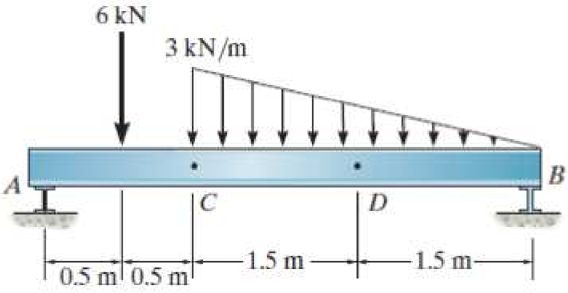
CE 203 – 121

Solution of HW # 1

Problem #1:

Given:

The beam shown



Required:

The internal forces at point E, located at 2 m from A

Solution:

First, we need to calculate the reaction at B. (Why only B?!)

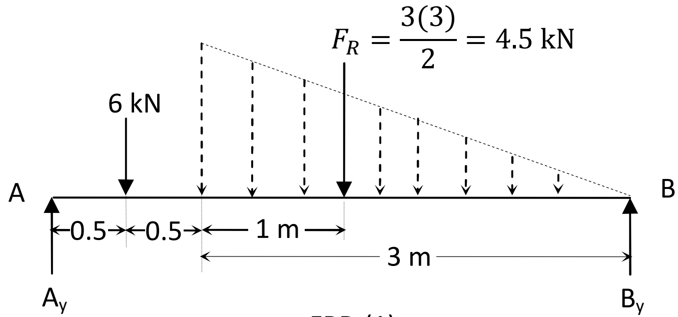
FBD (1) is drawn below.

$\Sigma M_A = 0$

(Why *ΣM, not ΣF, and why point A?!)*

$\Rightarrow 4B_y - 4.5(2) - 6(0.5) = 0$

$\Rightarrow B_y = 3\ kN\ (As\ shown)$

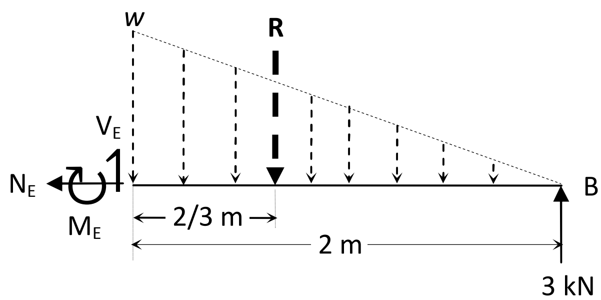


FBD (1)

*Now, we make a section through E and take the **right** part (Why?!). FBD (2) is drawn. Note the assumed directions of the internal forces (Why?!)*

$$\frac{w}{3} = \frac{2}{3} \Rightarrow w = 2\ kN/m$$

$$R = \frac{2(2)}{2} = 2\ kN$$



FBD (2)

*Note that the equivalent load R was calculated **after** cutting. (Why?!)*

$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \boxed{N_E = 0}$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow V_E - 2 + 3 = 0 \Rightarrow \boxed{V_E = -1\ kN = 1\ kN \downarrow}$$

$\Sigma M_E = 0$

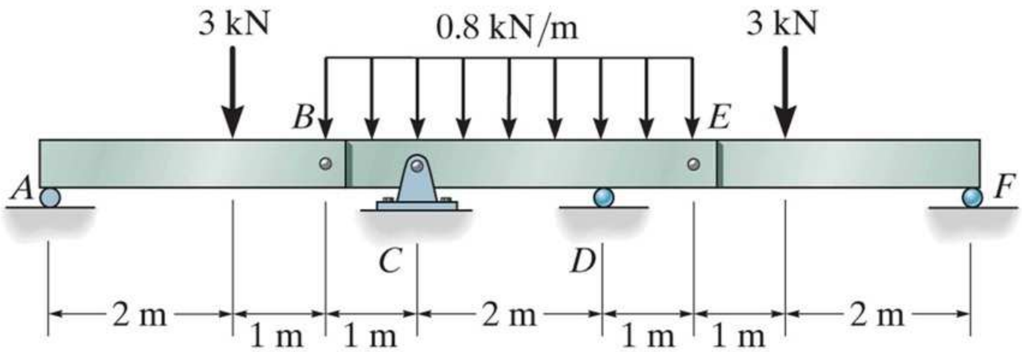
$$\Rightarrow -M_E - 2\left(\frac{2}{3}\right) + 3(2) = 0$$

$$\boxed{\Rightarrow M_E = 4\frac{2}{3}\ \text{kN.m (as shown)}}$$

Problem #2:

Given:

The beam shown



Required:

- *The reactions*
- *The internal forces at the center of the beam*

Solution:

To be able to find the reactions, we need to separate the beam at B or E. (Why?!)

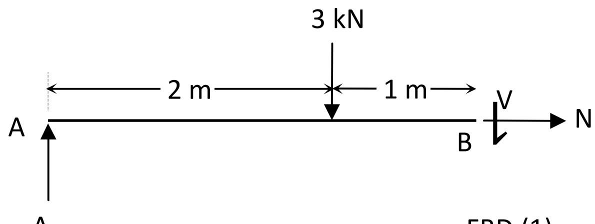
Taking AB, FBD (1) is drawn. Note that there's no moment at B. (Why?!).

$\Sigma M_B = 0$

(Why *not taking* $\Sigma F_y = 0$*?!)*

$\Rightarrow 3(1) - A_y(3) = 0$

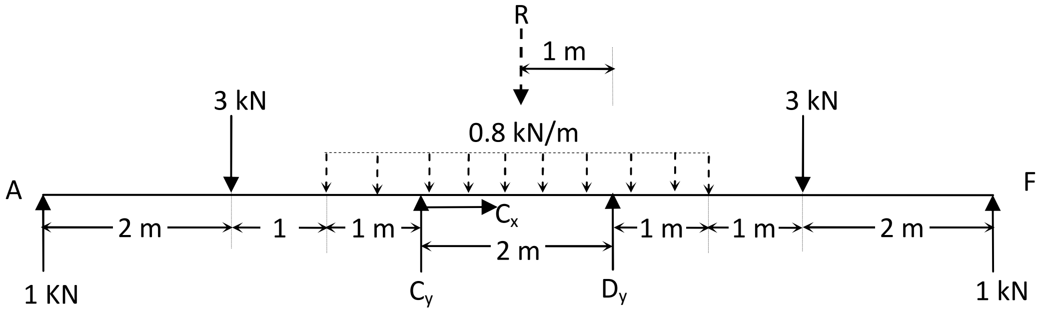
$\Rightarrow \boxed{A_y = 1\ kN\ \uparrow (as\ shown)}$



FBD (1)

Due to symmetry (in geometry, loads, supports, etc), $\boxed{F_y = A_y = 1\ kN\ \uparrow}$

Now, we take FBD (2) for the whole beam.



FBD (2)

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \boxed{C_x = 0}$

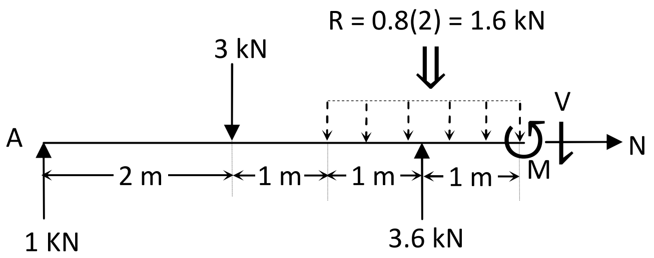
$\circlearrowleft + \; \Sigma M_D = 0$

$\Rightarrow -1(6) + 3(4) - 2C_y + 0.8(4)(1) - 3(2) + 1(4) = 0$

$\boxed{\Rightarrow C_y = \;3.6\text{ kN} \uparrow \;\text{(as shown)}}$

By symmetry, $\boxed{D_y = C_y = 3.6\; kN \;\uparrow}$

To determine the internal forces at the beam center, a cut (section) is made through that point, and FBD (3) (the left part) is drawn.



FBD (3)

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \boxed{N = 0}$

$+\uparrow \Sigma F_y = 0 \Rightarrow 1 - 3 - 1.6 + 3.6 - V = 0 \Rightarrow \boxed{V = 0\; kN}$

This is expected, as it is the value of the shear in the middle/center of a "symmetrical beam". (Why and how?!)

$\circlearrowleft + \; \Sigma M = 0$

$\Rightarrow M - 1(5) + 3(3) + 1.6(1) - 3.6(1) = 0$

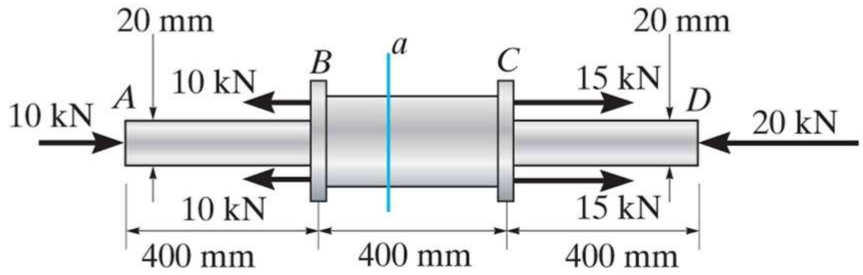
$\boxed{\Rightarrow M = \;-2\text{ kN.m} = \;2\text{ kN.m} \circlearrowright}$

Problem #3:

Given:

The figure shown

$D_{AB} = D_{CD} = 20\ mm;\ D_{BC} = 40\ mm$



Required:

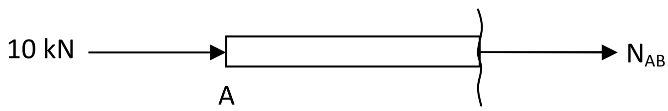
***σ** in AB, BC, and CD*

Solution:

$$\sigma = \frac{P}{A} = \frac{N}{A}$$

*N is the **internal** normal force, obtained by passing a cut (section) through the part of interest.*

A section through each of AB, BC, and CD is made, and the "easier" part (left or right) is taken and an FBD is drawn, as shown below.



(N_A may be assumed in the other direction ←, i.e, compression.)

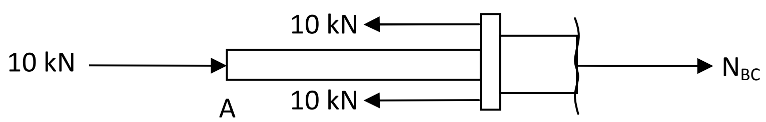
$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow 10 + N_{AB} = 0 \Rightarrow$$

$$N_{AB} = -10\ kN = 10\ kN\ \leftarrow = 10\ kN\ "C"$$

$$\sigma_{AB} = \left(\frac{N}{A}\right)_{AB} = \frac{-10(10)^3}{\pi/_4\,(20)^2} \Rightarrow$$

$$\boxed{\sigma_{AB} = -31.83\ MPa = 31.83\ MPa\ "C"}$$

**Be careful about the units (N, m, mm, Pa, MPa,….)*

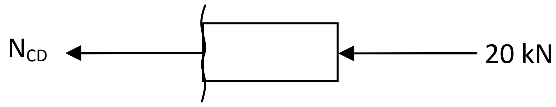


$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow 10 - 10 - 10 + N_{BC} = 0 \Rightarrow$$

$$N_{BC} = 10\ kN\ "T"$$

$$\sigma_{BC} = \left(\frac{N}{A}\right)_{BC} = \frac{10(10)^3}{\pi/_4\,(40)^2} \Rightarrow$$

$$\boxed{\sigma_{BC} = 7.958\ MPa\ "T"}$$



(Assumed “T”, may as well be assumed “C”)

$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow -20 - N_{CD} = 0 \Rightarrow$$

$$N_{CD} = -20\ kN = 20\ kN\ "C"$$

$$\sigma_{CD} = \left(\frac{N}{A}\right)_{CD} = \frac{-20(10)^3}{\pi/_4\,(20)^2} \Rightarrow$$

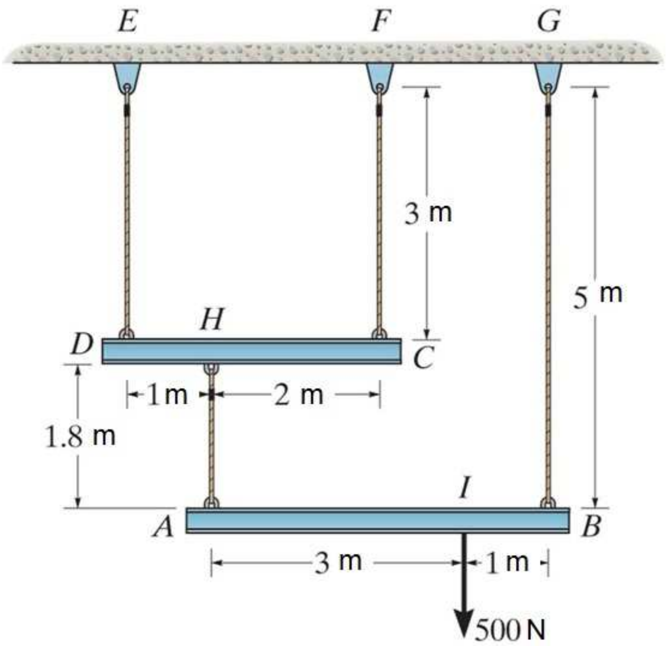
$$\boxed{\sigma_{CD} = -63.66\ MPa = 63.66\ MPa\ "C"}$$

Problem #4:

Given:

The figure shown

$A_{cable} = 20\ mm^2$



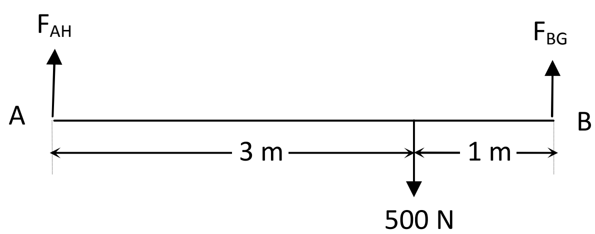
Required:

σ *in the cables.*

Solution:

In order to determine the stresses in the cables, we need to find the internal normal forces in these cables as $\sigma = \frac{N}{A}$ *where N is the* ***internal*** *force.*

FBD (1) is drawn for AB. (Why?!)



FBD (1)

+ $\Sigma M_B = 0$ *(Why?!)*

$\Rightarrow 500(1) - F_{AH}(4) = 0$

$\Rightarrow F_{AH} = 125\ \text{N "T"}$

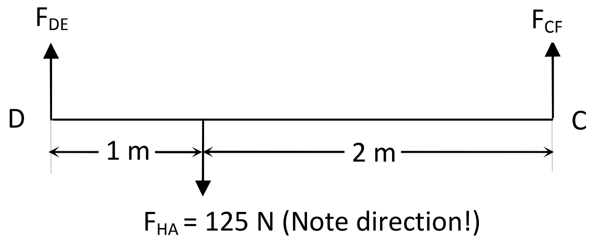
Solution of HW # 1

$+\uparrow \Sigma F_y = 0 \Rightarrow 125 - 500 + F_{BG} = 0$

$\Rightarrow F_{BG} = 375\ N$ "T"

Now, FBD (2) for DC is drawn.

(Another FBD for both AB and DC can be drawn. Why and how?!)



FBD (2)

$+ \circlearrowleft\ \Sigma M_C = 0$

$\Rightarrow 125(2) - F_{DE}(3) = 0 \Rightarrow F_{DE} = 83.333\ N$ "T"

$+\uparrow \Sigma F_y = 0 \Rightarrow 83.3333 - 125 + F_{CF} = 0$

$\Rightarrow F_{CF} = 41.667\ N$ "T"

$$\sigma_{AH} = \frac{F_{AH}}{A} = \frac{125}{20} \Rightarrow \boxed{\sigma_{AH} = 6.25\ MPa \text{ "T"}}$$

$$\sigma_{BG} = \frac{F_{BG}}{A} = \frac{375}{20} \Rightarrow \boxed{\sigma_{BG} = 18.75\ MPa \text{ "T"}}$$

$$\sigma_{DE} = \frac{F_{DE}}{A} = \frac{83.3333}{20} \Rightarrow \boxed{\sigma_{DE} = 4.167\ MPa \text{ "T"}}$$

$$\sigma_{CF} = \frac{F_{CF}}{A} = \frac{41.667}{20} \Rightarrow \boxed{\sigma_{CF} = 2.083\ MPa \text{ "T"}}$$