Stresses in Soil Mass

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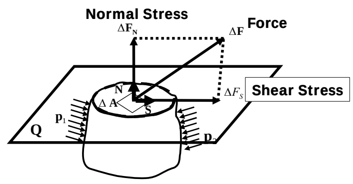
Chapter 6

Soil Stresses at a Point

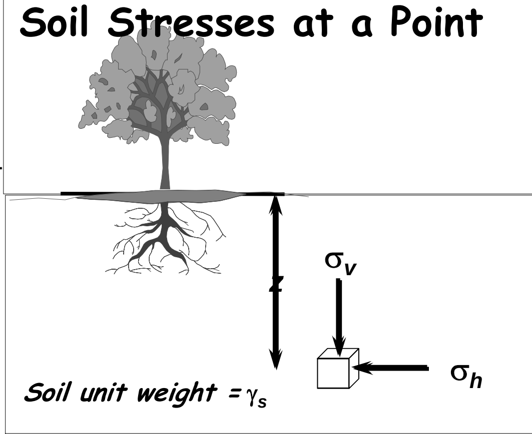
$$\text{Normal Stress} = \sigma = \lim_{\Delta A \to 0}\left(\frac{\Delta F_N}{\Delta A}\right)$$

$$\text{Shear Stress} = \tau = \lim_{\Delta A \to 0}\left(\frac{\Delta F_S}{\Delta A}\right)$$



Stress Acting on a Plane
Internal Forces Needed For Equilibrium

Soil Stresses at a Point



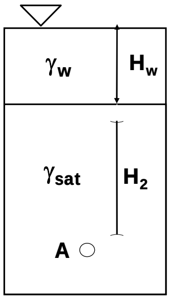
Stresses in Saturated Soil without Seepage

- Total vertical stress is called the body stress because it is generated by the mass in the body.
- To calculate the total vertical stress at a point in a soil mass, simply sum up the unit weights of all the material (soil solids & water) above that point: $\sigma_v = \int_0^H \gamma dz$
  - If the unit weight is constant throughout the depth, then $\sigma_v = \gamma H$
  - Because soil is typically layered, the total stress is computed using $\sigma_v = \sum_{i=1}^{n} \gamma_i z_i$

What is total vertical stress at point A

◈ **Solution**

$$\sigma_v = \gamma_w H_1 + \gamma_{sat} H_2$$



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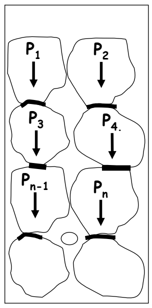
Total Stresses

(in Saturated Soil without Seepage)

- The **total stress** can be divided into two parts:
  1. a portion carried by water in the continuous void space (**neutral stress** or pore water pressure)
  2. a portion carried by the soil solids at their points of contact (**effective stress**)..
  - If we add up the vertical forces through the soil contacts and divide by the cross-sectional area:

$$\sigma' = \frac{P_1 + P_2 + P_3 + \cdots + P_n}{\overline{A}}$$



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Total Stresses

(in Saturated Soil without Seepage)

- Notice that the soil contacts have areas, a_1, a_2, (fraction of areas occupied by solid –to- solid contat)etc. So the area acted on by the pore pressure is

$$\overline{A} - \sum a_i = \overline{A} - a_s$$

- So we can write

$$\sigma = \sigma^{'} + \frac{u(\overline{A} - a_s)}{\overline{A}} = \sigma^{'} + u(1 - a_s^{'})$$

- But in actuality, the value of a_s' is very small and can be neglected, thus the resulting equation is

$$\sigma = \sigma' + u$$

- This is probably the most famous and significant equation developed by Karl Terzaghi (1925).

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Stresses in Saturated Soil with Seepage

- If water is seeping, the effective stresses will be different from the static case because the pore pressures are different. The effective stress can increase or decrease depending on the direction of seepage.
- Downward Seepage - see Fig a
- Upward Seepage - see Fig b
  - note that upward seepage reduces the effective stress. The minimum value corresponds to a critical gradient:
- Critical Conditions

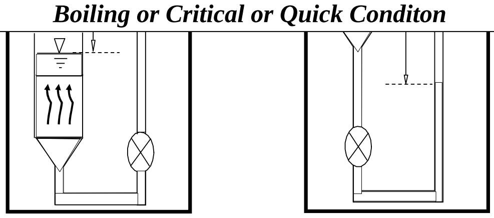
$$\sigma_c' = z\gamma' - i_{cr} z\gamma_w = 0$$

When the

Seepage Force = H γ_{sub} -- Effective Stress σ_{eff} = 0

This case is referred as

Boiling or Critical or Quick Conditon

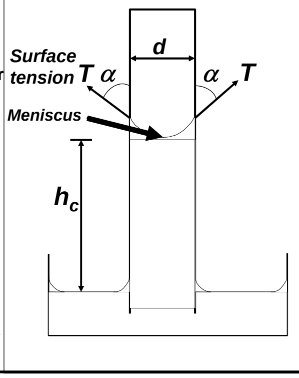


Upward Seepage (b) **Downward Seepage (a)**

Seepage Force

- **Another means of computing effective stress is through a seepage force per unit volume of soil.**
- **Compare the upward seepage forces to the static forces to determine the seepage force magnitude.**
- **Make sure that you understand Example 6.1, 6.2 & 6.3 page 192 in your book.**

Capillary Rise



$$h_c = \frac{4T}{\gamma_w d}\cos\alpha$$

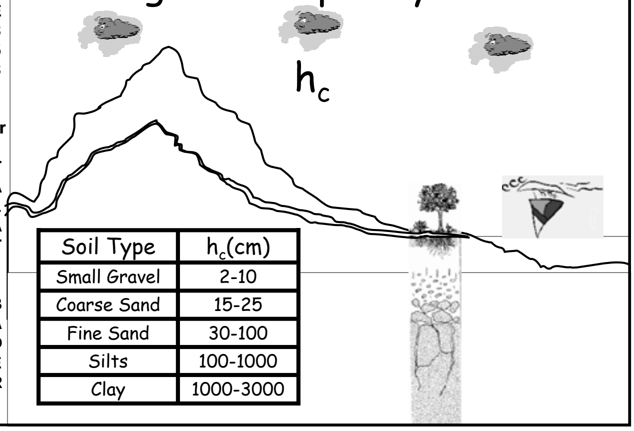
Factors Affecting Rise in Soil

- ***Shape***
- ***Size***
- ***Configuration of Voids***

- **α = 0**
- ***D = 10% Effective Particle Size***
- ***T = 75 dyn/cm***

$$h_c = 1.50 / D_{10}$$

Hight of Capillary Rise



Soil Type	h_c(cm)
Small Gravel	2-10
Coarse Sand	15-25
Fine Sand	30-100
Silts	100-1000
Clay	1000-3000

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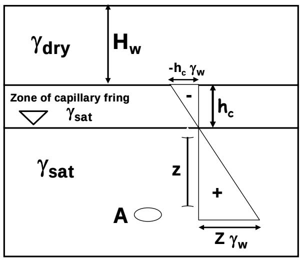
Calculation of Effective Stress in Capillary Zone

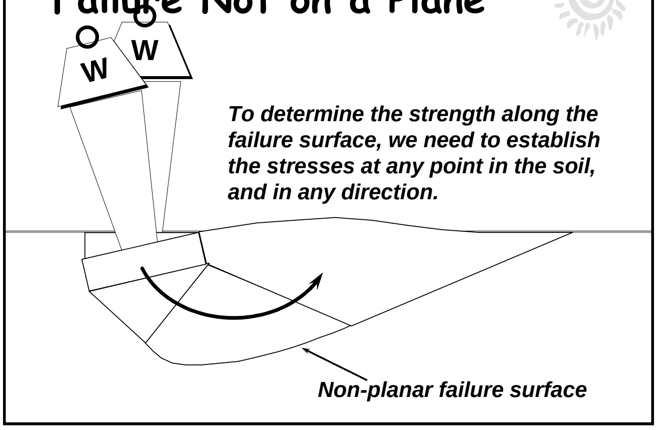
- The pore pressure at a point in a layer of soil fully saturated by capillary rise is equal to $u = -\gamma_w h_c$
  - Where h_c = distance of point above GWT.
- In the case of partially saturated soils (S = degree of saturation, %)

$$u = -\left(\frac{S}{100}\right)\gamma_w h_c$$

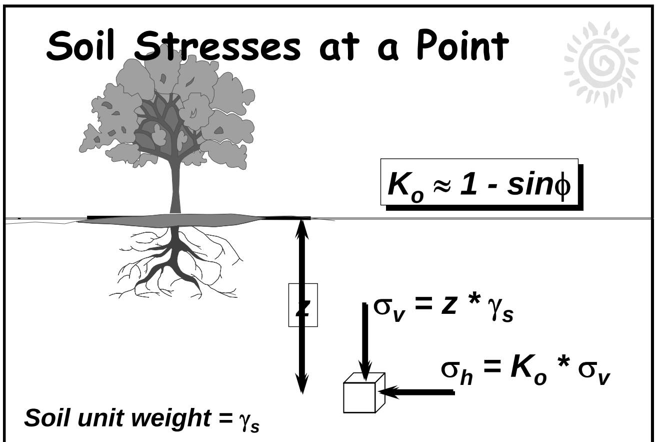


Failure Not on a Plane

To determine the strength along the failure surface, we need to establish the stresses at any point in the soil, and in any direction.



Soil Stresses at a Point



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Mhore Circle

- Read section 6.2 & Solve the Homework problem
- Mohre circle totorial
- http://www.science.ubc.ca/~geol307/mohr_intro.html

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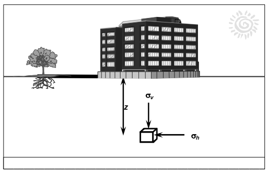
Basics of Surface Loads

- Categorize into groups based on areal extent
  - infinite extent (fills, surface surcharge)
  - finite extent
    - point load
    - line load
    - Infinite line load
    - strip load
    - uniformly loaded circular area
    - Uniformly loaded rectangular area

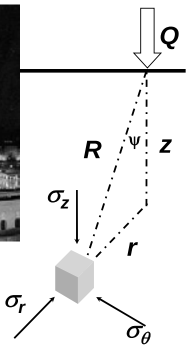


Basics of Surface Loads

- Load produces stress and strain
  - Stress and strain occur in all directions
  - Commonly focus only on vertical stress increase
- Analysis based on elastic theory
  - isotropic, homogeneous material
  - linear elastic behavior (spring-like)
  - material comprises a half-space

Stress Caused by a Point Load

- First solved by Boussinesq (1883)
- Stress is maximum nearest applied load and diminishes at distances away.



$$\Delta\sigma_z = \frac{Q}{z^2}\left\{\frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}\right\}$$

Distribution of Stress

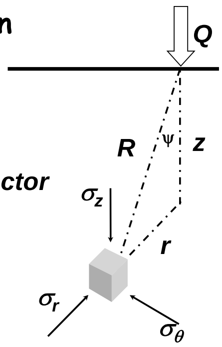
- Boussinesq solution
- By rearranging

e.g. $\sigma_z = \frac{Q\, I_\sigma}{z^2}$

I_σ is stress influence factor

$$I_\sigma = \frac{3}{2\pi[1+(r/z)^2]^{5/2}}$$

The Influence factor I_σ versus the r/z value is shown graphically



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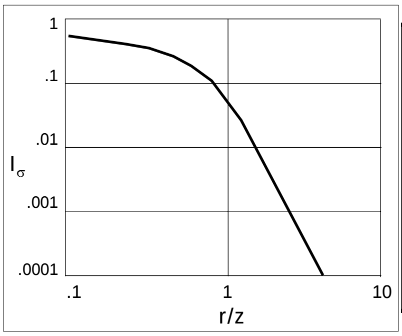
The Influence factor I_σ versus the r/z value is shown graphically in Figure 6.11 Page 206



r/z	Iσ	r/z	Iσ
0	0.4775	1.2	0.0513
0.1	0.4657	1.4	0.0317
0.2	0.4329	1.6	0.0200
0.3	0.3849	1.8	0.0129
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.2	0.0058
0.6	0.3314	2.4	0.0040
0.7	0.1762	2.6	0.0028
0.8	0.1386	2.8	0.0021
0.9	0.1083	3.0	0.0015
1.0	0.0844	4.0	0.0004

I_σ 21

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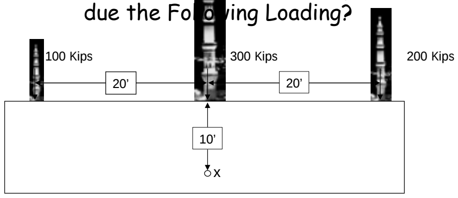
Example 1

Calculate the vertical Stress at point x due the Following Loading?



Load	r/z	Iσ	$\Delta\sigma_z$
100	20/10=2	0.0085	100(0.0085)/10²=0.0085
300	0	0.4775	300(0.4775)/10²=1.4325
200	20/10=2	0.0085	200(0.0085)/10²=0.017
		$\Sigma\Delta\sigma_z$	1.458 ksf

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Stress Caused by a Point Load

- ◈Slightly different problem solved by Westergaard (1938)
- ◈Based on thin layers sandwiched between reinforcement.

$$\Delta\sigma_z = \frac{PC}{2\pi z^2}\left\{\frac{1}{C^2 + (r/z)^2}\right\}^{3/2}$$

$$\text{where } C = \sqrt{\frac{1-2\nu}{2-2\nu}}$$

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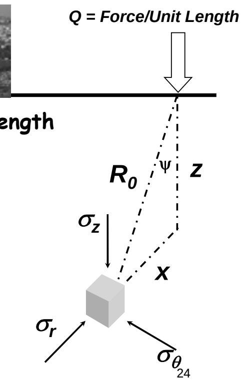
Stress Caused by Line Load



- ◈ Line load of magnitude p/unit length acts on half-space surface
- ◈ Vertical stress increase is:

$$\Delta\sigma_z = \frac{2Q}{z\pi\left[\left(\frac{x}{z}\right)^2 + 1\right]^2}$$

Solution is in Page 211

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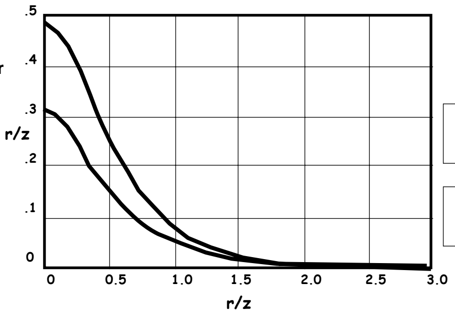
Point Load & Line Load Charts



$\sigma_z = \frac{Q}{z^2} I_\sigma$

Point Load

$I_\sigma = \frac{3}{2\pi\,[(r/z)^2+1]^{5/2}}$

Line Load

$I_\sigma = \frac{1}{\pi\,[2(r/z)^2+1]^{3/2}}$

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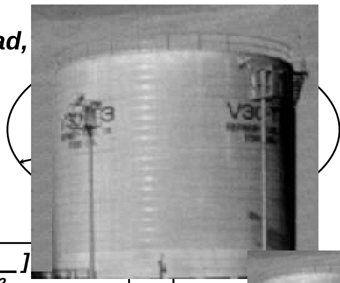
Uniformly Loaded Circular Area

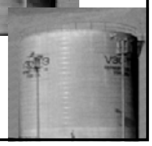
load,

By integration of Boussinesq solution over complete area:

$\sigma_z = q\left[1 - \frac{1}{[1+(a/z)^2]^{3/2}}\right]$

Solution Page 220





σ_z

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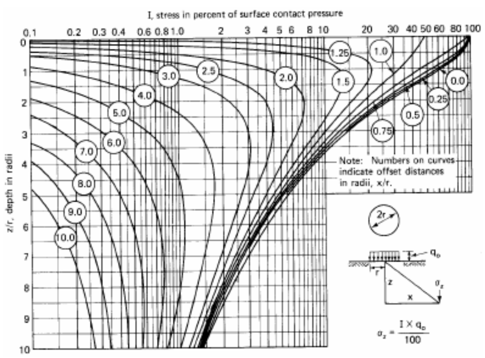
Chart for Uniformly Loaded Circular Area

A typical solution chart for circular footings is shown in this Figure. Note that q_o is in terms of stress (kPa).



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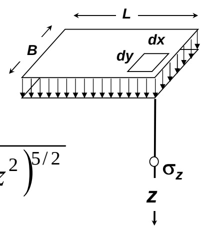
Stress Caused by Rectangularly Loaded Area

- Use Boussinesq point load on small parts of rectangle.

$$d\sigma = \frac{3q\,dx\,dy\,z^3}{2\pi\left(x^2+y^2+z^2\right)^{5/2}}$$

- Then integrate over rectangle area

$$\Delta\sigma = \int d\sigma = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3}{2\pi\left(x^2+y^2+z^2\right)^{5/2}}dx\,dy$$



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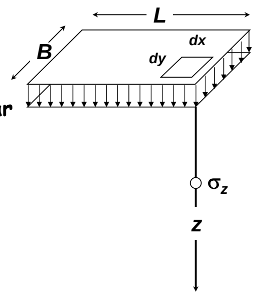
Stresses Under Rectangular Area

- Solution after Newmark for stresses under the corner of a uniformly loaded flexible rectangular area:
- Define *m = B/z* and *n = L/z*
- Solution by charts or numerically
- $\sigma_z = q.I_\sigma$



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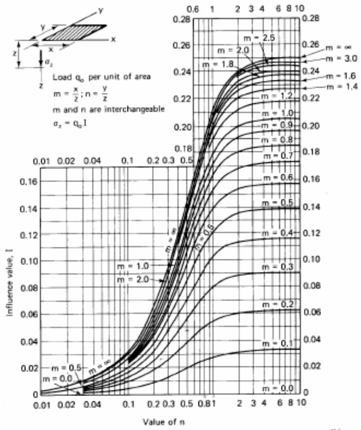
- $\sigma_z = q.I_\sigma$
- Define
  - *m = B/z*
  - *n = L/z*

Chart for Stresses Under Rectangular Area



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Stress Caused by Any-Shaped Loaded Area

- Use influence Newmark's chart:
  - requires scale drawing of area; scale is based on influence chart and depth at which stress is desired
  - basic principle is a graphical means of integrating effects of loaded area by counting squares on influence chart that are enclosed within the loaded area.

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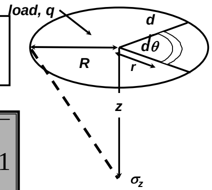
Stress Increment Approximation Using Newmark's Chart

This is a graphical procedure Developed by Newmark (1942)
Based on Stress developed under a circular loaded area

$$\sigma_z = q\left[1-\frac{1}{[1+(R/z)^2]^{3/2}}\right] = q.I_\sigma$$



Rearrange $$\frac{R}{z}=\sqrt{\left(1-\frac{\Delta\sigma_z}{q}\right)^{-2/3}-1}$$

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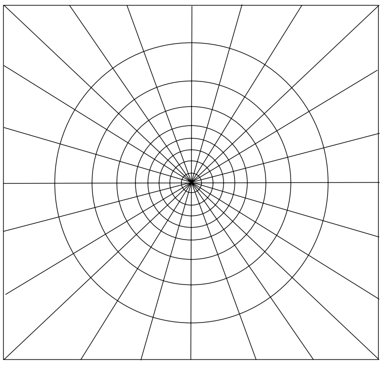
Newmark's Influence Chart for Vertical Stress Increment Approximation

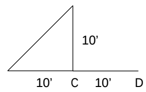


Scale: A ——— B

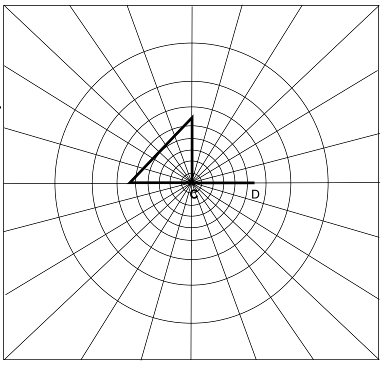
Influence Value: I_n=0.005

Example

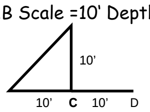
- Calculate the vertical increment at the 10 for depth under points C and D for the following profile. Assume a contact pressure of 2000 psf



Solution
Vertical Stress Increment Under Point C



- AB Scale =10' Depth

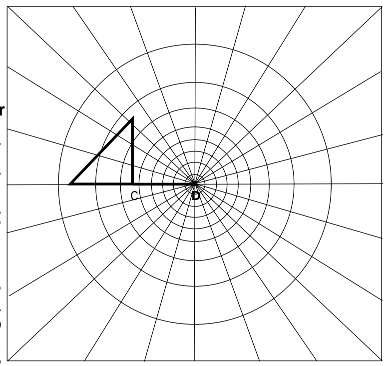


Scale: A ——— B

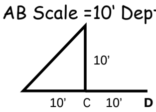
- AB Scale = 10' Depth
- Place point C in the center of the chart
- Count # of elements filling the Area = 25.7
- Then
- $\Delta\sigma$ = 25.7(0.005)(2000)
- $\Delta\sigma$ = 257 psf

Influence Value: I_n=0.005

Solution
Vertical Stress Increment Under Point D



- AB Scale =10' Depth



Scale: A ——— B

- AB Scale = 10' Depth
- Place point D in the center of the chart
- Count # of elements filling the Area = 3
- Then
- $\Delta\sigma$ = 3(0.005)(2000)
- $\Delta\sigma$ = 30 psf

Influence Value: I_n=0.005

Advantages and Disadvantages of Newmark's Influence Chart

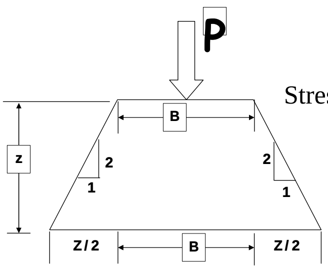
- Easy to use

- The plan must be redrawn for each depth where the stress increment is required (Time consuming)

Approximate Method

- In the absence of any charts a quick approximate way of determining the stress beneath a foundation is by using the 2:1 method.
- This is achieved by assuming that the stress dissipates at a ratio of 2:1 with foundation depth.
- This can be done for both strip footings and rectangular pad footings as shown

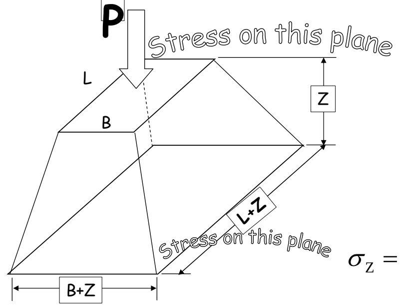
Strip Footing



$$\text{Stress } \sigma_0 = \frac{P}{B \times 1}$$

$$\text{Stress } \sigma_z = \frac{\sigma_0 B \times 1}{(B+z) \times 1}$$

Rectangular Footing



$$\sigma_0 = \frac{P}{B \times L}$$

$$\sigma_Z = \frac{\sigma_0 BL}{(B+Z)(L+Z)}$$

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Differential Pressures

- It is important to note that the above methods allow the calculation of vertical stress at the centre (maximum) of a footing and at the edge (minimum). This allows for the determination of two important identities:
- An estimate of the *average* pressure beneath the foundation
- An estimate of the differential stress, hence differential settlement, beneath a foundation.
- Note that the approximate method does not allow the differential settlement to be estimated.
- Also note that all of the above solutions assume rigid foundations.

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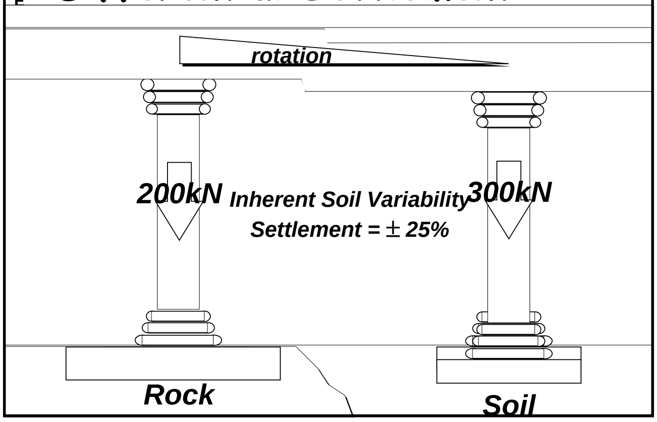
Differential Settlement



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Differential Settlement

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