Basic Equation for Fluid Flow in Soil

Chapter 5 - B

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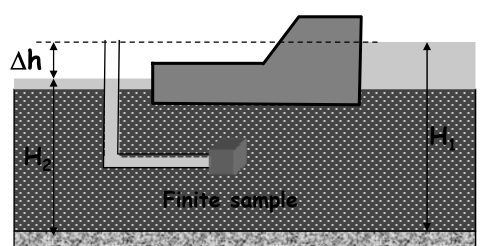
Flow Problems in Geotechnical Engineering

- ***e* = Void ratio**
- ***S_r* = Saturation**

- e & S_r = constant — ***Steady flow***
- e varies & S_r constant — ***Non-steady flow***
- e constant & S_r varies — ***Non-steady flow***
- e & S_r = varies — ***Non-steady flow***

- The derivation of a mathematical model needed for solving these problems is made by Finite element Analysis



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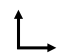
Flow can be classified

One Dimensional Flow

- Fluids parameters such as:
  - Pressure,
  - Velocity ,
  - Temperature, etc.
- Are constant in any cross section perpendicular to the direction of flow.

Two Dimensional Flow

- **Fluids parameters are the same in parallel planes.**



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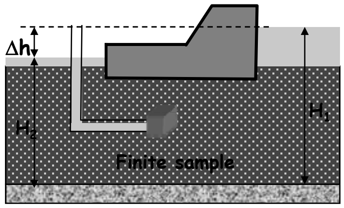
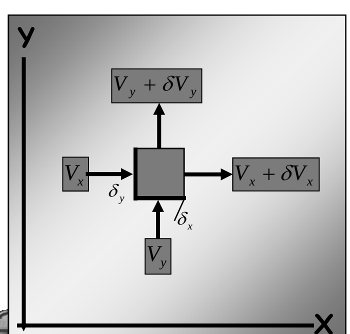
Two-Dimensional Flow, Laplace





- Two-dimensional steady flow of the incompressible pore fluid is governed by Laplace's equation which indicates simply that any imbalance in flows into and out of an element in the x direction must be compensated by a corresponding opposite imbalance in the y direction.

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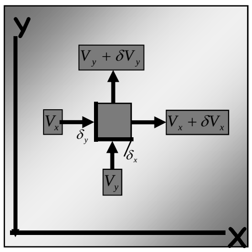
Two-Dimensional Flow, Laplace



- For a rectangular element with dimensions d_x, d_y and unit thickness, in the x direction the velocity of flow into the element is

$$v_x = -k\frac{\partial h}{\partial x}$$

- the negative sig uired because flow occurs down the hydraulic gradient.
- The velocity of flow out of the element is

$$v_x + \delta v_x = -k\left(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2}\delta x\right)$$

Two-Dimensional Flow, Laplace

- Similar expressions can be written for the y direction.
- Balance of flow requires that

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

- Laplace's equation can be solved graphically, analytically, numerically, or analogically.

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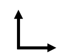
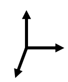


Three Dimensional Flow

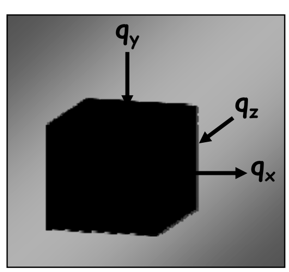
♦ **the fluid parameters vary in the three coordinates directions.**



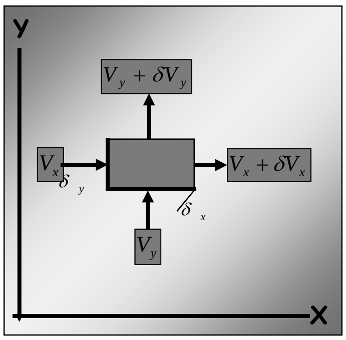
Three-Dimensional Flow, Laplace

- Laplace's equation becomes

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$



Anisotropic soil



- For a soil with permeability k_x and k_y in the x and y directions respectively, Laplace's equation for two-dimensional seepage becomes

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$$

- This can be solved by applying a scale factor to the x dimensions so that transformed coordinates x_t are used

$$x_t = x\sqrt{\frac{k_y}{k_x}}$$

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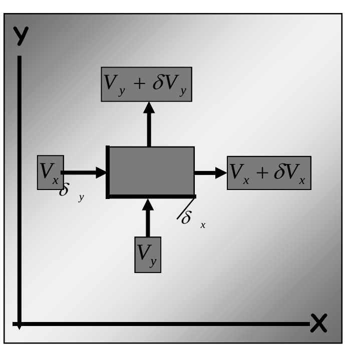
Anisotropic soil



- **It may be preferable in some cases to transform the y coordinates using:**

$$y_t = y\sqrt{\frac{k_x}{k_y}}$$

- The equivalent permeability remains unchanged.
- For many natural sedimentary soils seasonal variations in the depositional regime have resulted in horizontal macroscopic permabilities significantly greater than vertical permeabilities.
- Transformation of coordinates lends itself to analysis of seepage in such situations.

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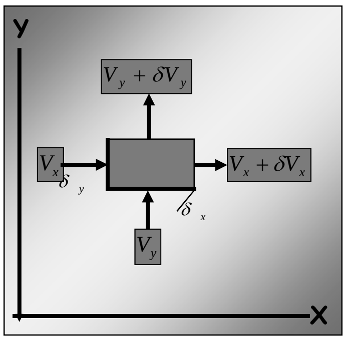
Anisotropic soil

- In the transformed coordinates the equation regains its simple form

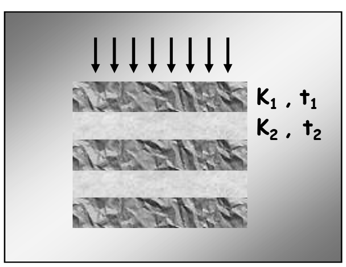


$$\frac{\partial^2 h}{\partial {x_t}^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$$

- **and flownet generation can proceed as usual.**
- **Calculations of flow are made using an equivalent permeability**

$$k_t = \sqrt{k_x k_y}$$

Stratified Soil Vertical Flow



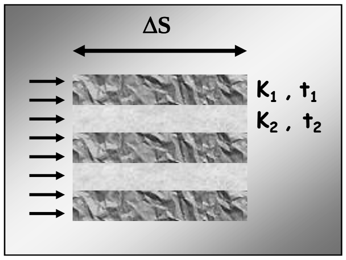
- The flow rate q through area A of each layer is the same. Hence the head drop across a series of layers is

$$\Delta h = \frac{qt_1}{Ak_1} + \frac{qt_2}{Ak_2} + \frac{qt_3}{Ak_3} + \ldots \frac{qt_n}{Ak_n}$$

- **The average coefficient of permeability is**

$$K_v = \frac{t_1 + t_2 + t_3 + \ldots.. + t_n}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + \ldots.. + \frac{t_n}{k_n}}$$

Stratified Soil, Horizontal Flow



- The head drop Δh over the same flow path length Δs will be the same for each layer So $i_1 = i_2 = i_3$ etc.
- The flow rate through a layered block of soil of breadth B is therefore

$$q = Bt_1k_1i_1 + Bt_2k_2i_2 + Bt_3k_3i_3 + \ldots + Bt_nk_ni_n$$

- **The average coefficient of permeability is**

$$K_v = \frac{t_1k_1 + t_2k_2 + t_3k_3 + \ldots t_nk_n}{t_1 + t_2 + t_3 + \ldots + t_n}$$

Solving the Laplace Equation

- Analytical, closed form or series solutions of the PDE (Partial Deferential Equation).
  - quite mathematical, and not very general.
- Numerical solution methods
  - Typically, the finite element method or the finite difference method.
  - very powerful and easy to apply
  - can deal with heterogeneity, anisotropy, 2D, 3D
- Graphical Techniques - Flow-net Methods
  - commonly used in engineering practice to solve 2D flow problems.

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TWO DIMENSIONAL FLOW
FLOW NETS

Two dimensional flow: fluid parameters are the same in parallel planes.