•**3–1.** A concrete cylinder having a diameter of 6.00 in. and gauge length of 12 in. is tested in compression. The results of the test are reported in the table as load versus contraction. Draw the stress–strain diagram using scales of 1 in. = 0.5 ksi and 1 in. = $0.2(10^{-3})$ in./in. From the diagram, determine approximately the modulus of elasticity.

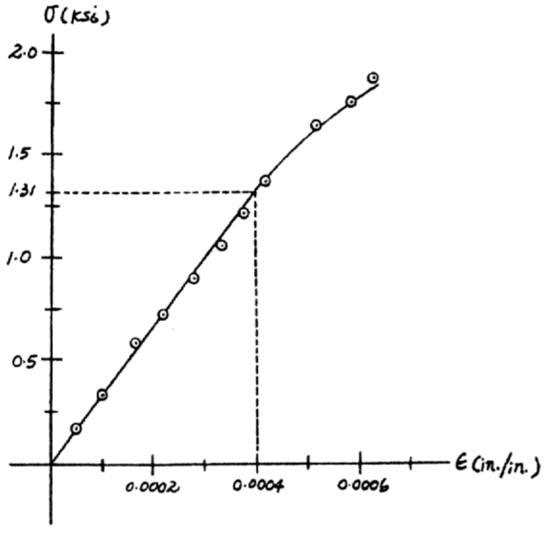
Load (kip)	Contraction (in.)
0	0
5.0	0.0006
9.5	0.0012
16.5	0.0020
20.5	0.0026
25.5	0.0034
30.0	0.0040
34.5	0.0045
38.5	0.0050
46.5	0.0062
50.0	0.0070
53.0	0.0075

Stress and Strain:

$\sigma = \dfrac{P}{A}$(ksi)	$\varepsilon = \dfrac{\delta L}{L}$(in./in.)
0	0
0.177	0.00005
0.336	0.00010
0.584	0.000167
0.725	0.000217
0.902	0.000283
1.061	0.000333
1.220	0.000375
1.362	0.000417
1.645	0.000517
1.768	0.000583
1.874	0.000625

Modulus of Elasticity: From the stress–strain diagram

$$E_{\text{approx}} = \frac{1.31 - 0}{0.0004 - 0} = 3.275\left(10^3\right)\ \text{ksi}$$ **Ans.**



3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

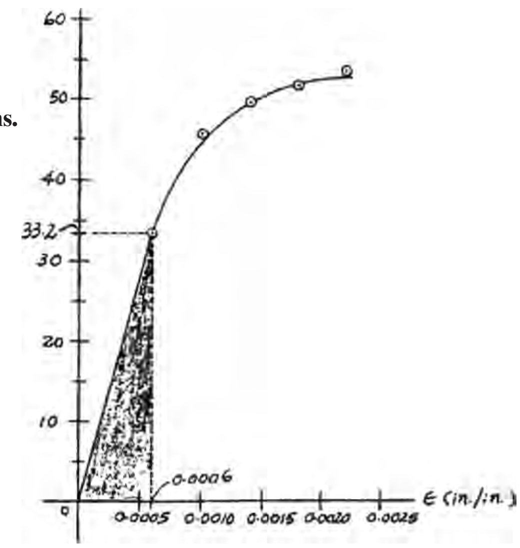
σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Elasticity: From the stress–strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3\left(10^3\right)\ \text{ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram (shown shaded).

$$u_t = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\ \frac{\text{in.}}{\text{in.}}\right) = 9.96\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 53.4$ ksi.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Toughness: The modulus of toughness is equal to the area under the stress–strain diagram (shown shaded).

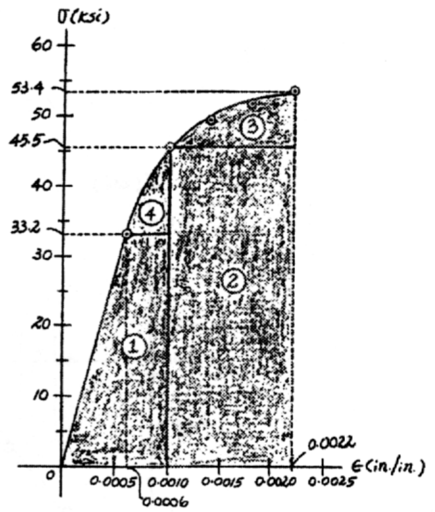
$$(u_t)_{\text{approx}} = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004 + 0.0010)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+ 45.5\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+ \frac{1}{2}(7.90)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+ \frac{1}{2}(12.3)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$= 85.0\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



***3–4.** A tension test was performed on a specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. The data are listed in the table. Plot the stress–strain diagram, and determine approximately the modulus of elasticity, the ultimate stress, and the fracture stress. Use a scale of $20\text{ mm} = 50\text{ MPa}$ and $20\text{ mm} = 0.05\text{ mm/mm}$. Redraw the linear-elastic region, using the same stress scale but a strain scale of $20\text{ mm} = 0.001\text{ mm/mm}$.

Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

Stress and Strain:

$\sigma = \frac{P}{A}(\text{MPa}) \quad \varepsilon = \frac{\delta L}{L}(\text{mm/mm})$

0	0
90.45	0.00035
259.9	0.00120
308.0	0.00204
333.3	0.00330
355.3	0.00498
435.1	0.02032
507.7	0.06096
525.6	0.12700
507.7	0.17780
479.1	0.23876

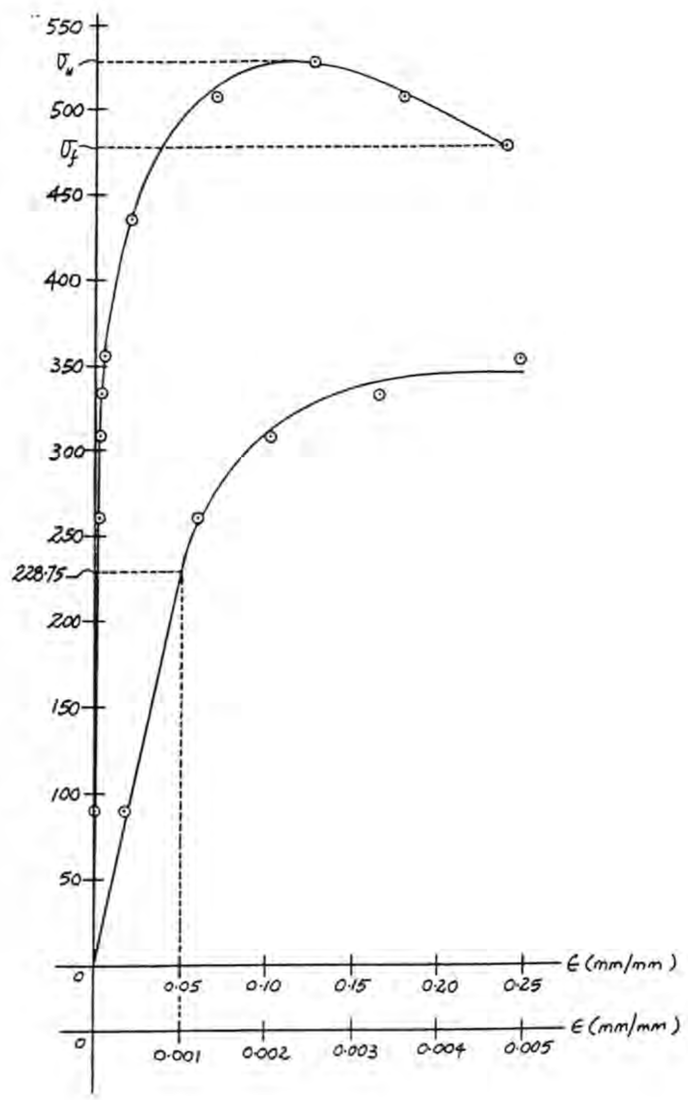
Modulus of Elasticity: From the stress–strain diagram

$$(E)_{\text{approx}} = \frac{228.75(10^6) - 0}{0.001 - 0} = 229\text{ GPa} \qquad \textbf{Ans.}$$

Ultimate and Fracture Stress: From the stress–strain diagram

$$(\sigma_\mu)_{\text{approx}} = 528\text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_f)_{\text{approx}} = 479\text{ MPa} \qquad \textbf{Ans.}$$



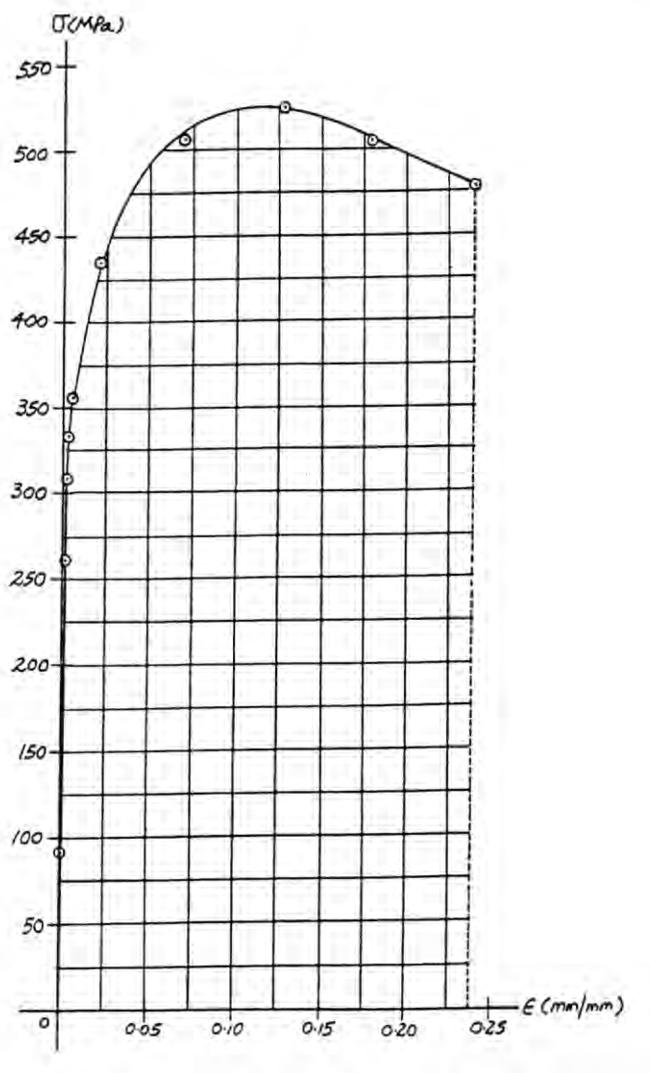
3–5. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and gauge length of 50 mm. Using the data listed in the table, plot the stress–strain diagram, and determine approximately the modulus of toughness. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm.

Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

Stress and Strain:

$$\sigma = \frac{P}{A}(\text{MPa}) \quad \varepsilon = \frac{\delta L}{L}(\text{mm/mm})$$

0	0
90.45	0.00035
259.9	0.00120
308.0	0.00204
333.3	0.00330
355.3	0.00498
435.1	0.02032
507.7	0.06096
525.6	0.12700
507.7	0.17780
479.1	0.23876



Modulus of Toughness: The modulus of toughness is equal to the total area under the stress–strain diagram and can be approximated by counting the number of squares. The total number of squares is 187.

$$(u_t)_{\text{approx}} = 187(25)\left(10^6\right)\left(\frac{\text{N}}{\text{m}^2}\right)\left(0.025\ \frac{\text{m}}{\text{m}}\right) = 117\ \text{MJ/m}^3 \quad \textbf{Ans.}$$

3–6. A specimen is originally 1 ft long, has a diameter of 0.5 in., and is subjected to a force of 500 lb. When the force is increased from 500 lb to 1800 lb, the specimen elongates 0.009 in. Determine the modulus of elasticity for the material if it remains linear elastic.

Normal Stress and Strain: Applying $\sigma = \dfrac{P}{A}$ and $\varepsilon = \dfrac{\delta L}{L}$.

$$\sigma_1 = \frac{0.500}{\frac{\pi}{4}(0.5^2)} = 2.546 \text{ ksi}$$

$$\sigma_2 = \frac{1.80}{\frac{\pi}{4}(0.5^2)} = 9.167 \text{ ksi}$$

$$\Delta\varepsilon = \frac{0.009}{12} = 0.000750 \text{ in./in.}$$

Modulus of Elasticity:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{9.167 - 2.546}{0.000750} = 8.83\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$

3–7. A structural member in a nuclear reactor is made of a zirconium alloy. If an axial load of 4 kip is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 relative to yielding. What is the load on the member if it is 3 ft long and its elongation is 0.02 in.? $E_{zr} = 14(10^3)$ ksi, $\sigma_Y = 57.5$ ksi. The material has elastic behavior.

Allowable Normal Stress:

$$\text{F.S.} = \frac{\sigma_y}{\sigma_{\text{allow}}}$$

$$3 = \frac{57.5}{\sigma_{\text{allow}}}$$

$$\sigma_{\text{allow}} = 19.17 \text{ ksi}$$

$$\sigma_{\text{allow}} = \frac{P}{A}$$

$$19.17 = \frac{4}{A}$$

$$A = 0.2087 \text{ in}^2 = 0.209 \text{ in}^2 \qquad \textbf{Ans.}$$

Stress–Strain Relationship: Applying Hooke's law with

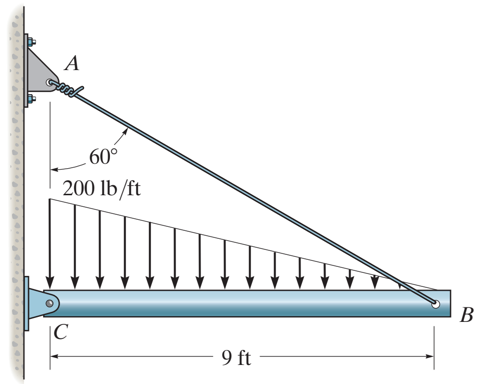
$$\varepsilon = \frac{\delta}{L} = \frac{0.02}{3\,(12)} = 0.000555 \text{ in./in.}$$

$$\sigma = E\varepsilon = 14\left(10^3\right)(0.000555) = 7.778 \text{ ksi}$$

Normal Force: Applying equation $\sigma = \dfrac{P}{A}$.

$$P = \sigma A = 7.778\,(0.2087) = 1.62 \text{ kip} \qquad \textbf{Ans.}$$

***3–8.** The strut is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when the distributed load acts on the strut.



Here, we are only interested in determining the force in wire AB.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}\cos 60°(9) - \frac{1}{2}(200)(9)(3) = 0 \qquad F_{AB} = 600 \text{ lb}$$

The normal stress the wire is

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{600}{\frac{\pi}{4}(0.2^2)} = 19.10(10^3) \text{ psi} = 19.10 \text{ ksi}$$

Since $\sigma_{AB} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in wire.

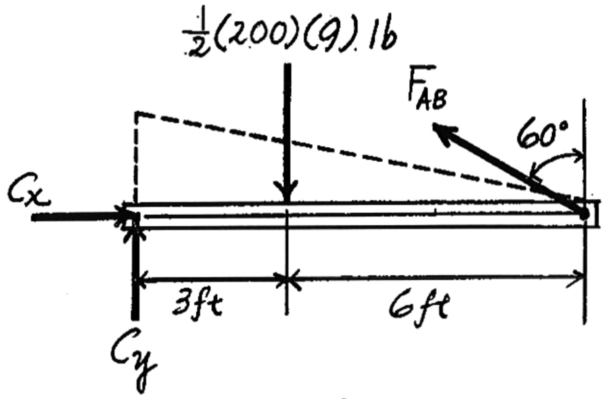
$$\sigma_{AB} = E\epsilon_{AB}; \qquad 19.10 = 29.0(10^3)\epsilon_{AB}$$

$$\epsilon_{AB} = 0.6586(10^{-3}) \text{ in/in}$$

The unstretched length of the wire is $L_{AB} = \dfrac{9(12)}{\sin 60°} = 124.71$ in. Thus, the wire stretches

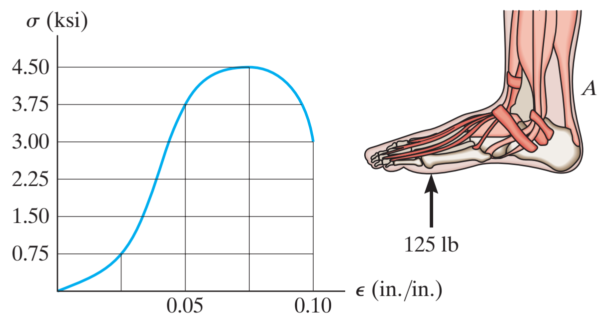
$$\delta_{AB} = \epsilon_{AB} L_{AB} = 0.6586(10^{-3})(124.71)$$

$$= 0.0821 \text{ in.}$$ **Ans.**



(a)

•3–9. The $\sigma-\epsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 6.5 in. and an approximate cross-sectional area of 0.229 in^2, determine its elongation if the foot supports a load of 125 lb, which causes a tension in the tendon of 343.75 lb.

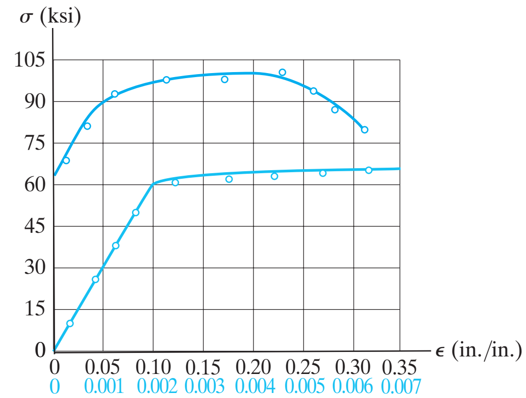


$$\sigma = \frac{P}{A} = \frac{343.75}{0.229} = 1.50 \text{ ksi}$$

From the graph $\varepsilon = 0.035$ in./in.

$$\delta = \varepsilon L = 0.035(6.5) = 0.228 \text{ in.}$$ **Ans.**

3–10. The stress–strain diagram for a metal alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



From the stress–strain diagram, Fig. a,

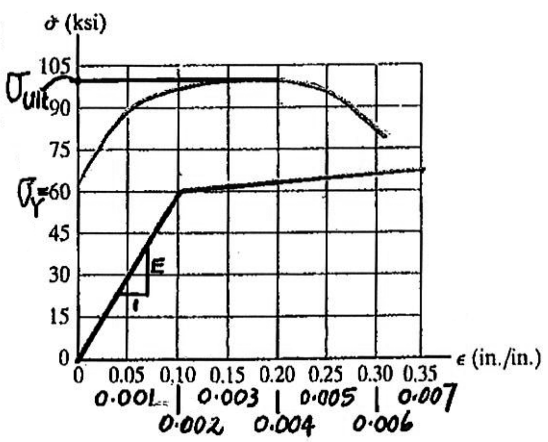
$$\frac{E}{1} = \frac{60 \text{ ksi} - 0}{0.002 - 0}; \qquad E = 30.0(10^3) \text{ ksi}$$ **Ans.**

$$\sigma_y = 60 \text{ ksi} \qquad \sigma_{u/t} = 100 \text{ ksi}$$

Thus,

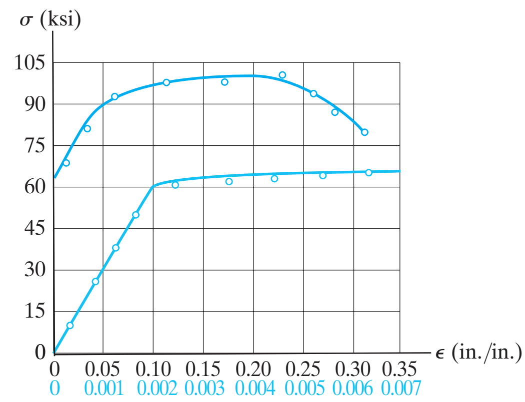
$$P_Y = \sigma_Y A = 60\left[\tfrac{\pi}{4}(0.5^2)\right] = 11.78 \text{ kip} = 11.8 \text{ kip}$$ **Ans.**

$$P_{u/t} = \sigma_{u/t}\, A = 100\left[\tfrac{\pi}{4}(0.5^2)\right] = 19.63 \text{ kip} = 19.6 \text{ kip}$$ **Ans.**



(a)

3–11. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 90 ksi, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.



From the stress–strain diagram Fig. *a*, the modulus of elasticity for the steel alloy is

$$\frac{E}{1} = \frac{60 \text{ ksi} - 0}{0.002 - 0}; \qquad E = 30.0(10^3) \text{ ksi}$$

when the specimen is unloaded, its normal strain recovered along line AB, Fig. *a*, which has a gradient of E. Thus

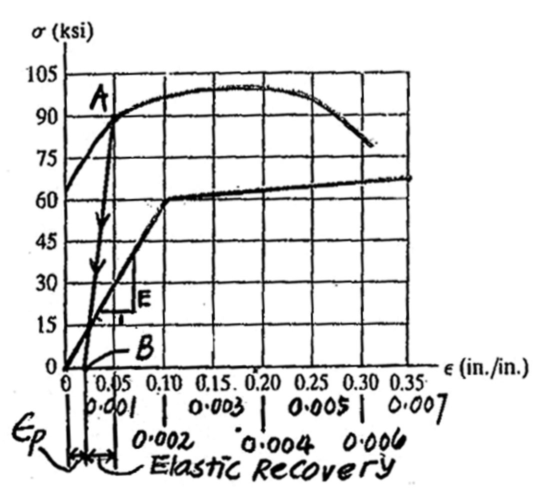
$$\textit{Elastic Recovery} = \frac{90}{E} = \frac{90 \text{ ksi}}{30.0(10^3) \text{ ksi}} = 0.003 \text{ in/in} \qquad \textbf{Ans.}$$

Thus, the permanent set is

$$\epsilon_P = 0.05 - 0.003 = 0.047 \text{ in/in}$$

Then, the increase in gauge length is

$$\Delta L = \epsilon_P L = 0.047(2) = 0.094 \text{ in} \qquad \textbf{Ans.}$$



(a)

***3–12.** The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.

The Modulus of resilience is equal to the area under the stress–strain diagram up to the proportional limit.

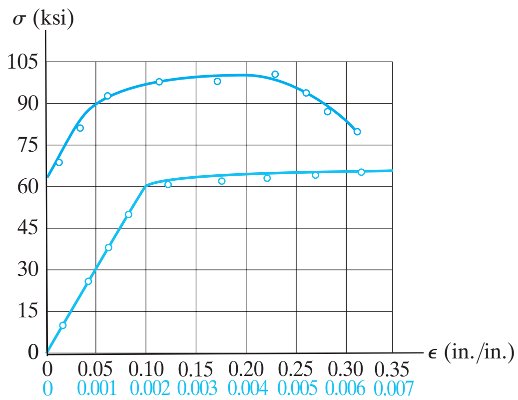
$$\sigma_{PL} = 60 \text{ ksi} \qquad \epsilon_{PL} = 0.002 \text{ in/in.}$$

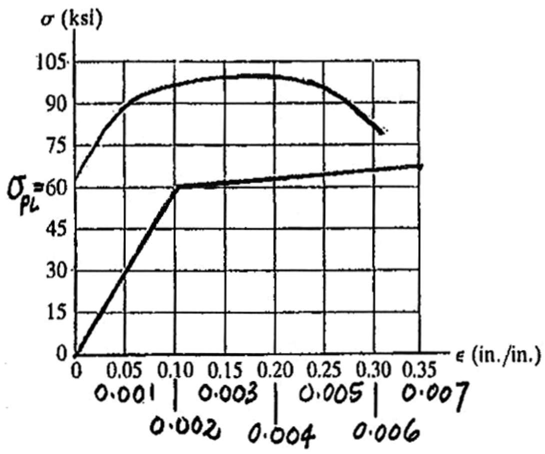
Thus,

$$(u_i)_r = \frac{1}{2}\sigma_{PL}\epsilon_{PL} = \frac{1}{2}\left[60(10^3)\right](0.002) = 60.0\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number is 38. Thus,

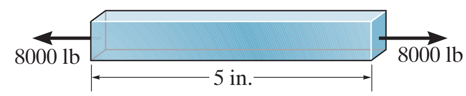
$$\left[(u_i)_t\right]_{\text{approx}} = 38\left[15(10^3)\ \frac{\text{lb}}{\text{in}^2}\right]\left(0.05\ \frac{\text{in}}{\text{in}}\right) = 28.5(10^3)\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$





(a)

•3–13. A bar having a length of 5 in. and cross-sectional area of 0.7 in^2 is subjected to an axial force of 8000 lb. If the bar stretches 0.002 in., determine the modulus of elasticity of the material. The material has linear-elastic behavior.



Normal Stress and Strain:

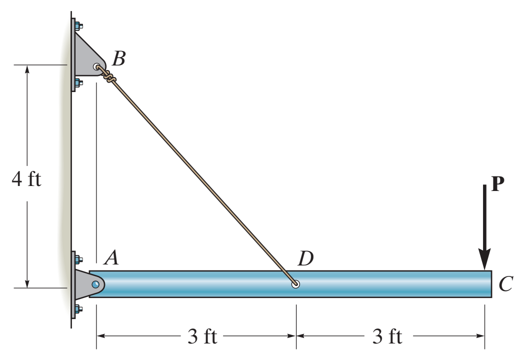
$$\sigma = \frac{P}{A} = \frac{8.00}{0.7} = 11.43 \text{ ksi}$$

$$\varepsilon = \frac{\delta L}{L} = \frac{0.002}{5} = 0.000400 \text{ in./in.}$$

Modulus of Elasticity:

$$E = \frac{\sigma}{\varepsilon} = \frac{11.43}{0.000400} = 28.6(10^3) \text{ ksi}$$ **Ans.**

3–14. The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 0.25 in., determine how much it stretches when a load of $P = 600$ lb acts on the pipe.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - 600(6) = 0 \qquad F_{BD} = 1500 \text{ lb}$$

The normal stress developed in the wire is

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{1500}{\frac{\pi}{4}(0.25^2)} = 30.56(10^3) \text{ psi} = 30.56 \text{ ksi}$$

Since $\sigma_{BD} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in the wire.

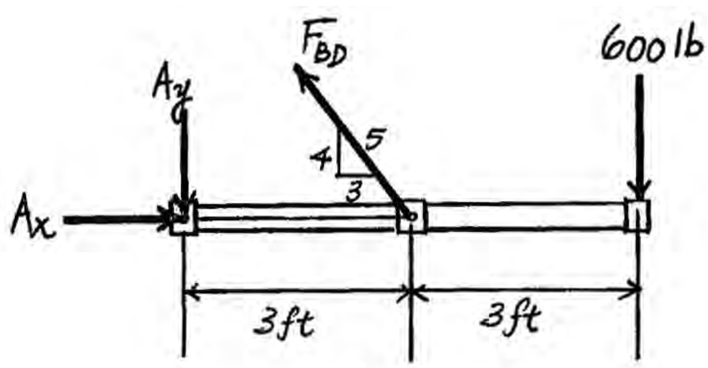
$$\sigma_{BD} = E\epsilon_{BD}; \qquad 30.56 = 29.0(10^3)\epsilon_{BD}$$

$$\epsilon_{BD} = 1.054(10^{-3}) \text{ in./in.}$$

The unstretched length of the wire is $L_{BD} = \sqrt{3^2 + 4^2} = 5\text{ft} = 60$ in. Thus, the wire stretches

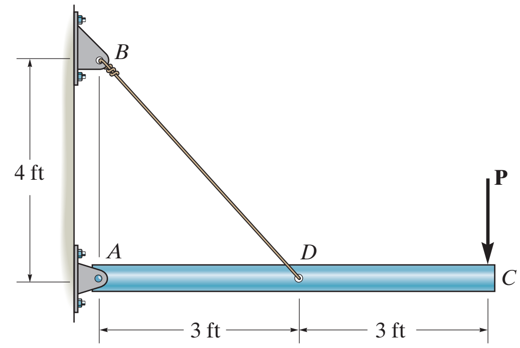
$$\delta_{BD} = \epsilon_{BD} L_{BD} = 1.054(10^{-3})(60)$$

$$= 0.0632 \text{ in}$$ **Ans.**



(a)

3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 0.25 in., determine the load P if the end C is displaced 0.075 in. downward.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. a

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - P(6) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{3^2 + 4^2} = 5 \text{ ft} = 60 \text{ in.}$ From the geometry shown in Fig. b, the stretched length of wire BD is

$$L_{BD'} = \sqrt{60^2 + 0.075^2 - 2(60)(0.075)\cos 143.13^\circ} = 60.060017$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{60.060017 - 60}{60} = 1.0003(10^{-3}) \text{ in./in.}$$

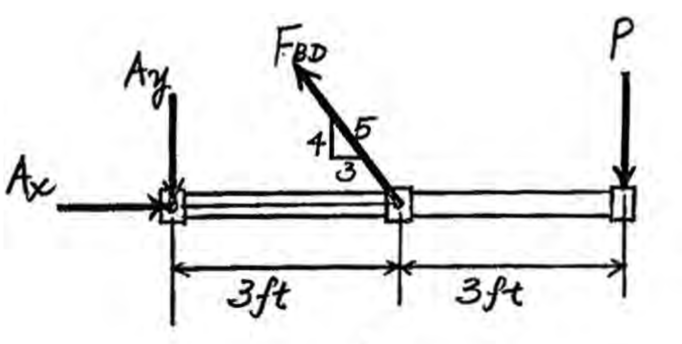
Then, the normal stress can be obtain by applying Hooke's Law.

$$\sigma_{BD} = E\epsilon_{BD} = 29(10^3)\left[1.0003(10^{-3})\right] = 29.01 \text{ ksi}$$

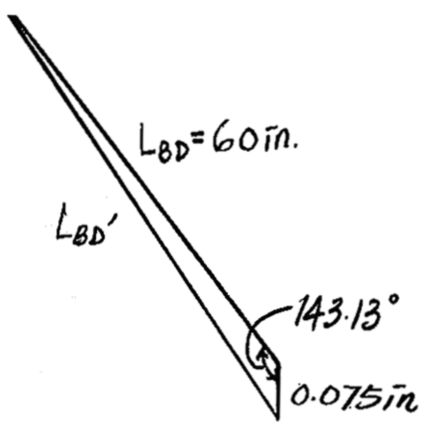
Since $\sigma_{BD} < \sigma_y = 36$ ksi, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \qquad 29.01(10^3) = \frac{2.50\,P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 569.57 \text{ lb} = 570 \text{ lb} \qquad \textbf{Ans.}$$

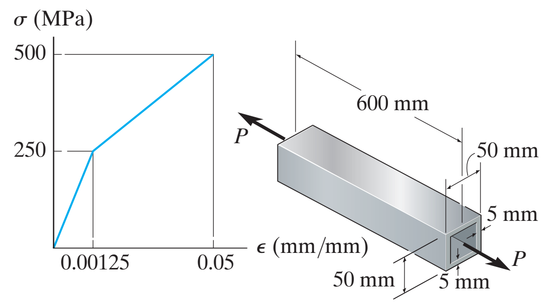


(a)



(b)

***3–16.** Determine the elongation of the square hollow bar when it is subjected to the axial force $P = 100$ kN. If this axial force is increased to $P = 360$ kN and released, find the permanent elongation of the bar. The bar is made of a metal alloy having a stress–strain diagram which can be approximated as shown.



Normal Stress and Strain: The cross-sectional area of the hollow bar is $A = 0.05^2 - 0.04^2 = 0.9(10^{-3})\text{m}^2$. When $P = 100\,\text{kN}$,

$$\sigma_1 = \frac{P}{A} = \frac{100(10^3)}{0.9(10^{-3})} = 111.11 \text{ MPa}$$

From the stress–strain diagram shown in Fig. *a*, the slope of the straight line OA which represents the modulus of elasticity of the metal alloy is

$$E = \frac{250(10^6) - 0}{0.00125 - 0} = 200 \text{ GPa}$$

Since $\sigma_1 < 250$ MPa, Hooke's Law can be applied. Thus

$$\sigma_1 = E\varepsilon_1; \quad 111.11(10^6) = 200(10^9)\varepsilon_1$$

$$\varepsilon_1 = 0.5556(10^{-3}) \text{ mm/mm}$$

Thus, the elongation of the bar is

$$\delta_1 = \varepsilon_1 L = 0.5556(10^{-3})(600) = 0.333 \text{ mm} \qquad \textbf{Ans.}$$

When $P = 360$ kN,

$$\sigma_2 = \frac{P}{A} = \frac{360(10^3)}{0.9(10^{-3})} = 400 \text{ MPa}$$

From the geometry of the stress–strain diagram, Fig. *a*,

$$\frac{\varepsilon_2 - 0.00125}{400 - 250} = \frac{0.05 - 0.00125}{500 - 250} \qquad \varepsilon_2 = 0.0305 \text{ mm/mm}$$

When $P = 360$ kN is removed, the strain recovers linearly along line BC, Fig. *a*, parallel to OA. Thus, the elastic recovery of strain is given by

$$\sigma_2 = E\varepsilon_r; \qquad 400(10^6) = 200(10^9)\varepsilon_r$$

$$\varepsilon_r = 0.002 \text{ mm/mm}$$

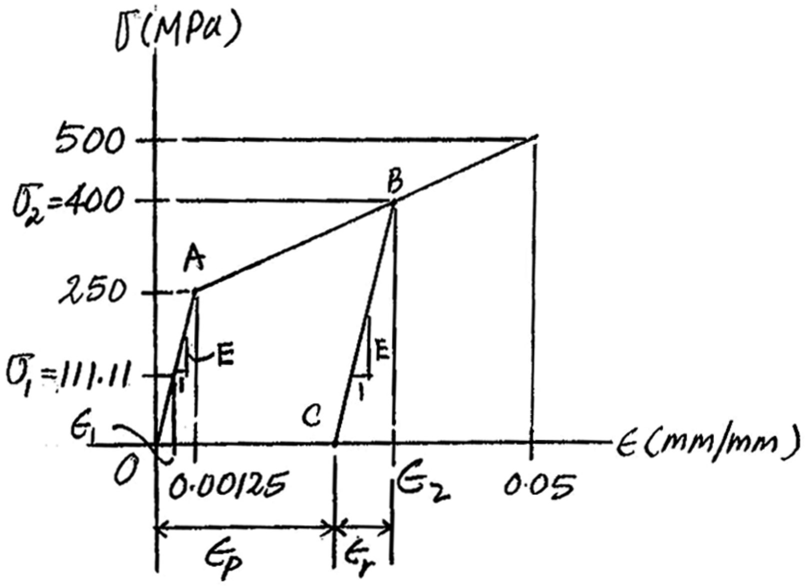
The permanent set is

$$\varepsilon_P = \varepsilon_2 - \varepsilon_r = 0.0305 - 0.002 = 0.0285 \text{ mm/mm}$$

Thus, the permanent elongation of the bar is

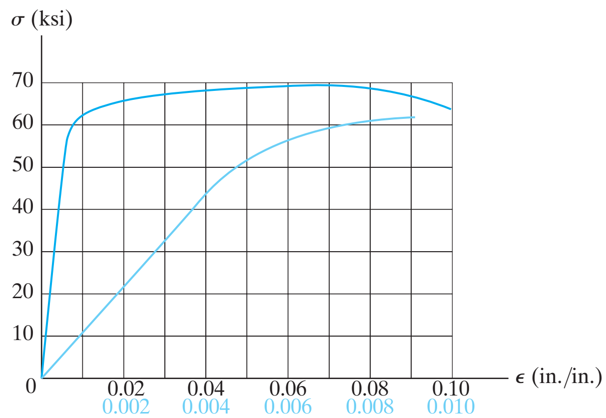
$$\delta_P = \varepsilon_P L = 0.0285(600) = 17.1 \text{ mm} \qquad \textbf{Ans.}$$

3–16. Continued



(a)

3–17. A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the proportional limit, (b) the modulus of elasticity, and (c) the yield strength based on a 0.2% strain offset method.



Proportional Limit and Yield Strength: From the stress–strain diagram, Fig. *a*,

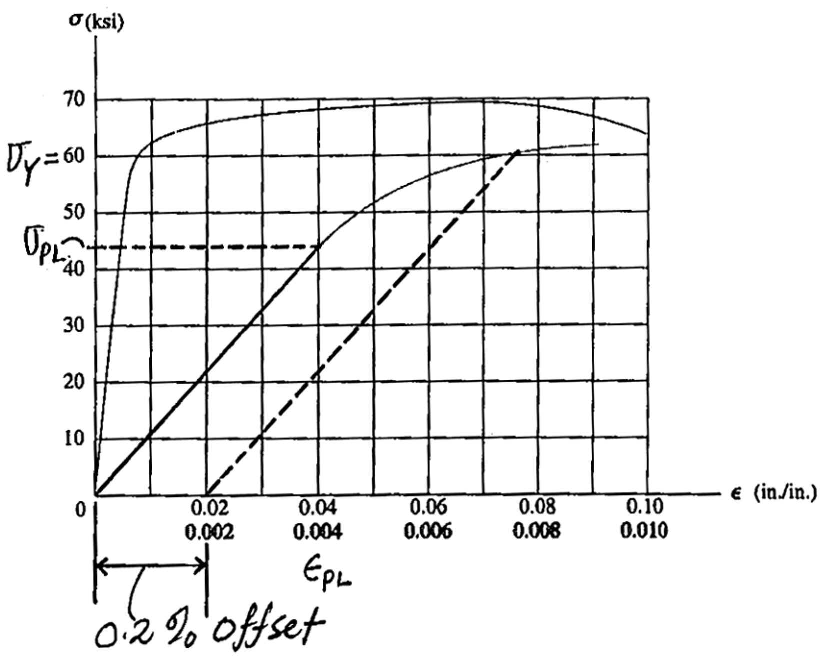
$$\sigma_{pl} = 44 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_Y = 60 \text{ ksi} \qquad \textbf{Ans.}$$

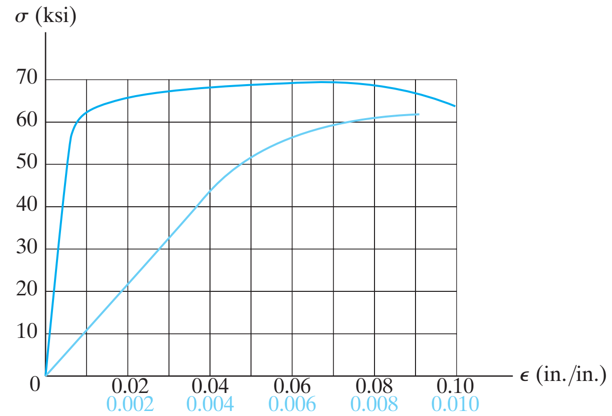
Modulus of Elasticity: From the stress–strain diagram, the corresponding strain for $\sigma_{PL} = 44$ ksi is $\varepsilon_{pl} = 0.004$ in./in. Thus,

$$E = \frac{44 - 0}{0.004 - 0} = 11.0(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the



3–18. A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the modulus of resilience; and (b) modulus of toughness.



stress–strain diagram up to the proportional limit. From the stress–strain diagram,

$$\sigma_{pl} = 44 \text{ ksi} \qquad \varepsilon_{pl} = 0.004 \text{ in./in.}$$

Thus,

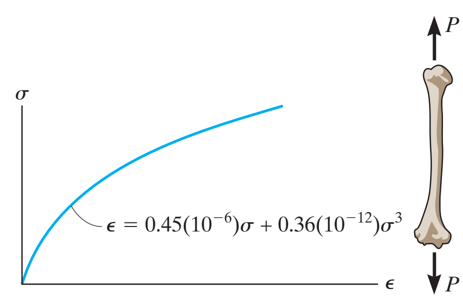
$$(U_i)_r = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}(44)(10^3)(0.004) = 88\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

Modulus of Toughness: The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number of squares is 65. Thus,

$$\left[(U_i)_t\right]_{\text{approx}} = 65\left[10(10^3)\,\frac{\text{lb}}{\text{in}^2}\right]\left[0.01\frac{\text{in.}}{\text{in.}}\right] = 6.50(10^3)\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

The stress–strain diagram for a bone is shown, and can be described by the equation

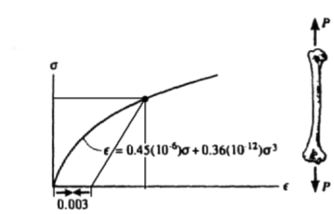
3–19. The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



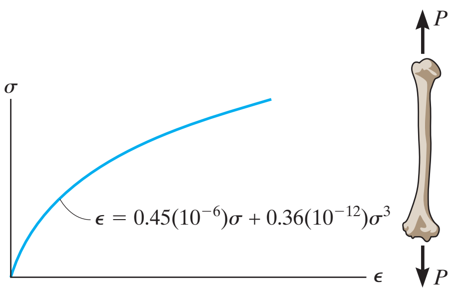
$$\varepsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3,$$

$$d\epsilon = \left(0.45(10^{-6}) + 1.08(10^{-12})\,\sigma^2\right)d\sigma$$

$$E = \left.\frac{d\sigma}{d\epsilon}\right|_{\sigma=0} = \frac{1}{0.45(10^{-6})} = 2.22 \text{ MPa} \qquad \textbf{Ans.}$$



***3–20.** The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



When $\varepsilon = 0.12$

$$120(10^3) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root:

$$\sigma = 6873.52 \text{ kPa}$$

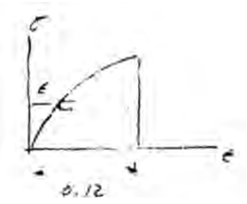
$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \varepsilon)d\sigma$$

$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)d\sigma$$

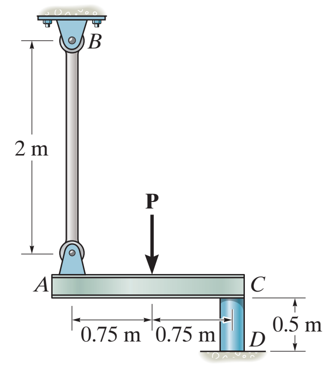
$$= 0.12\,\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4\Big|_0^{6873.52}$$

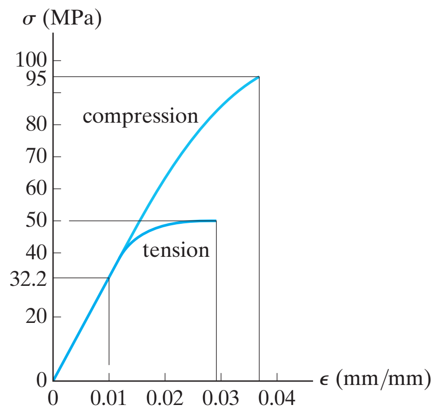
$$= 613 \text{ kJ/m}^3 \qquad \textbf{Ans.}$$

$$\delta = \varepsilon L = 0.12(200) = 24 \text{ mm} \qquad \textbf{Ans.}$$



•3–21. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.





From the stress–strain diagram,

$$E = \frac{32.2(10)^6}{0.01} = 3.22(10^9)\text{ Pa}$$

Thus,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40(10^3)}{\frac{\pi}{4}(0.04)^2} = 31.83\text{ MPa}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83(10^6)}{3.22(10^9)} = 0.009885\text{ mm/mm}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40(10^3)}{\frac{\pi}{4}(0.08)^2} = 7.958\text{ MPa}$$

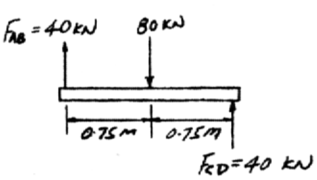
$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958(10^6)}{3.22(10^9)} = 0.002471\text{ mm/mm}$$

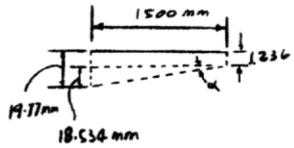
$$\delta_{AB} = \varepsilon_{AB}L_{AB} = 0.009885(2000) = 19.771\text{ mm}$$

$$\delta_{CD} = \varepsilon_{CD}L_{CD} = 0.002471(500) = 1.236\text{ mm}$$

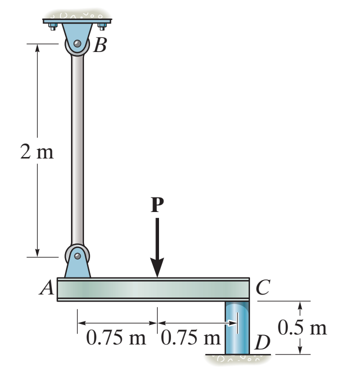
Angle of tilt α:

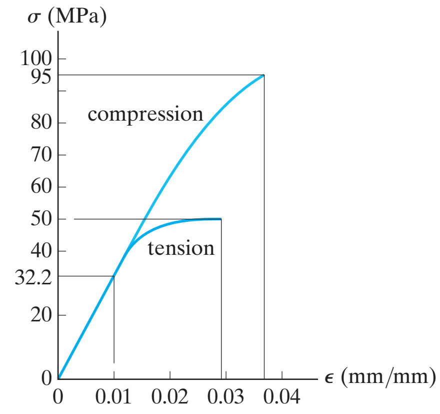
$$\tan\alpha = \frac{18.535}{1500}; \qquad \alpha = 0.708^\circ$$ **Ans.**





3–22. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.





Rupture of strut AB:

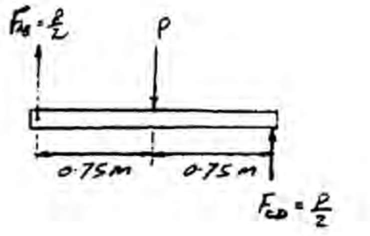
$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

$$P = 11.3 \text{ kN (controls)}$$ **Ans.**

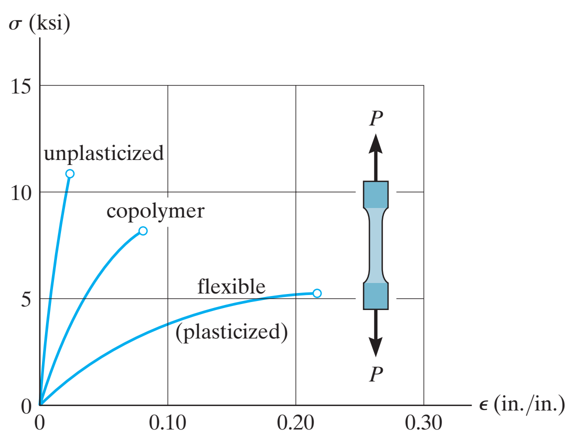
Rupture of post CD:

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239 \text{ kN}$$



3–23. By adding plasticizers to polyvinyl chloride, it is possible to reduce its stiffness. The stress–strain diagrams for three types of this material showing this effect are given below. Specify the type that should be used in the manufacture of a rod having a length of 5 in. and a diameter of 2 in., that is required to support at least an axial load of 20 kip and also be able to stretch at most $\frac{1}{4}$ in.



Normal Stress:

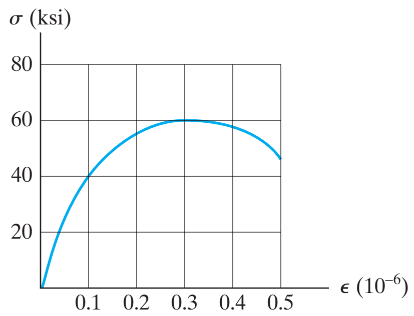
$$\sigma = \frac{P}{A} = \frac{20}{\frac{\pi}{4}(2^2)} = 6.366 \text{ ksi}$$

Normal Strain:

$$\varepsilon = \frac{0.25}{5} = 0.0500 \text{ in./in.}$$

From the stress–strain diagram, the *copolymer* will satisfy both stress and strain requirements. **Ans.**

***3–24.** The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 30(10^3)$ ksi and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



Choose,

$$\sigma = 40 \text{ ksi}, \quad \varepsilon = 0.1$$

$$\sigma = 60 \text{ ksi}, \quad \varepsilon = 0.3$$

$$0.1 = \frac{40}{30(10^3)} + k(40)^n$$

$$0.3 = \frac{60}{30(10^3)} + k(60)^n$$

$$0.098667 = k(40)^n$$

$$0.29800 = k(60)^n$$

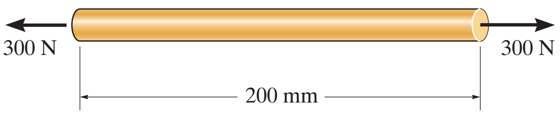
$$0.3310962 = (0.6667)^n$$

$$\ln(0.3310962) = n \ln(0.6667)$$

$$n = 2.73 \qquad \textbf{Ans.}$$

$$k = 4.23(10^{-6}) \qquad \textbf{Ans.}$$

•3–25. The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.697 \text{ MPa}$$

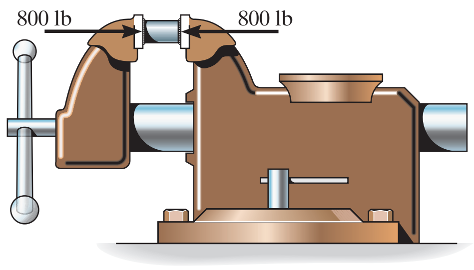
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{1.697(10^6)}{2.70(10^9)} = 0.0006288$$

$$\delta = \varepsilon_{long} L = 0.0006288\,(200) = 0.126 \text{ mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{lat} = -V\varepsilon_{long} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \varepsilon_{lat} d = -0.0002515\,(15) = -0.00377 \text{ mm} \qquad \textbf{Ans.}$$

3–26. The short cylindrical block of 2014-T6 aluminum, having an original diameter of 0.5 in. and a length of 1.5 in., is placed in the smooth jaws of a vise and squeezed until the axial load applied is 800 lb. Determine (a) the decrease in its length and (b) its new diameter.



a)

$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}(0.5)^2} = 4074.37 \text{ psi}$$

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{-4074.37}{10.6(10^6)} = -0.0003844$$

$$\delta = \varepsilon_{long}\, L = -0.0003844\,(1.5) = -0.577\,(10^{-3}) \text{ in.}$$ **Ans.**

b)

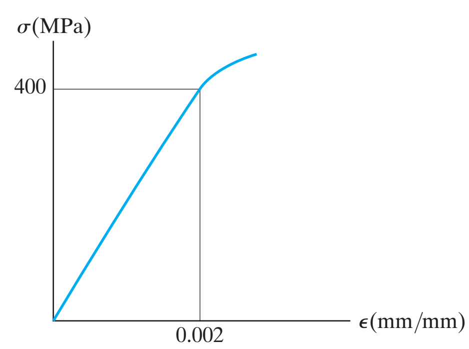
$$V = \frac{-\varepsilon_{lat}}{\varepsilon_{long}} = 0.35$$

$$\varepsilon_{lat} = -0.35\,(-0.0003844) = 0.00013453$$

$$\Delta d = \varepsilon_{lat}\, d = 0.00013453\,(0.5) = 0.00006727$$

$$d' = d + \Delta d = 0.5000673 \text{ in.}$$ **Ans.**

3–27. The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. When the applied load on the specimen is 50 kN, the diameter is 12.99265 mm. Determine Poisson's ratio for the material.



Normal Stress:

$$\sigma = \frac{P}{A} = \frac{50(10^3)}{\frac{\pi}{4}(0.013^2)} = 376.70 \text{ Mpa}$$

Normal Strain: From the stress–strain diagram, the modulus of elasticity $E = \frac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's law

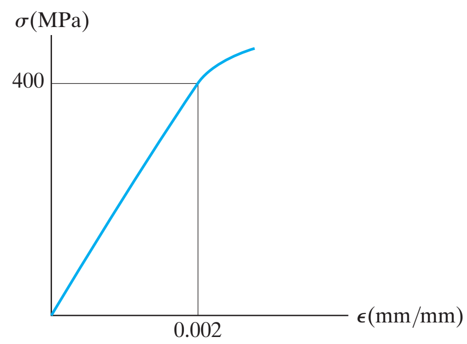
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{376.70(10^6)}{200(10^4)} = 1.8835\left(10^{-3}\right) \text{ mm/mm}$$

$$\varepsilon_{lat} = \frac{d - d_0}{d_0} = \frac{12.99265 - 13}{13} = -0.56538\left(10^{-3}\right) \text{ mm/mm}$$

Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio.

$$V = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{-0.56538(10^{-3})}{1.8835(10^{-3})} = 0.300$$ **Ans.**

***3–28.** The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. If a load of $P = 20$ kN is applied to the specimen, determine its diameter and gauge length. Take $\nu = 0.4$.



Normal Stress:

$$\sigma = \frac{P}{A} = \frac{20(10^3)}{\frac{\pi}{4}(0.013^2)} = 150.68\text{Mpa}$$

Normal Strain: From the Stress–Strain diagram, the modulus of elasticity $E = \dfrac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's Law

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{150.68(10^6)}{200(10^9)} = 0.7534\left(10^{-3}\right)\text{ mm/mm}$$

Thus,

$$\delta L = \varepsilon_{\text{long}} L_0 = 0.7534\left(10^{-3}\right)(50) = 0.03767\text{ mm}$$

$$L = L_0 + \delta L = 50 + 0.03767 = 50.0377\text{ mm} \qquad \textbf{Ans.}$$

Poisson's Ratio: The lateral and longitudinal can be related using poisson's ratio.

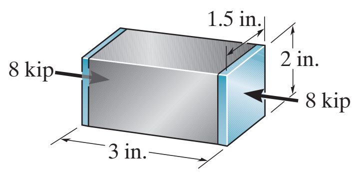
$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -0.4(0.7534)\left(10^{-3}\right)$$

$$= -0.3014\left(10^{-3}\right)\text{ mm/mm}$$

$$\delta d = \varepsilon_{\text{lat}}\, d = -0.3014\left(10^{-3}\right)(13) = -0.003918\text{ mm}$$

$$d = d_0 + \delta d = 13 + (-0.003918) = 12.99608\text{ mm} \qquad \textbf{Ans.}$$

•3–29. The aluminum block has a rectangular cross section and is subjected to an axial compressive force of 8 kip. If the 1.5-in. side changed its length to 1.500132 in., determine Poisson's ratio and the new length of the 2-in. side. $E_{al} = 10(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{8}{(2)(1.5)} = 2.667\text{ ksi}$$

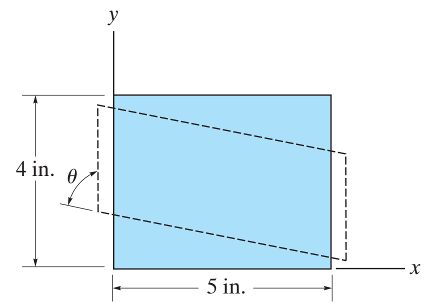
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{-2.667}{10(10^3)} = -0.0002667$$

$$\varepsilon_{\text{lat}} = \frac{1.500132 - 1.5}{1.5} = 0.0000880$$

$$\nu = \frac{-0.0000880}{-0.0002667} = 0.330 \qquad \textbf{Ans.}$$

$$h' = 2 + 0.0000880(2) = 2.000176\text{ in.} \qquad \textbf{Ans.}$$

3–30. The block is made of titanium Ti-6A1-4V and is subjected to a compression of 0.06 in. along the y axis, and its shape is given a tilt of $\theta = 89.7°$. Determine ϵ_x, ϵ_y, and γ_{xy}.



Normal Strain:

$$\varepsilon_y = \frac{\delta L_y}{L_y} = \frac{-0.06}{4} = -0.0150 \text{ in./in.} \qquad \textbf{Ans.}$$

Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio.

$$\varepsilon_x = -\nu\varepsilon_y = -0.36(-0.0150)$$

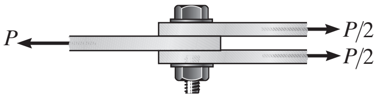
$$= 0.00540 \text{ in. /in.} \qquad \textbf{Ans.}$$

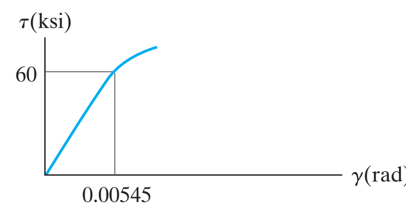
Shear Strain:

$$\beta = 180° - 89.7° = 90.3° = 1.576032 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - \beta = \frac{\pi}{2} - 1.576032 = -0.00524 \text{ rad} \qquad \textbf{Ans.}$$

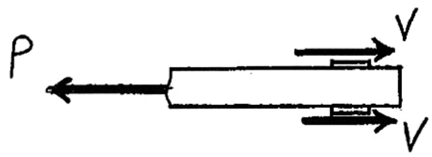
3–31. The shear stress–strain diagram for a steel alloy is shown in the figure. If a bolt having a diameter of 0.75 in. is made of this material and used in the double lap joint, determine the modulus of elasticity E and the force P required to cause the material to yield. Take $\nu = 0.3$.





The shear force developed on the shear planes of the bolt can be determined by considering the equilibrium of the FBD shown in Fig. *a*

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V + V - P = 0 \qquad V = = \frac{P}{2}$$



(a)

From the shear stress–strain diagram, the yield stress is $\tau_y = 60$ ksi. Thus,

$$\tau_y = \frac{V_y}{A}; \qquad 60 = \frac{P/2}{\frac{\pi}{4}\left(0.75^2\right)}$$

$$P = 53.01 \text{ kip} = 53.0 \text{ kip} \qquad \textbf{Ans.}$$

From the shear stress–strain diagram, the shear modulus is

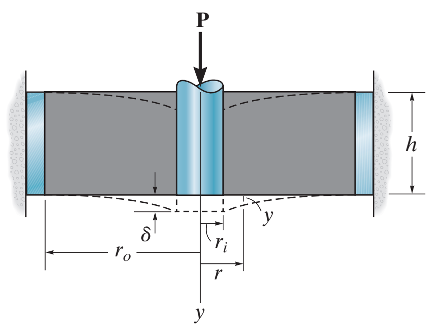
$$G = \frac{60 \text{ ksi}}{0.00545} = 11.01(10^3) \text{ ksi}$$

Thus, the modulus of elasticity is

$$G = \frac{E}{2(1 + \upsilon)}; \qquad 11.01(10^3) = \frac{E}{2(1 + 0.3)}$$

$$E = 28.6(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

***3–32.** A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -\tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.



Shear Stress–Strain Relationship: Applying Hooke's law with $\tau_A = \dfrac{P}{2\pi\, r\, h}$.

$$\gamma = \frac{\tau_A}{G} = \frac{P}{2\pi\, h\, G\, r}$$

$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(\frac{P}{2\pi\, h\, G\, r}\right) \qquad \text{(Q.E.D)}$$

If γ is small, then $\tan\gamma = \gamma$. Therefore,

$$\frac{dy}{dr} = -\frac{P}{2\pi\, h\, G\, r}$$

$$y = -\frac{P}{2\pi\, h\, G}\int\frac{dr}{r}$$

$$y = -\frac{P}{2\pi\, h\, G}\ln r + C$$

At $r = r_o$, $\quad y = 0$

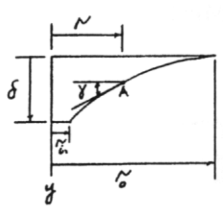
$$0 = -\frac{P}{2\pi\, h\, G}\ln r_o + C$$

$$C = \frac{P}{2\pi\, h\, G}\ln r_o$$

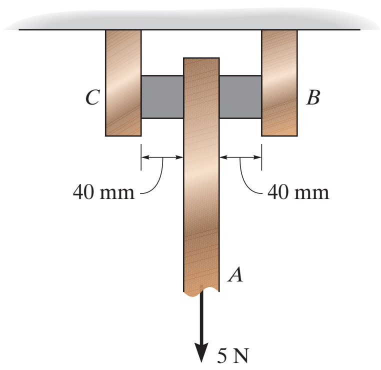
Then, $y = \dfrac{P}{2\pi\, h\, G}\ln\dfrac{r_o}{r}$

At $r = r_i$, $\quad y = \delta$

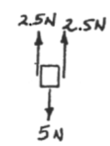
$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i} \qquad \textbf{Ans.}$$



•3–33. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 5 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 30 mm and 20 mm. $G_r = 0.20$ MPa.



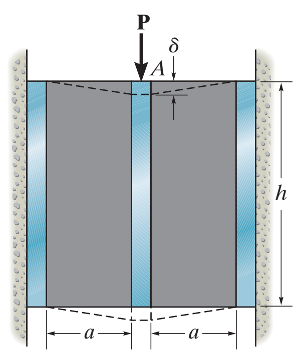
$$\tau_{avg} = \frac{V}{A} = \frac{2.5}{(0.03)(0.02)} = 4166.7 \text{ Pa}$$



$$\gamma = \frac{\tau}{G} = \frac{4166.7}{0.2(10^6)} = 0.02083 \text{ rad}$$

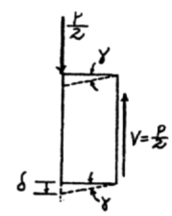
$$\delta = 40(0.02083) = 0.833 \text{ mm}$$ **Ans.**

3–34. A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A if a vertical load $\mathbf{P}$ is applied to this plate. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



Average Shear Stress: The rubber block is subjected to a shear force of $V = \frac{P}{2}$.

$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{b\,h} = \frac{P}{2\,b\,h}$$



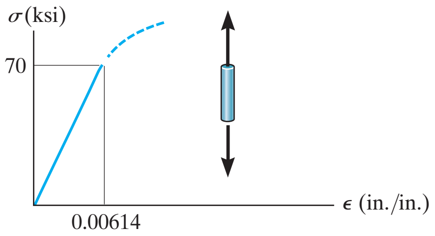
Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2\,b\,h}}{G} = \frac{P}{2\,b\,h\,G}$$

Thus,

$$\delta = a\,\gamma = \; = \frac{P\,a}{2\,b\,h\,G}$$ **Ans.**

3–35. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. When the applied load is 9 kip, the new diameter of the specimen is 0.49935 in. Compute the shear modulus G_{al} for the aluminum.



From the stress–strain diagram,

$$E_{al} = \frac{\sigma}{\varepsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

When specimen is loaded with a 9 - kip load,

$$\sigma = \frac{P}{A} = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

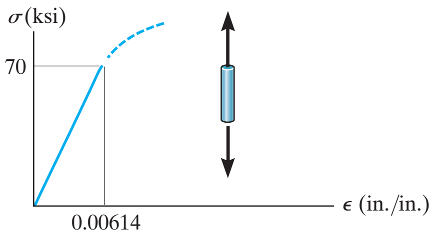
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040208 \text{ in./in.}$$

$$\varepsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{0.49935 - 0.5}{0.5} = -0.0013 \text{ in./in.}$$

$$V = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{-0.0013}{0.0040208} = 0.32332$$

$$G_{al} = \frac{E_{at}}{2(1+v)} = \frac{11.4(10^3)}{2(1+0.32332)} = 4.31(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

***3–36.** The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip, determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.5)^2} = 50.9296 \text{ ksi}$$

From the stress–strain diagram

$$E = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{50.9296}{11400.65} = 0.0044673 \text{ in./in.}$$

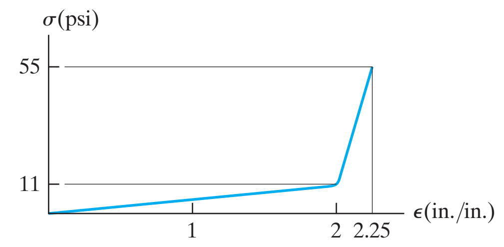
$$G = \frac{E}{2(1+v)}; \qquad 3.8(10^3) = \frac{11400.65}{2(1+v)}; \qquad v = 0.500$$

$$\varepsilon_{\text{lat}} = -v\varepsilon_{\text{long}} = -0.500(0.0044673) = -0.002234 \text{ in./in.}$$

$$\Delta d = \varepsilon_{\text{lat}}\, d = -0.002234(0.5) = -0.001117 \text{ in.}$$

$$d' = d + \Delta d = 0.5 - 0.001117 = 0.4989 \text{ in.} \qquad \textbf{Ans.}$$

3–37. The σ–ϵ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.



$$E = \frac{11}{2} = 5.5 \text{ psi}$$ **Ans.**

$$u_t = \frac{1}{2}(2)(11) + \frac{1}{2}(55 + 11)(2.25 - 2) = 19.25 \text{ psi}$$ **Ans.**

$$u_t = \frac{1}{2}(2)(11) = 11 \text{ psi}$$ **Ans.**

3–38. A short cylindrical block of 6061-T6 aluminum, having an original diameter of 20 mm and a length of 75 mm, is placed in a compression machine and squeezed until the axial load applied is 5 kN. Determine (a) the decrease in its length and (b) its new diameter.

a) $$\sigma = \frac{P}{A} = \frac{-5(10^3)}{\frac{\pi}{4}(0.02)^2} = -15.915 \text{ MPa}$$

$$\sigma = E\,\varepsilon_{long}; \qquad -15.915(10^6) = 68.9(10^9)\,\varepsilon_{long}$$

$$\varepsilon_{long} = -0.0002310 \text{ mm/mm}$$

$$\delta = \varepsilon_{long} L = -0.0002310(75) = -0.0173 \text{ mm}$$ **Ans.**

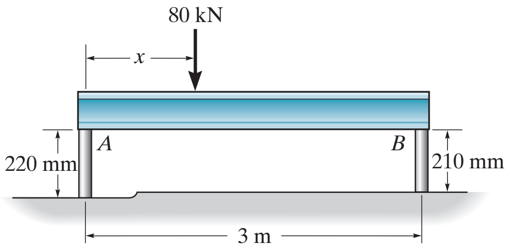
b) $$v = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \qquad 0.35 = -\frac{\varepsilon_{lat}}{-0.0002310}$$

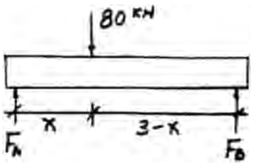
$$\varepsilon_{lat} = 0.00008085 \text{ mm/mm}$$

$$\Delta d = \varepsilon_{lat}\, d = 0.00008085(20) = 0.0016 \text{ mm}$$

$$d' = d + \Delta d = 20 + 0.0016 = 20.0016 \text{ mm}$$ **Ans.**

3–39. The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.





$$\zeta + \Sigma M_A = 0; \qquad F_B(3) - 80(x) = 0; \qquad F_B = \frac{80x}{3} \qquad \textbf{(1)}$$

$$\zeta + \Sigma M_B = 0; \qquad -F_A(3) + 80(3 - x) = 0; \qquad F_A = \frac{80(3 - x)}{3} \qquad \textbf{(2)}$$

Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \qquad \varepsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \varepsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \qquad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$$x = 1.53 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (2),

$$F_A = 39.07 \text{ kN}$$

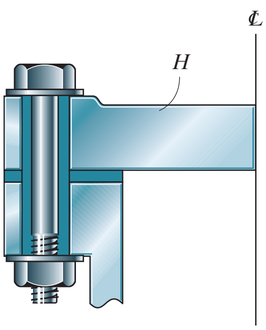
$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

$$\varepsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

$$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$$d'_A = d_A + d\,\varepsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm} \qquad \textbf{Ans.}$$

***3–40.** The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 800 lb, determine the normal strain in the bolts. Each bolt has a diameter of $\frac{3}{16}$ in. If $\sigma_Y = 40$ ksi and $E_{st} = 29(10^3)$ ksi, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



Normal Stress:

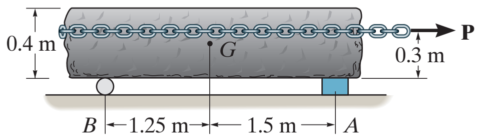
$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}\left(\frac{3}{16}\right)^2} = 28.97 \text{ ksi} < \sigma_\gamma = 40 \text{ ksi}$$

Normal Strain: Since $\sigma < \sigma_\gamma$, Hooke's law is still valid.

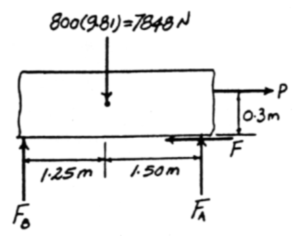
$$\varepsilon = \frac{\sigma}{E} = \frac{28.97}{29(10^3)} = 0.000999 \text{ in./in.} \qquad \textbf{Ans.}$$

If the nut is unscrewed, the load is zero. Therefore, the strain $\varepsilon = 0$ **Ans.**

•3–41. The stone has a mass of 800 kg and center of gravity at G. It rests on a pad at A and a roller at B. The pad is fixed to the ground and has a compressed height of 30 mm, a width of 140 mm, and a length of 150 mm. If the coefficient of static friction between the pad and the stone is $\mu_s = 0.8$, determine the approximate horizontal displacement of the stone, caused by the shear strains in the pad, before the stone begins to slip. Assume the normal force at A acts 1.5 m from G as shown. The pad is made from a material having $E = 4$ MPa and $\nu = 0.35$.



Equations of Equilibrium:



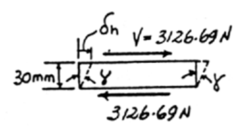
$$\curvearrowleft +\Sigma M_B = 0; \qquad F_A(2.75) - 7848(1.25) - P(0.3) = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F = 0 \qquad [2]$$

Note: The normal force at A does not act exactly at A. It has to shift due to friction.

Friction Equation:

$$F = \mu_s F_A = 0.8\, F_A \qquad [3]$$



Solving Eqs. [1], [2] and [3] yields:

$$F_A = 3908.37 \text{ N} \qquad F = P = 3126.69 \text{ N}$$

Average Shear Stress: The pad is subjected to a shear force of $V = F = 3126.69$ N.

$$\tau = \frac{V}{A} = \frac{3126.69}{(0.14)(0.15)} = 148.89 \text{ kPa}$$

Modulus of Rigidity:

$$G = \frac{E}{2(1 + \nu)} = \frac{4}{2(1 + 0.35)} = 1.481 \text{ MPa}$$

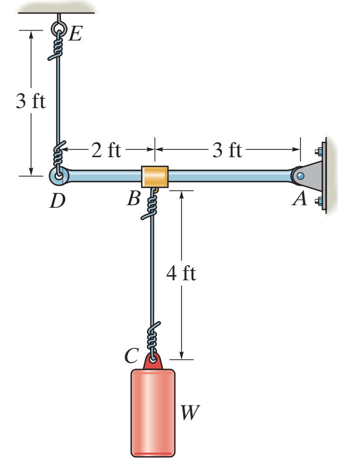
Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{148.89(10^3)}{1.481(10^6)} = 0.1005 \text{ rad}$$

Thus,

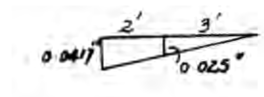
$$\delta_h = h\gamma = 30(0.1005) = 3.02 \text{ mm} \qquad \textbf{Ans.}$$

3–42. The bar DA is rigid and is originally held in the horizontal position when the weight W is supported from C. If the weight causes B to be displaced downward 0.025 in., determine the strain in wires DE and BC. Also, if the wires are made of A-36 steel and have a cross-sectional area of 0.002 in^2, determine the weight W.



$$\frac{3}{0.025} = \frac{5}{\delta}$$

$$\delta = 0.0417 \text{ in}$$

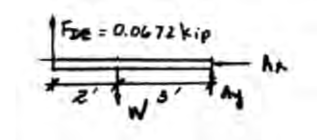


$$\varepsilon_{DE} = \frac{\delta}{L} = \frac{0.0417}{3(12)} = 0.00116 \text{ in./in.} \qquad \textbf{Ans.}$$

$$\sigma_{DE} = E\varepsilon_{DE} = 29(10^3)(0.00116) = 33.56 \text{ ksi}$$

$$F_{DE} = \sigma_{DE}A_{DE} = 33.56\,(0.002) = 0.0672 \text{ kip}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -(0.0672)\,(5) + 3(W) = 0$$

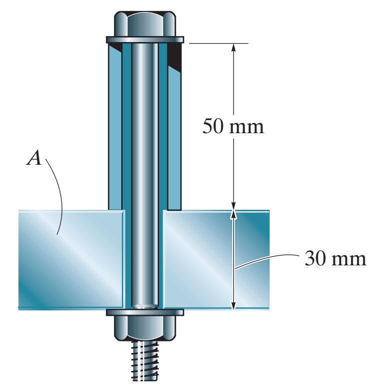


$$W = 0.112 \text{ kip} = 112 \text{ lb} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{W}{A_{BC}} = \frac{0.112}{0.002} = 55.94 \text{ ksi}$$

$$\varepsilon_{BC} = \frac{\sigma_{BC}}{E} = \frac{55.94}{29\,(10^3)} = 0.00193 \text{ in./in.} \qquad \textbf{Ans.}$$

3–43. The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



Normal Stress:

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15 \text{ MPa}$$

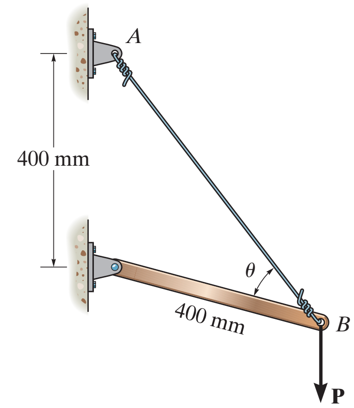
$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79 \text{ MPa}$$

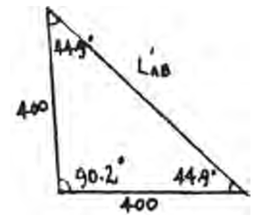
Normal Strain: Applying Hooke's Law

$$\varepsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\varepsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884 \text{ mm/mm} \qquad \textbf{Ans.}$$

***3–44.** The A-36 steel wire AB has a cross-sectional area of 10 mm^2 and is unstretched when $\theta = 45.0°$. Determine the applied load P needed to cause $\theta = 44.9°$.





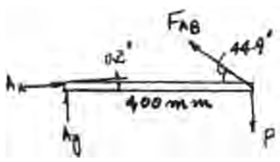
$$\frac{L'_{AB}}{\sin 90.2°} = \frac{400}{\sin 44.9°}$$

$$L'_{AB} = 566.67 \text{ mm}$$

$$L_{AB} = \frac{400}{\sin 45°} = 565.69$$

$$\varepsilon = \frac{L'_{AB} - L_{AB}}{L_{AB}} = \frac{566.67 - 565.69}{565.69} = 0.001744$$

$$\sigma = E\varepsilon = 200(10^9)\,(0.001744) = 348.76 \text{ MPa}$$



$$\circlearrowleft + \Sigma M_A = 0$$

$$P(400 \cos 0.2°) - F_{AB} \sin 44.9° \,(400) = 0 \qquad \textbf{(1)}$$

However,

$$F_{AB} = \sigma A = 348.76(10^6)(10)(10^{-6}) = 3.488 \text{ kN}$$

From Eq. (1),

$$P = 2.46 \text{ kN} \qquad \textbf{Ans.}$$