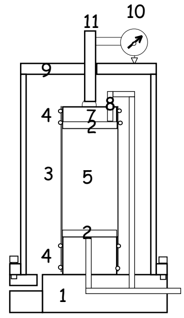
Drained Test

Drained Triaxial set-up
(D test)



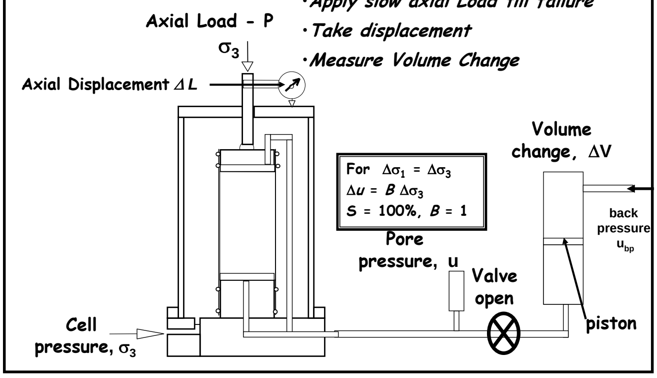
Triaxial Sample Preparation

1 - Cell Base
2 – Porous Stone
3 – Membrane
4 – O-Ring
5 – Soil Specimen
7 – Top Cap
8 – Connect Top Drainage
10 – Displacement Gage
11 – Axial Load Piston



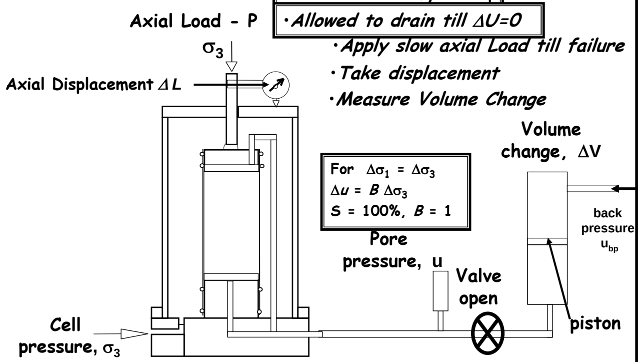
Drained Test

- *Apply σ_3 by filling the cell with water*
- *Axial load σ_3*
- *Dry or Saturated (Test Saturation)*
- *Apply slow axial Load till failure*
- *Take displacement*
- *Measure Volume Change*



Consolidated Drained Test

- *Apply σ_3 by filling the cell with water*
- *Axial load σ_3*
- *Test for Saturation*
- *Consolidate sample $\Delta\sigma_1$*
- *Allowed to drain till $\Delta U=0$*
- *Apply slow axial Load till failure*
- *Take displacement*
- *Measure Volume Change*



Effective (Drained) strength

- Mohr - Coulomb equation appropriate - use effective stresses and strength parameters

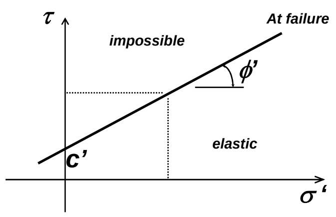
$$s' = \tau_f = c' + \sigma' \tan\phi'$$

$$\sigma' = \sigma - u$$

- Present as τ - σ' plot (use Mohr circles or stress paths)

Effect of confining stress Coulomb's Equation

$$s = \tau_f = c' + (\sigma - u)\tan\phi'$$



s = shear strength

τ_f = shear stress at failure

c' = soil cohesion (effective)

ϕ' = effective angle of shearing resistance

$\sigma' = \sigma - u$ = effective normal stress on failure plane

Strength Parameter Interpretation

Mohr Circle

- plot Mohr circles of effective stress
- draw tangent
- measure c', ϕ'

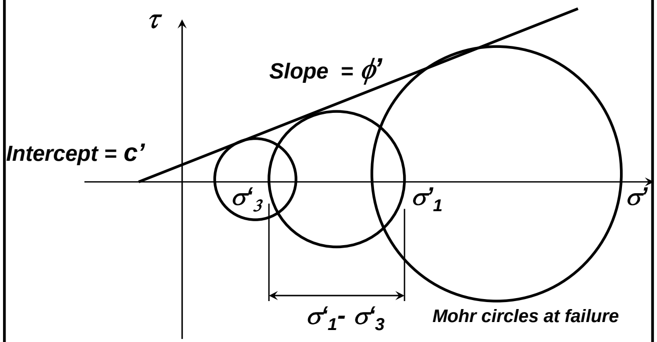
$c' \cot\phi' = d \cot\psi'$

$\sin\phi' = \tan\psi'$

Stress path

- determine
  $p' = (\sigma_1' + \sigma_3')/2 = \sigma'$
  $q' = (\sigma_1' - \sigma_3')/2$
  $= (\sigma_1 - \sigma_3)/2 = \tau$
  for every data point
- plot on τ *vs* σ plot (plots as top of Mohr circle)
- measure d and ψ' from tangent to curves

Mohr-Coulomb failure envelope



Stress Path Method

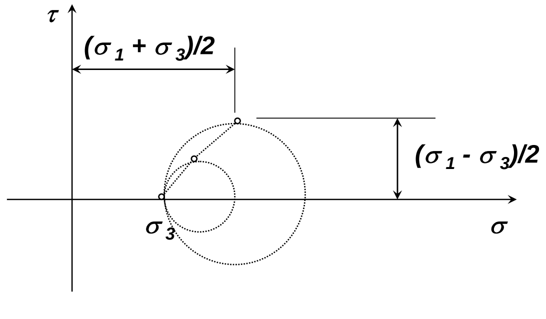
Stress paths

- Alternative way of plotting results
- follows stress state (top of Mohr's circle) during shearing (loading)
- plot (p,q) for each data point, where

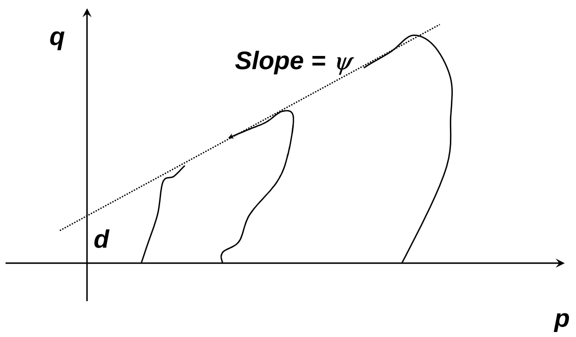
  $p = (\sigma_1 + \sigma_3)/2$ = centre of circle

  $q = (\sigma_1 - \sigma_3)/2$ = radius of circle = τ

Basis of stress path plot



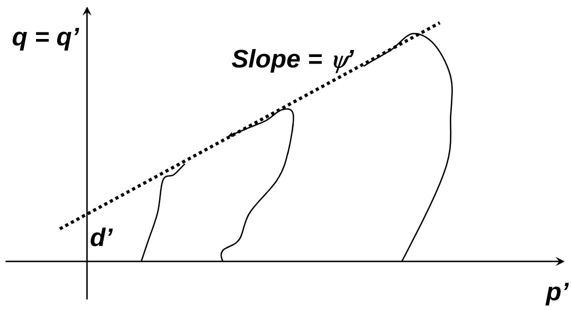
Stress path plot



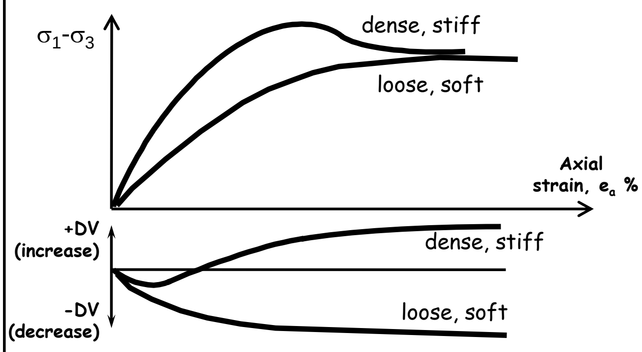
Note $c \neq d$, $\phi \neq \psi$ but are related

End Stress Path Method

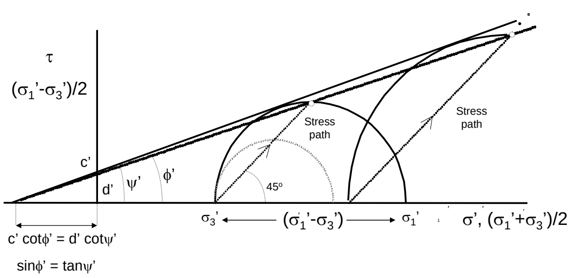
Stress path plot



Typical D test results



Failure envelopes



For cohesionless soils and soft clays c' = 0 and all strength is 'frictional'

Determination of deformation parameters (drained)

- Plot (σ'_1 - σ'_3) against ε_a % and ε_v against ε_a
- drained => volume change $\Delta V \neq 0$
  - => determine ν' from "elastic" portion of (σ'_1 - σ'_3) vs ε % curve using ε_v and ε_a
  - ($\nu' = -\varepsilon_3/\varepsilon_1$ $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ => $\varepsilon_3 = (\varepsilon_v - \varepsilon_a)/2$)
  - => Young's modulus, E' = slope of "elastic" portion of (σ'_1 - σ'_3) vs ε % curve (use 3D's Hooke's Law)

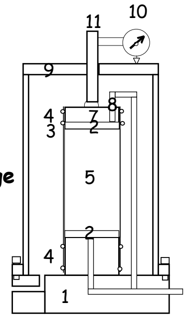
Typical Results for Silty Clay soil (Drained Test)

ϕ_Δ	Soil Plasticity	Plasticity index PI
30°	low plasticity	5–10
12°	high plasticity	50–100
$12^o \le \phi \le 30^o$	moderate plasticity	$10 \le \Pi \le 50$

Consolidated Undrained Test

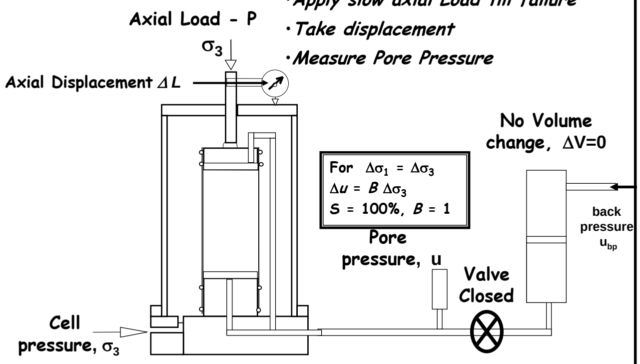
Triaxial Sample Preparation

1 - Cell Base
2 – Porous Stone
3 – Membrane
4 – O-Ring
5 – Soil Specimen
7 – Top Cap
8 – Connect Top Drainage
10 – Displacement Gage
11 – Axial Load Piston



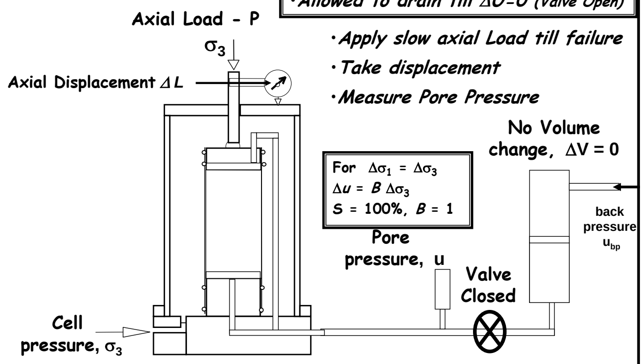
Undrained Test

- Apply σ_3 by filling the cell with water
- Axial load σ_3
- Test for Saturation
- Apply slow axial Load till failure
- Take displacement
- Measure Pore Pressure



Consolidated Undrained Test

- Apply σ_3 by filling the cell with water
- Axial load σ_3
- Test for Saturation
- Allowed to drain till $\Delta U=0$ (Valve Open)
- Apply slow axial Load till failure
- Take displacement
- Measure Pore Pressure



Pore Pressure Development

- No volume change allowed □ pore pressures develop
- $^{+ve}$ pore pressures if sample tries to consolidate (usually loose or soft)
- $^{-ve}$ pore pressures if sample tries to dilate (usually dense or stiff)
- The "A" parameter (after Skempton) is a measure of how much pore pressures will change during loading

Skempton's Pore Pressure Parameters A & B

- Skempton (1954) proposed the following relationship relating pore pressures to total stresses

$$\Delta u = B\,[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

- *A* and *B* are called Skempton's pore pressure parameters

Pore Pressure Parameters cont.

$$\Delta u = B\,[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

- For $\Delta\sigma_1 = \Delta\sigma_3$

$\Delta u = B\,\Delta\sigma_3$

- For S = 100%, B = 1

$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$

- For CU triaxial test

$\Delta\sigma_3 = 0$

$\Delta\sigma_1 - \Delta\sigma_3 = \sigma_1 - \sigma_3$

$\Delta u_f = A_f(\sigma_1 - \sigma_3)$

Skempton's A Parameter

Type of Clay	A_f
Highly sensitive clays	*0.75 to 1.5*
Normally consolidated clays	*0.5 to 1*
compacted sandy clays	*0.25 to 0.75*
lightly overconsolidated clays	*0 to 0.5*
compacted clay-gravels	*-0.25 to 0.25*
heavily overconsolidated clays	*-0.5 to 0*

What dos a negative value imply ?

Pore Pressure Parameter, B
(Skempton)

For undrained, all-round compression of a soil

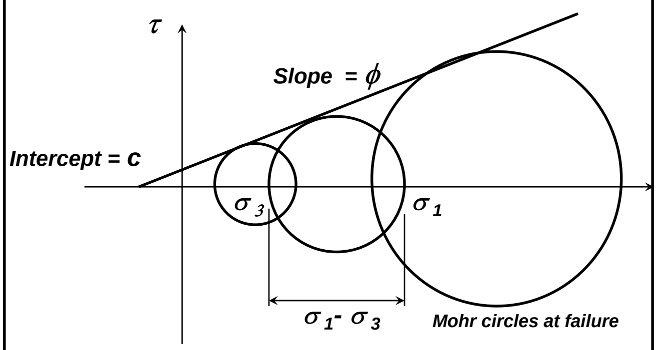
$$\Delta u = B\,\Delta\sigma$$

for S = 100 %, $B \approx 1$, $\Delta u = \Delta\sigma$, $\Delta\sigma' = 0$

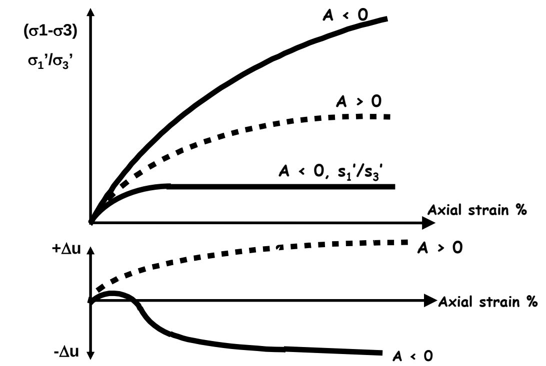
For rock, the rock skeleton is significantly stiffer and as a result, on undrained, all-round compression, the skeleton takes more of the applied load and hence B is less than 1.

Can measure B is a triaxial test. A "B test" is often performed to test saturation of a sample.

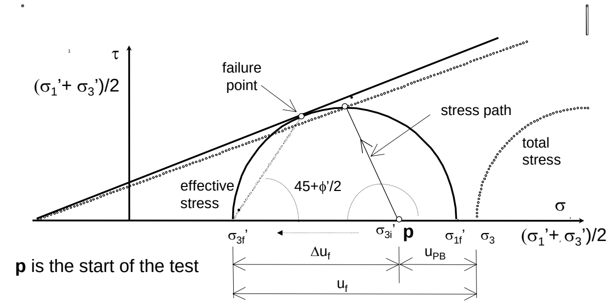
Mohr-Coulomb failure envelope



Typical CUPP test results



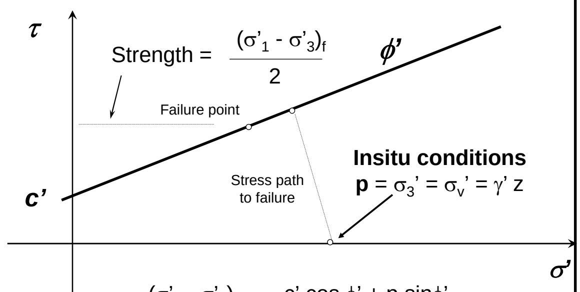
Failure envelopes



p is the start of the test

$$\Delta u_f = A\,(\Delta\sigma_1 - \Delta\sigma_3) = A\,(\sigma_1 - \sigma_3)$$

Estimating strength from drained strength parameters



$$\frac{(\sigma'_1 - \sigma'_3)_f}{2} = \frac{c' \cos\phi' + p \sin\phi'}{1 + (2a - 1)\sin\phi'}$$

Determination of deformation parameters (undrained)

- Plot $(\sigma'_1 - \sigma'_3)$ against ε_a %
- undrained => no volume change, $\Delta V = 0$
  => $\nu_u = 0.5$ (undrained Poisson's ratio)
- Young's modulus, E_u = slope of "elastic" portion of $(\sigma_1 - \sigma_3)$ or $(\sigma'_1 - \sigma'_3)$ vs ε % curve

Summary : Strength

Undrained (total)	Drained (effective)
• total stresses	• effective stresses ($\sigma' = \sigma - u$)
• c_u and ϕ_u	• c' and ϕ', A and B
• not fundamental	• fundamental
• saturation & pore pressures unknown	• saturation and pore pressures known
• short term	• long term

Analysis of Undrained Test

Determination of stresses

$(\sigma_1 - \sigma_3) = P/A$

Where

P = load

A = cross-sectional area

Correct for change in cross-sectional area (volume remains constant)

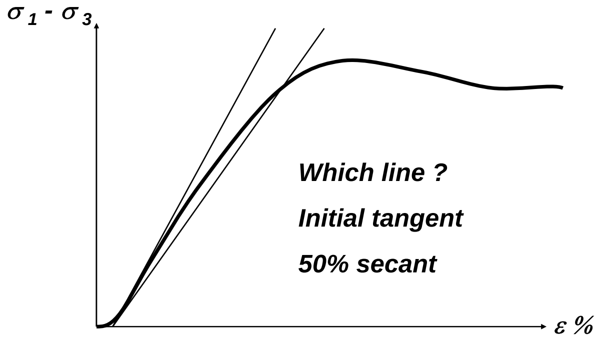
$A = A_o/(1-\varepsilon)$

ε = axial strain = $\Delta L/L_o$

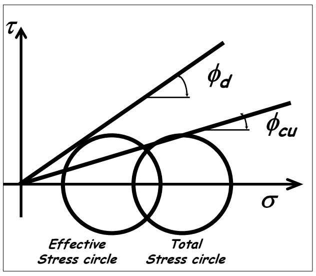
Determination of deformation parameters

- Plot $(\sigma_1 - \sigma_3)$ against ε %
- undrained => no volume change, $\Delta V = 0$
  => $\nu_u = 0.5$ (undrained Poisson's ratio)
- Young's modulus, E_u = slope of "elastic" portion of $(\sigma_1 - \sigma_3)$ vs ε % curve
  (use 3D's Hooke's Law to show this)

Determination of E_u



CU & D Triaxial Tests



- The effective stresses give ϕ the drained angle of friction.
- The total stresses give ϕ_{XY}, the consolidate-undrained angle of friction.
- Typically ϕ_{XY} is about half of ϕ_{Δ}

Total : $\sigma_1 = \sigma_3 + \Delta\sigma$ $\quad$ $\sigma_3 = \sigma_3$ $\quad$ $\sigma_1 - \sigma_3 = \Delta\sigma$

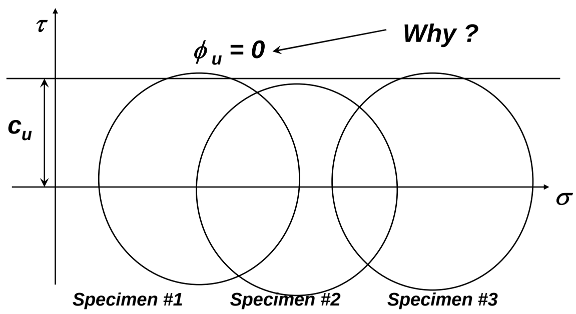
Effectiv: $\sigma'_1 - \sigma'_3 = \Delta\sigma$ $\quad$ $\sigma'_1 = \sigma_3 + \Delta\sigma - \upsilon$ $\quad$ $\sigma'_3 = \sigma_3 - \upsilon$

Unconsolidated Undrained Strength Saturated clay

Undrained (Saturated Clay)

- Remove "undisturbed" sample from tube
- place in cell, apply cell pressure (to re-instate field conditions)
- no drainage allowed at any stage
- increase axial load through constant displacement of ram at relatively fast rate
- measure axial load and strain
- repeat for 2 more samples (or multi-stage test)
- plot results in terms of total stresses (as pore pressures are unknown)

Undrained Test Results
(100% saturated)



Why does ϕ_u = 0 ?

Consider two tests with different cell pressures

$\Delta\sigma_3$ = 100 kPa	$\Delta\sigma_3$ = 200 kPa

Skeleton relatively compressible, increase in cell pressure is therefore carried by pore water

=> Δu =100 kPa	=> Δu = 200 kPa
therefore, initially	therefore, initially
$\Delta\sigma_3' = \Delta\sigma_3 - \Delta u$	$\Delta\sigma_3' = \Delta\sigma_3 - \Delta u$
= 100 - 100	= 200 - 200
= 0	= 0

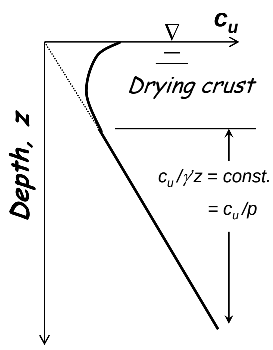
- Actual initial pore pressures are unknown (and are probably negative - why ?)
- pore pressures at failure are also unknown but they will differ by 100 kPa
- hence, at failure, major and minor principal effective stresses will be the same => both specimens plot as identical effective stress Mohr circles
- c_u and ϕ_u are not fundamental soil properties, but are products of testing method and interpretation

Undrained strength

- Doesn't matter what cell pressure is used for testing
- Why carry out more than one test ?
- For samples obtained from different depths - will undrained strength be different ?
- Used for short term analyses (also called total stress and undrained) of bearing capacity, slope stability etc problems

Undrained Strength Versus Depth

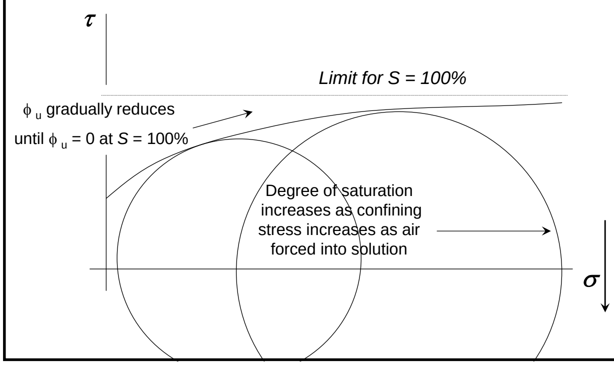
- **In soft clays**, c_u varies with depth
- **In stiff clays**, c_u is approx. constant over limited depth range



Unsaturated soil/rock

- Most soil/rock above water table is partially saturated; S ~ 80 to 90 %
- why isn't it dry ? (capillary action) => suctions
- B < 1 => for undrained conditions we will get some increase in effective stress (why ?)
- therefore may measure a friction angle

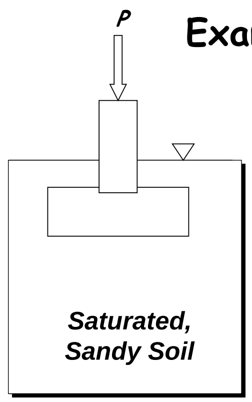
Friction angle - why ?



Sources of Error in Triaxial Test

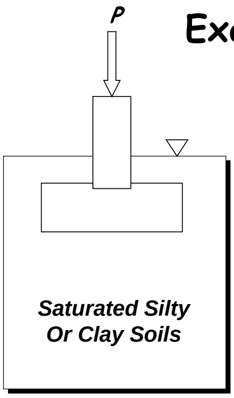
- Sources of error within the triaxial test are:
  - sample disturbance
  - poor preparation
  - air bubbles in sample and in pore water lines in drained tests
  - punctured membranes and poor seals
  - unsaturated soil
  - loading rate not appropriate for test
  - calibration of volume change
  - insensitivity of load frame....stiffness

Example 1 (Sandy Soil)



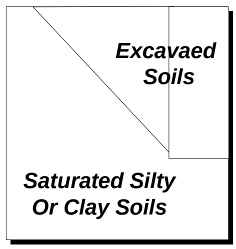
- Since the stresses from the foundation loads are quickly transferred to the soil skeleton, the foundation loads are carried by effective stresses.
- To determine whether or not shear failure would occur in the soil from the foundation loads, we would use a drained shear strength criterion with respect to the effective stresses in the soil mass.

Example 2 (Clay Soil)



- In the short term the increased stresses from the foundation loads are quickly transferred to the soil skeleton and the pore fluid. In the short term, we would use an undrained shear strength failure criterion $(\chi = \chi_{\upsilon}, \phi = 0)$ το *assess* possible shear failure.
- In the long term, the increased stresses from the foundation loads are carried by the soil skeleton via effective stresses. In the long term, we would use a drained shear strength failure criterion to assess possible shear failure.

Example 3



- **Assume that we are quickly *cutting* a slope in a saturated clay soil deposit.**
- **In the short term, we would use an** undrained shear strength **model to determine whether or not shear failure (or a slope failure) would occur.**
- **In the long term, we would use a** drained shear strength **model to determine**
- **whether or not shear failure (or a slope failure) would occur.**

Final Thoughts on Shear Strengths

- Fine-grained soils
  - ↓ high SSA's
  - ↓ low permeability's (drain slowly)
  - ↓ relevant strength is typically undrained
- Coarse-grained soils
  - ↓ lowSSA's
  - ↓ High permeability's (drain quickly)
  - ↓ relevant strength is typically drained

How To Determine Shear Strength in the Field?

In-situ Test Methods

In-situ Testing

- Soil is tested without extraction from the ground
- Provide:
  - preliminary or approximate design data
  - in-situ properties where undisturbed sampling is not possible
- Advantages:
  - cheap
  - "On the spot" results

Standard Penetration Test (SPT)

- Used primarily in granular soils
- Crude form of testing but results are widely accepted globally
- Correlations
  - relative density and friction angle
  - undrained cohesion
  - direct estimate of settlement
- Corrections - overburden pressure and PWP build-up

Cone Penetrometer Test (CPT)

- Instrumented probe jacked into ground at constant rate of penetration (2cm/sec)
- Cone resistance (q_c) and sleeve friction (f_s) measured
- cone resistance (q_c) correlates with strength, and friction ratio (q_c/f_s) with material type
- Should always be correlated with borehole information

Settlement and Bearing Correlations

Pressuremeter Testing

- Probe is inserted in the ground and inflated
- Prebore (Menard) and self boring
- Strength and modulus can be determined (drained or undrained?)
- Convenient test method but some aspects are not yet fully understood

DilatometerTest

- Probe is jacked/driven in the ground and membrane is inflated
- Membrane displaced and required pressure recorded
- Correlates with:
    - material type
    - undrained shear strength
    - lateral earth pressure co-efficient
    - soil modulus

More In-situ Tests

- Vane shear tests
- Pocket penetrometer
- Dynamic (driven) Cone Penetrometer
- Permeability testing
    - packer test
    - falling head test
    - talsma tubes

IN SITU TESTING

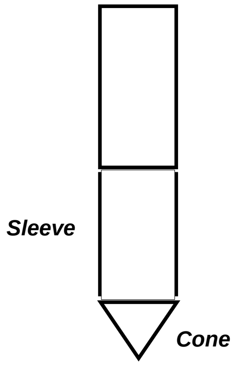
- Undisturbed material
- Large volume of material
- Inclusion of defects
- Mechanical / Geophysical testing
- uncertain test configuration

- *Complements sampling*

Mechanical in situ tests

- Vane shear - torque on vane gives undrained cohesive strength of soft clay
- Hammering - Standard Penetration Test (64 kg), Dynamic cone (9kg) -pavements.
- Static (Dutch) Cone penetrometer.
- Lateral pressure tests - pressuremeter in bore hole, Camkometer, Flat plate dilatometer
- Trial footing / pile tests, CBR

Cone Penetrometer



- continuous record to define stratigraphy
- q_c and f_s
- Friction angle of sands
- Undrained strength of clays
- Material type from friction ratio = f_s / q_c

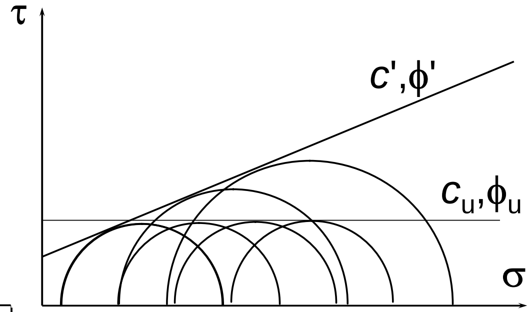
Measurement in borehole (i)

- Standard Penetration Test (SPT) very common world-wide
  - $N < 10$ very loose to loose
  - $30 < N < 50$ dense
  - $N > 50$ very dense
  - correlations available between N and ϕ
  - beware - many correction factors proposed see Kulhawy for comprehensive treatment

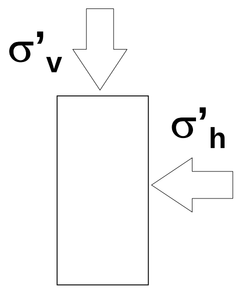
Measurement in-situ (i)

- Cone penetration test (CPT) common in Europe, Australia, parts of US.
  - Correlations are available for ϕ based on cone resistance, q_c
  - Correlations also for soil type, based on q_c and friction ratio

Triaxial



Triaxial



- Physical disturbance
- Does sample have same m.c. as in field?
- What is *K*? i.e. is σ'_h correct?
- Does $\sigma'_{h1} = \sigma'_{h2}$?
- Is σ_1 vertical ?