بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

Second Semester 1432-33 / 2011-12 (112)

CE 203 STRUCTURAL MECHANICS I

Major Exam 2

KEY SOLUTION

Problem	Prepared by
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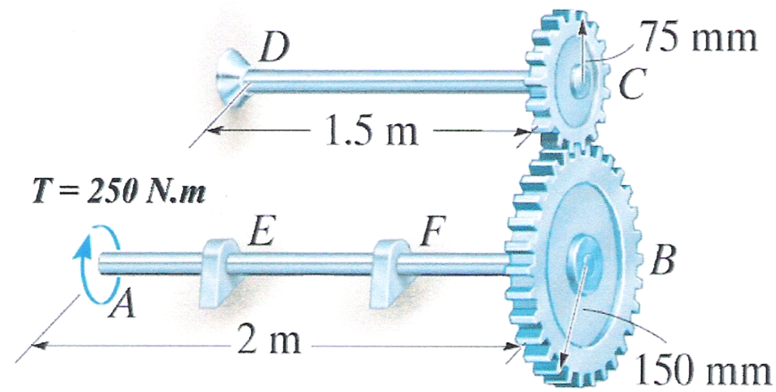
Remarks	

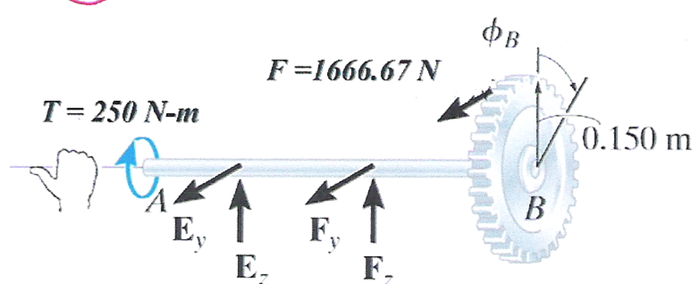
Problem # 1 (__/20)

In the assembly shown below, determine the maximum shear stress in shaft *AB* and shaft *CD*. Also, determine the angle of twist of *gear B* and the angle of twist of *end A*. Note that shaft *AB* has a diameter of **30** mm and shaft *CD* has a diameter of **25** mm. *E and F are* smooth bearings. ***G*** = ***75*** **GPa.**

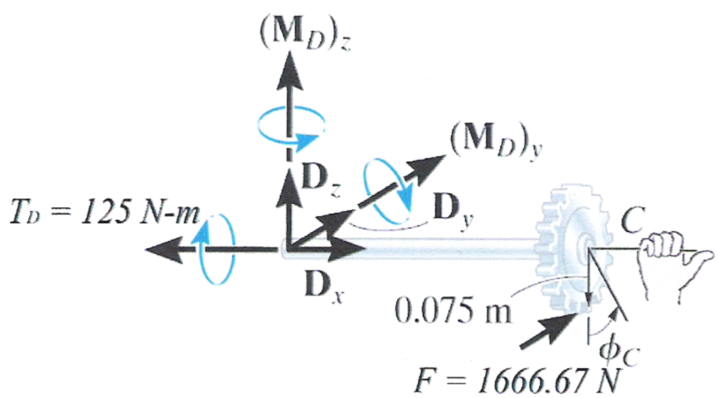


Solution:

From the *FBD* of shaft *AB*, the tangential force between the gears is $F = (250 \text{ N-m})/(0.15 \text{ m}) = 1,666.67$ N. (2)



Summing moment about the x-axis of shaft DC, the force $F = 1{,}666.67$ N creates the torque react at the fixed end D; $T_D = (1{,}666.67\text{ N})(0.075\text{ m})$. 2



Maximum shear stress due to an applied torque; $\tau_{max} = Tc/J$

Shaft AB: $T_{AB} = 250$ N-m, $C = (30/2)$ mm $= 0.0125$ m,

$$J_{AB} = (\pi/2)\times c^4 = (\pi/2)\times(0.015)^4 = 7.952\times10^{-8}\text{ m}^4$$

4

$$(\tau_{max})_{AB} = (250\text{ N-m})(0.015\text{ m})/(7.952\times10^{-8}\text{ m}^4) = 47.158\text{ MPa}$$

Shaft DC: $T_{DC} = 125$ N-m, $C = (25/2)$ mm $= 0.0125$ m,

$$J_{DC} = (\pi/2)\times c^4 = (\pi/2)\times(0.0125)^4 = 3.835\times10^{-8}\text{ m}^4$$

4

$$(\tau_{max})_{DC} = (125\text{ N-m})(0.0125\text{ m})/(3.835\times10^{-8}\text{ m}^4) = 40.743\text{ MPa}$$

Angle of Twist: $\Phi = TL/JG$

We first determine the angle of twist at gear C due to $T_{DC} = 125$ N-m, in shaft DC;

$$\Phi_C = T_{DC}L_{DC}/J_{DC}G$$

$$= [(125\text{ N-m})(1.5\text{ m})]/[(3.835x10^{-8}\text{ m}^4)(75x10^9\text{ N/m}^2)]$$

$$= 0.06519\text{ Rad}$$

2

The rotation Φ_C of gear C causes gear B to rotate Φ_B;

$$\Phi_B(0.15\text{ m}) = \Phi_C(0.075\text{ m})$$

$$\Phi_B = \Phi_C(0.075\text{ m})/(0.15\text{ m})$$

$$= (0.06519\text{ Rad})\ (0.075\text{ m})/(0.15\text{ m})$$

$$= +0.0326\text{ Rad}$$

2

The angle of twist of end A with respect to end B caused by the 250 N-m torque;

$$\Phi_{A/B} = T_{AB}L_{AB}/J_{AB}G$$

$$= [(250\text{ N-m})(2.0\text{ m})]/[(7.952x10^{-8}\text{ m}^4)(75x10^9\text{ N/m}^2)]$$

$$= +0.08384\text{ Rad}$$

2

The rotation of end A is therefore the sum of Φ_B and $\Phi_{A/B}$;

$$\Phi_A = \Phi_B + \Phi_{A/B} = +0.0326\text{ Rad} + 0.08384\text{ Rad}$$

$$= +0.1164\text{ Rad}$$

2

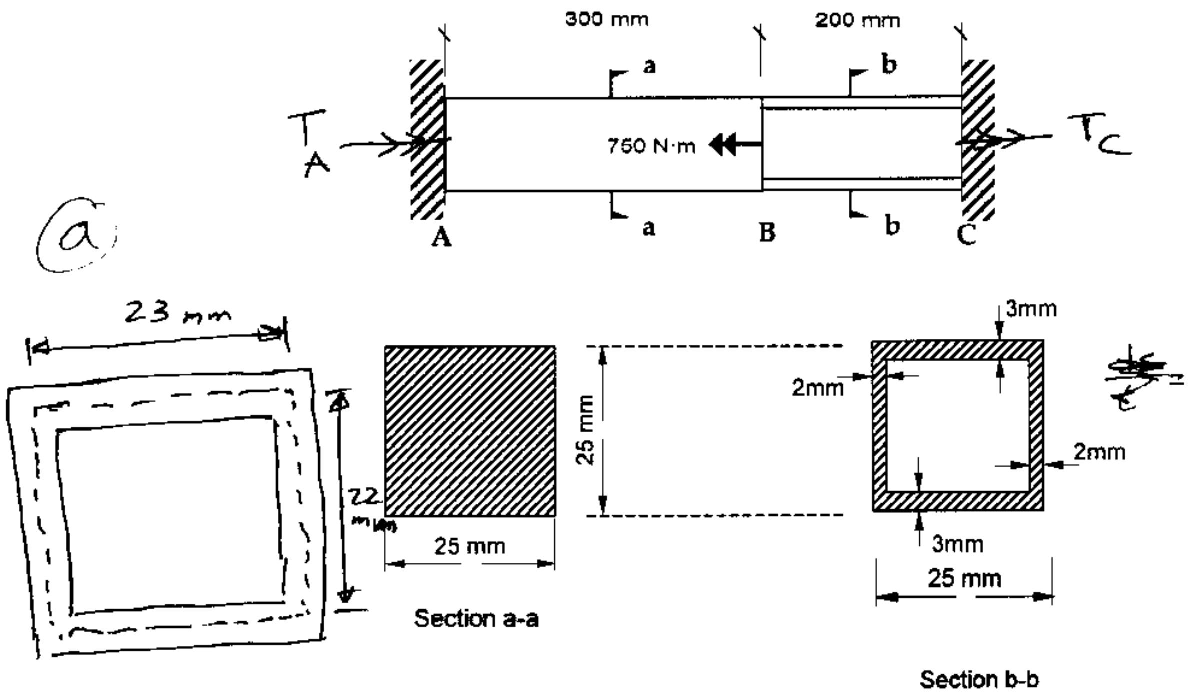
Problem # 2

The shaft is made from two segments: AB is a solid section, and BC is a thin tube.

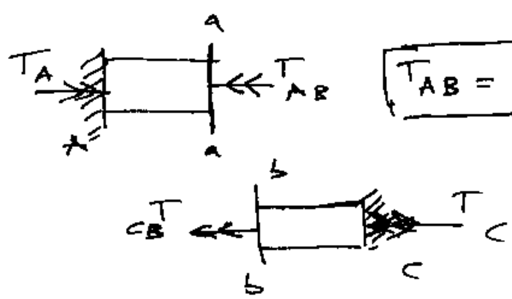
a) Determine the maximum shear stress in the whole shaft and indicate its location.
b) Determine the angle of twist of B.

$G_{steel} = 75\ GPa$

(a)



This problem is statically indeterminate.



$\boxed{T_{AB} = -T_A}$

$\boxed{T_{CB} = T_C}$

From Equilibrium

$$\boxed{T_A + T_C - 750 = 0} \quad (1)$$

$A_m = 22 \times 23 = 506\ mm^2$
$= 5.06 \times 10^{-4}\ m^2$

From compatibility

$$\phi_{A/B} + \phi_{B/C} = 0$$

$$\frac{7.1\,(-T_A)\,(0.3)}{(0.025)^4 (75 \times 10^9)} + \frac{T_C\,(0.2)}{4\,(5.06 \times 10^{-4})^2 (75 \times 10^9)} \left[2\left(\frac{22}{2}\right) + 2\left(\frac{23}{2}\right)\right] = 0$$

$$-7.2704 \times 10^{-5}\, T_A + 9.7199 \times 10^{-5}\, T_C = 0 \quad (2)$$

solving (1) & (2)

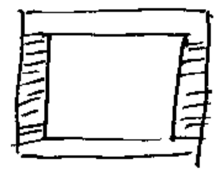
$$\boxed{\begin{aligned} T_C &= 320.94\ N.m \\ T_A &= 429.06\ N.m \end{aligned}}$$

$$\tau_{AB(max)} = \frac{4.81T}{a^3} = \frac{4.81(429.06)}{(0.025)^3} = \boxed{132.08\ MPa}$$

$$\tau_{BC_{(max)}} = \frac{T}{2tA_m} = \frac{320.94}{2(0.002)(5.06\times10^{-4})} = \boxed{157.57\ MPa}$$

Max. τ stress = 157.57 MPa in segment BC

Located at
the sides where
$t = 2mm$.

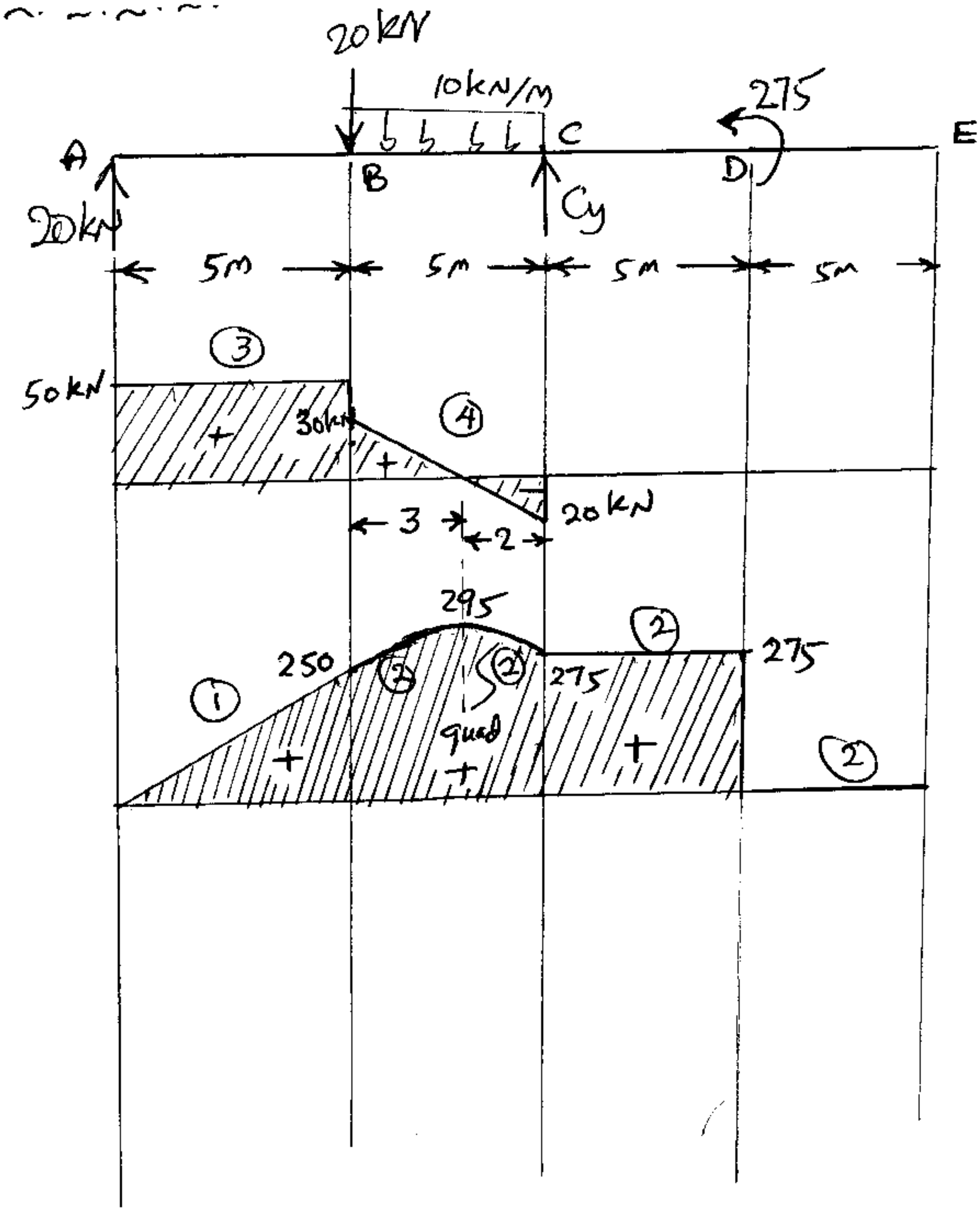


(b) $\phi_B = \phi_{A/B} = \phi_{B/C}$

$$\phi_{A/B} = \frac{7.1(429.06)(0.3)}{(.025)^4(75\times10^9)} = 0.0312\ rad$$

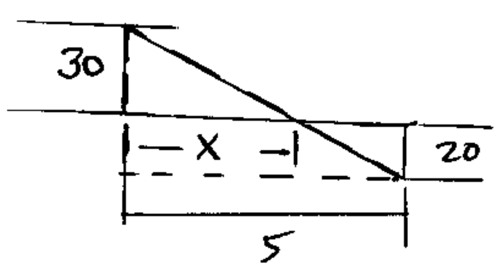
$$\phi_{B/C} = \frac{320.94(0.2)}{4(5.06\times10^{-4})^2(75\times10^9)}[37.33] = 0.0312\ rad$$

Problem # 3



③ $\Sigma M @ A = 0 + \circlearrowleft \quad -20(5) - 10(5)(7.5) + C_y(10) + 275 \Rightarrow$

$C_y = 20\ kN \uparrow$

① 

$$\frac{x}{30} = \frac{5}{50}$$

$\therefore\ x = 3\ m.$

O marks distribution

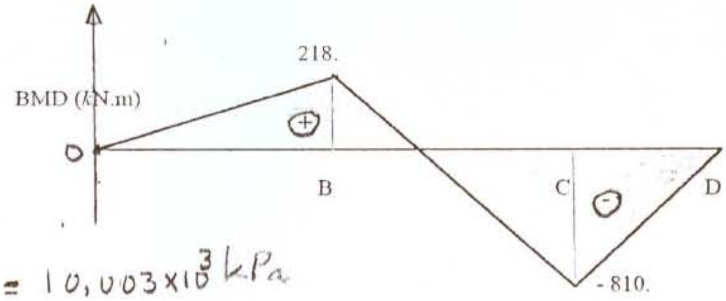
Problem # 4 **Key Solution**

The beam ABCD shown has a cross-section that is composed of two identical vertical boards and one horizontal board (all with common thickness t = 200 mm) and has the given bending moment diagram.

a) Compute the maximum tensile stress σ_t and *clearly* specify its location in the beam.

b) Compute the resultant force on the left vertical board at the location of maximum positive moment.

Given: $\bar{y} = 325$ mm; and $I_{CA} = 7.083 \times 10^{-3}\ \text{m}^4$



a)

$$\sigma(y) = -\frac{My}{I_{CA}} \quad (1\%)$$

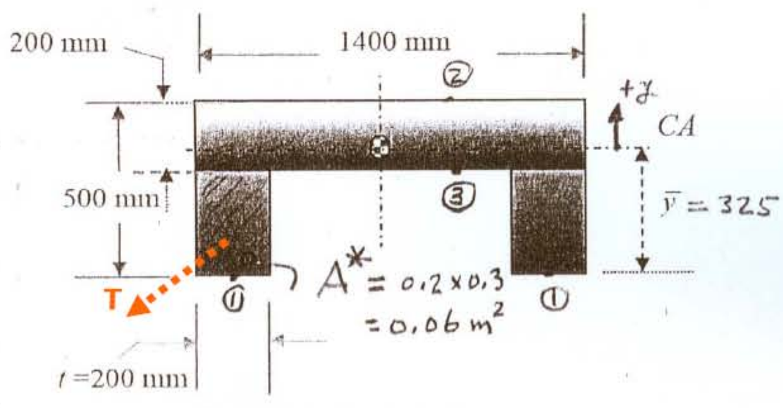
$$\sigma_t^{①} = -\frac{218.(0.325)}{7.083\times10^{-3}} = 10,003\times10^{3}\ kPa$$

(4%) $\therefore\ \sigma_t^{①} \simeq 10.\ MPa$

$$\sigma_t^{②} = -\frac{-810(0.175)}{7.083\times10^{-3}}$$

(4%) $\simeq 20.0\ MPa$

$$\therefore\ \sigma_t^{(max)} = 20.\ MPa$$



Cross-section

Caused by negative moment in cross section at C. This tensile stress is acting on (1%) all material points at $y = +0.175$ m from C.A.

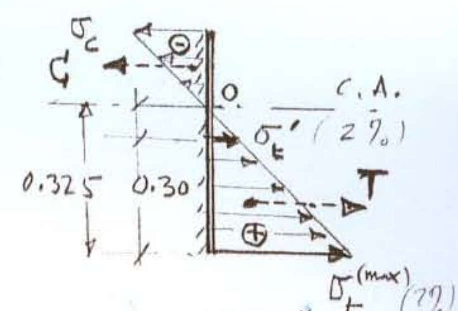
b) since $N = \int \sigma\, dA$, then on area A^* with

$M(x) = 218\ kN.m$

$$\sigma_t' = \sigma_t^{③} = \sigma(y=-0.025)$$

$$(4\%) \quad = -\frac{218(-0.025)}{7.083\times10^{-3}}$$

$$\simeq 0.769\ MPa$$



Resultant force on left board with area $A^* = T^*$

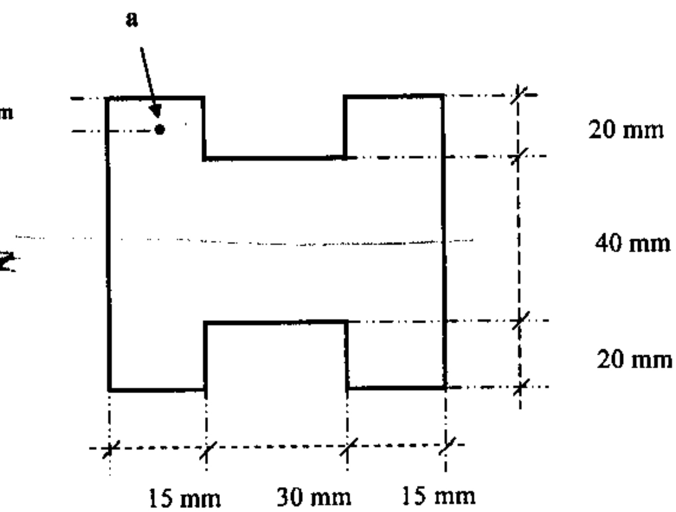
(6%) $\therefore\ T = \frac{1}{2}[\sigma_t^{①} + \sigma_t^{③}] * A^* = \frac{1}{2}[10. + 0.769] * 0.06$

$= 0.3231\ MN \Rightarrow T^* \cong 323.1\ kN$

Problem # 5

The beam with the shown cross-section is subjected to a vertical shear force of 20 kN.

a) Determine the moment of inertia about the neutral axis.
b) Determine the shear stress at point a.
c) Determine the maximum shear stress and indicate where it acts.



(a) divide into 3 rectangles

$$I_z = 2\left(\frac{1}{12} \times 15 \times 80^3\right) + \frac{1}{12} \times 30 \times 40^3 \quad (3)$$

$$I_z = 144 \times 10^4 \ mm^4 \quad (2)$$

(b) $\tau_a = \dfrac{VQ}{It}$ (2)

$Q = 15 \times 10 \times 35$ (1)

$= 5250 \ mm^3$

$\therefore \tau_a = \dfrac{20 \times 10^3 \times 15 \times 10 \times 35}{144 \times 10^4 \times 15}$ (1)

10 mm

15 mm

$40 - 5 = 35$ mm

z

$\tau_a = 4.9$ MPa (1)

(c) realizing that (2)

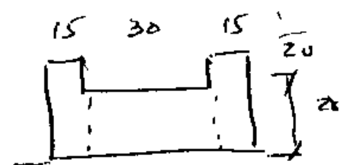
τ varies with $\frac{Q}{t}$

we have to check two points (locations)

(1) max Q → at N/A

(2) min t → at 20 mm above N.A

$\tau_{N/A} = \dfrac{VQ}{It}$ (2)



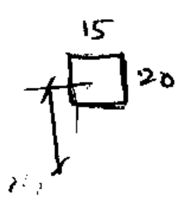
$Q = 2 \times 15 \times \frac{40^2}{2}$ (1)

$+ 30 \times \frac{20^2}{2} = 30{,}000 \ mm^3$

$\therefore \tau_{N/A} = \dfrac{20 \times 10^3 (\quad)}{144 \times 10^4 \times 60} = 6.9$ MPa (1) (1)

$\tau_{20^+ mm} = \dfrac{VQ}{It}$

above N.A.



$Q = 15 \times 20 \times 30$ (1)

$t = 15$ mm

$\therefore \tau_{20^+ mm} = \dfrac{20 \times 10^3 (9000)}{144 \times 10^4 \times 15} = 8.3$ MPa

$\therefore \tau_{max} = 8.3$ MPa acts 20 mm from top of section (2)