CIVIL ENGINEERING DEPARTEMNT - KFUPM

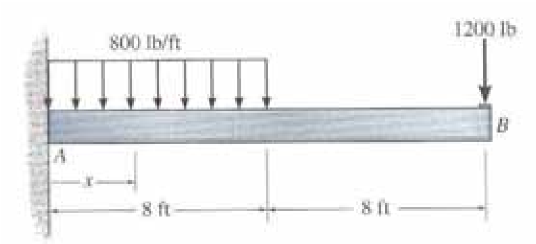
CE 202 Statics and Strength of Materials

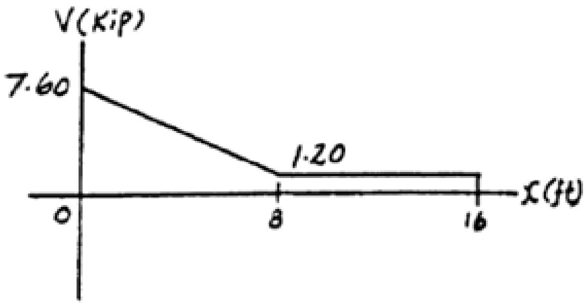
First Semester: ’101
Instructor: Dr. Salah Al-Dulaijan

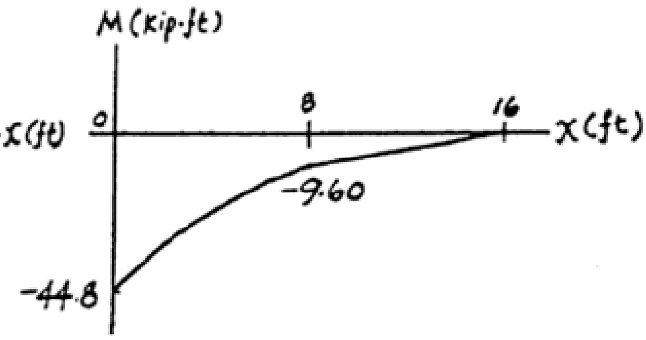
HOME WORK ASSIGNMENT NO. 10.

- **Subject material covered:** Flexural Formula, shear formula, and combined loadings
- DUE DATE: **Wednesday, January 19th, 2011.**

1. If the beam has a square cross-section of 9 in. on each side, determine the absolute maximum bending stress in the beam.



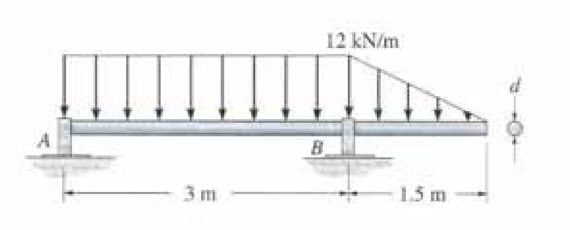


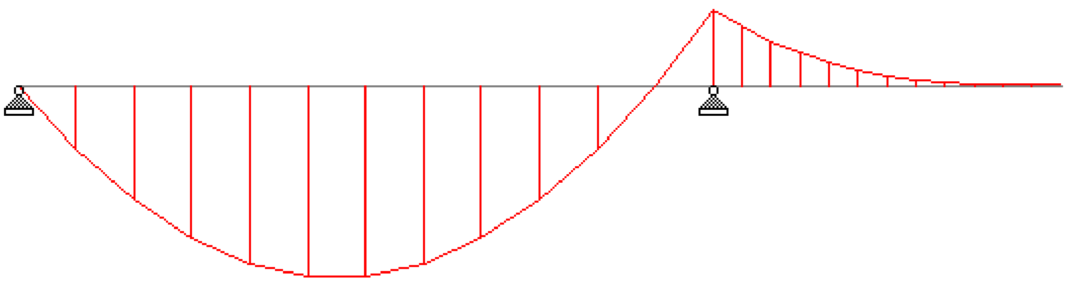


Absolute Maximum bending Stress: The maximum moment is $M_{max} = 44.8$ kip.ft as indicated on the bending moment diagram. Applying the flexural formula

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{44.8 \times 12 \times 4.5}{1/12 \times 9 \times 9^3} = 4.42\ ksi$$

2. The rod is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



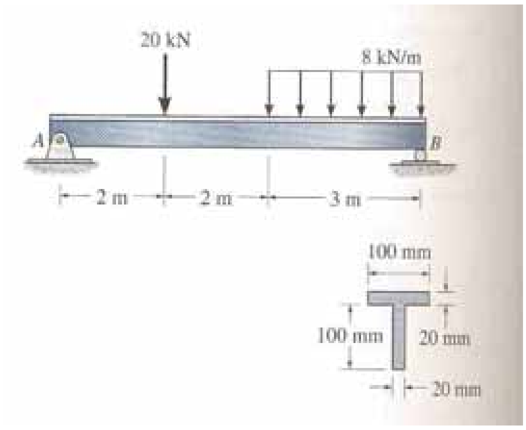


Mmax = 8.37 KNm @1.38m

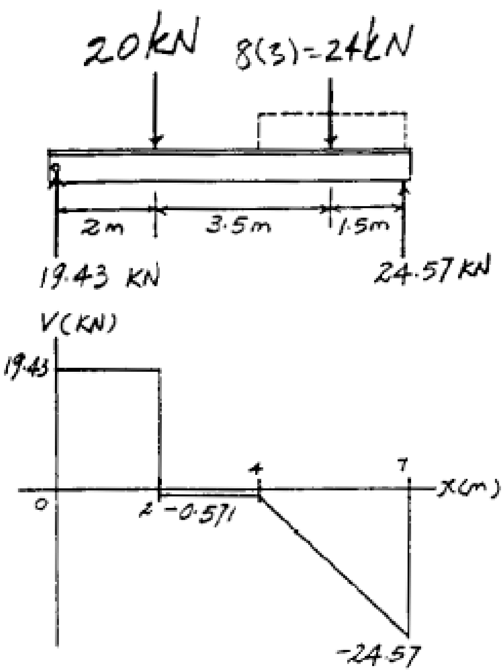
$$\sigma_{allow} = \frac{M_{max}c}{I} \quad ; \quad c = r_{min} = \frac{\sigma_{allow}\pi r^4}{4M_{max}} \quad , \quad or \quad r_{min} = \left(\frac{4M_{max}}{\pi\sigma_{allow}}\right)^{\frac{1}{3}}$$

$$d_{min} = 2r_{min} = \quad 2\left(\frac{4 \times 8.37 \times 10^3}{\pi \times 180 \times 10^6}\right)^{\frac{1}{3}} = 2 \times 0.03898\, m \ \approx 78\, mm$$

3. The T-beam is subjected to the loading shown. Determine the maximum transverse shear stress in the beam at the critical section.



Internal Shear Force: As shown on Shear diagram, $V_{max} = 24.57$ kN.



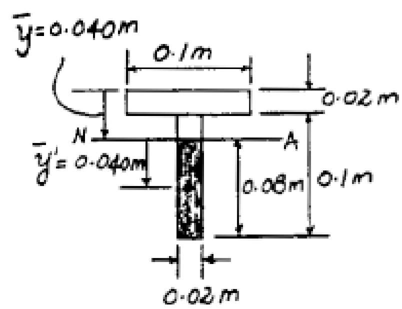
Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.01(0.1)(0.02) + 0.07\,(0.1)(0.02)}{0.1(0.02) + 0.1(0.02)} = 0.0400 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.1)\left(0.02^3\right) + 0.1(0.02)(0.0400 - 0.01)^2 + \frac{1}{12}(0.02)\left(0.1^3\right) + (0.02)(0.1)(0.07 - 0.0400)^2 = 5.3333\left(10^{-6}\right) \text{ m}^4$$

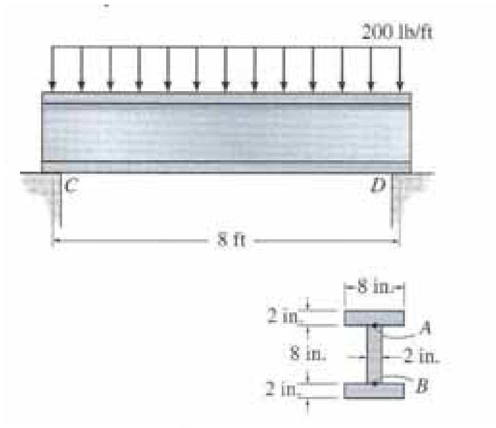
$$Q_{max} = \bar{y}'A' = 0.04(0.02)(0.08) = 64.0\left(10^{-6}\right) \text{ m}^3$$

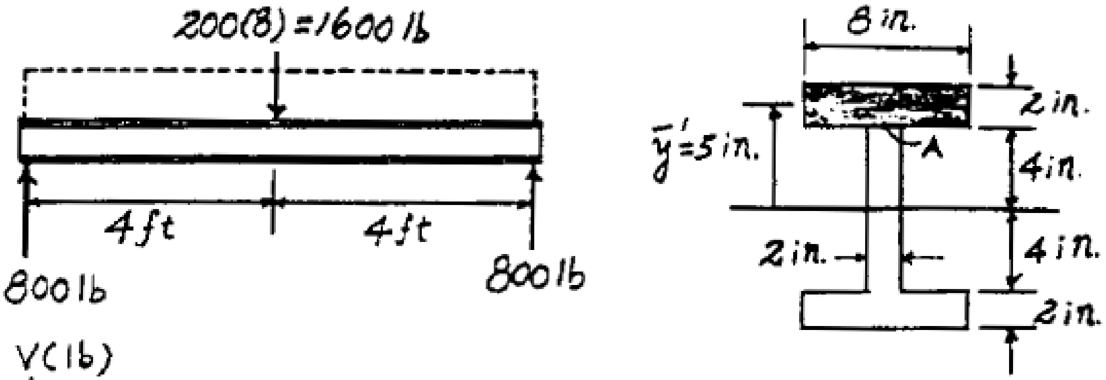
Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

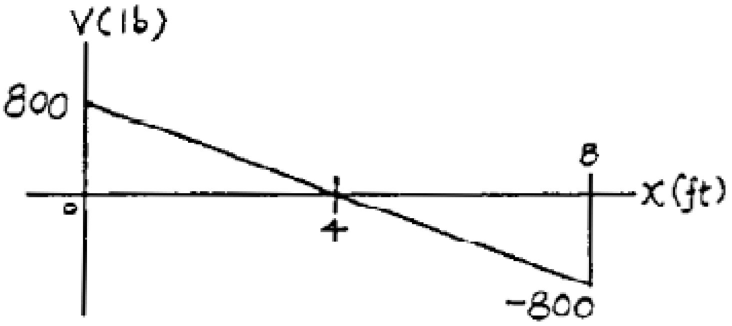


$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{24.57(10^3)\,64.0(10^{-6})}{5.3333(10^{-6})\,(0.02)} = 14.7 \text{ MPa}$$

4. The beam is made from three plastic pieces glued together at the seams A and B. If it is subjected to the loading shown, determine the shear stress developed in the glued joints at the critical section. The supports at C and D exert only vertical reactions on the beam.







Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{max} = 800$ lb.

Section Properties:

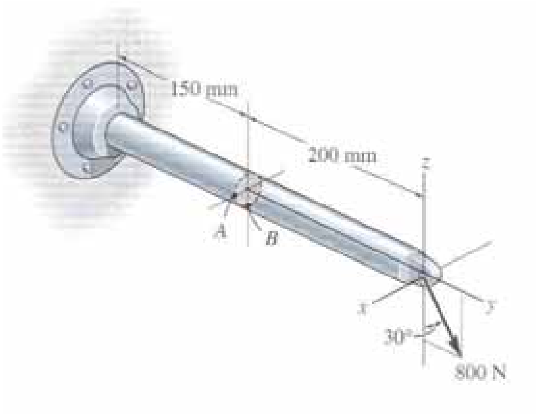
$$I_{NA} = \frac{1}{12}(8)\left(12^3\right) - \frac{1}{12}(6)\left(8^3\right) = 896 \text{ in}^4$$

$$Q_A = \bar{y}'A' = 5(8)(2) = 80.0 \text{ in}^3$$

Shear Stress: Applying the shear formula

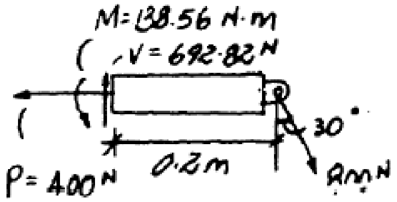
$$\tau_A = \frac{VQ_A}{It} = \frac{800(80.0)}{896(2)} = 35.7 \text{ psi}$$

5. The bar has a diameter of 40 mm. If it is subjected to a force of 800 N as shown, determine the stress components that act at points A and B.



$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\ (10^{-6})\ \text{m}^4$$

$$A = \pi r^2 = \pi(0.02^2) = 1.256637\ (10^{-3})\ \text{m}^2$$

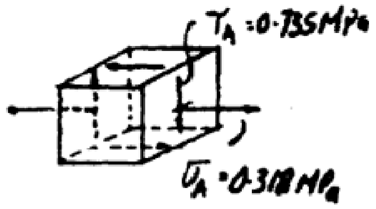


$$Q_A = \bar{y}' A' = \left(\frac{4\,(0.02)}{3\pi}\right)\left(\frac{\pi\,(0.02)^2}{2}\right) = 5.3333\ (10^{-6})\ \text{m}^3$$

$$\sigma_A = \frac{P}{A} + \frac{Mz}{I}$$

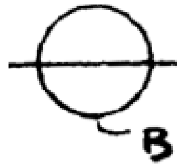
$$= \frac{400}{1.256637\ (10^{-3})} + 0 = 0.318\ \text{MPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{692.82\ (5.3333)\ (10^{-6})}{0.1256637\ (10^{-6})(0.04)} = 0.735\ \text{MPa}$$



$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\ (10^{-6})\ \text{m}^4$$

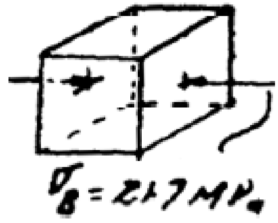
$$A = \pi r^2 = \pi(0.02^2) = 1.256637\ (10^{-3})\ \text{m}^2$$



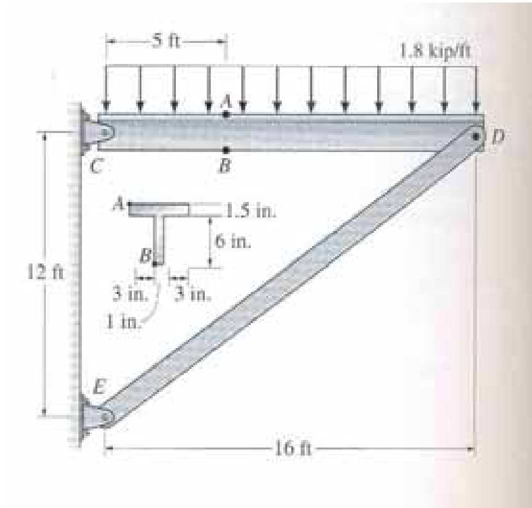
$$Q_B = 0 \quad ; \quad \tau_B = 0$$

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{1.256637\ (10^{-3})} - \frac{138.56\ (0.02)}{0.1256637\ (10^{-6})}$$

$$= -\ 21.7\ \text{MPa}$$



6. The frame supports a centrally applied distributed load of 1.8 kip/ft. Determine state of stress developed at points A and B on member CD. The pins at C and D are at the same location as the neutral axis for the cross-section.



Part	A_i	y_i	$A_i\,y_i$	$A_i(y_i-\underline{y}_i)^2$	I_{xi}
1	10.5	0.75	7.875	19.5247934	1.96875
2	6	4.5	27	34.1683884	18
Σ	16.5 in^2		34.875	53.6931818	19.96875

$\underline{y}$= 2.113636in I_x = 73.66 in^4

Neutral axis = 2.11 in from the top (A)

So y_A=2.11 in and y_B = (6+1.5-2.11) = 5.39 in

The stresses at any section of member CD consist of tensile stress (σ^a) from the truss action and bending stresses (σ^b) from the bending action, both resulting from the distributed load on this member.

Points A and B are both at points where $Q = 0,\ so\ \tau_A = \tau_B = VQ/It = 0$

Bending Stresses, σ^b:

$$R_C = R_C = 1.8 \times 16 \times 0.5 = 14.4\ Kips$$

$$M_{A-B} = 14.4 \times 5 - 1.8 \times 5 \times 2.5 = 49.5\ Kip - ft$$

Top fibres of member CD are in compression $(so, \sigma_A^b = -ve)$, while the bottom fibres of are in tension $(so, \sigma_A^b = -ve)$.

$$\sigma_A^b = -\frac{My_A}{I} = -\frac{49.5 \times 12 \times 2.11}{73.66} = -17.0152\ Ksi,$$

$$\sigma_B^b = +\frac{My_B}{I} = \frac{49.5 \times 12 \times 5.39}{73.66} = +43.4654\ Ksi$$

Axial Stresses, σ^a:

Let <CDE = ß, then Sin ß = 0.6 and Cos ß =0.8

Equilibrium of joint D gives

$$F_{DE} = \frac{R_D}{sin ß} = \frac{14.4}{0.6} = 24\ Kips\ , \qquad and$$

$$F_{CD} = F_{DE} cos ß = 24 \times 0.6 = 19.2\ Kips.$$

$$\sigma_A^a = \sigma_B^a = \sigma^a \ = \frac{F_{CD}}{A_{CD}} = \frac{19.2\ Kips}{16.5\ in^2} = +1.1636\ Ksi\ (tension)$$

$$\sigma_A = \sigma_A^b + \sigma_A^a = -17.0152 + 1.1636 = -15.8516\ Ksi \approx -15.85\ Ksi$$

$$\sigma_B = \sigma_B^b + \sigma_B^a = 43.4654 + 1.1636 = +44.629\ Ksi \approx +44.63\ Ksi$$

Therefore The states of stress

$$at\ A, (\sigma_A, \tau_A) \ = \ (-15.85\ Ksi, 0.00\ Ksi), \qquad and$$

$$at\ B, (\sigma_B, \tau_B) \ = \ (+44.63\ Ksi, 0.00\ Ksi)$$

Sketch each one on an element