1. The down-wards speed of a Parachutist of mass $m=75\ kg$ and with drag coefficient $C_D= 13\ kg/sec$ (due to air resistance) may be determined from either one of the following equations:

   - *Analytical* expression: $\mathbf{v(t) = m\ g/C_D\ [1 - exp(-C_D\ t/m)]}$.
   - *Iterative* expression: $\mathbf{v_{i+1} = v_i + [\ g - C_D/m\ v_i]\ (t_{i+1} - t_i)}$.

   Use $g=9.81\ m/s^2$ and compare the values of speed [i.e.: $v(t)$ *and* v_{i+1}] at $t=3$ seconds assuming the initial value $v(t=0) = v_0 = 0.0\ m/sec$ for two values of $\Delta t = (t_{i+1} - t_i)$: 0.5 *and* 1.0 sec.

Solution:-

- First, the **analytical solution**:- at t = 3 sec.

$$V(3) = 75 * 9.81 * \frac{\left\{1-e^{\frac{-13*3}{75}}\right\}}{13} = 22.95\frac{m}{sec}.$$

- Second, **the iterative solution or numerical solution**:-

Using the given equation, $V_0 = 0$ and $\Delta t = 0.5\ sec. \rightarrow V_{0+0.5} = V_0 + \left\{9.81 - 13 * \frac{V_0}{75}\right\} * \Delta t$

$\rightarrow V_{0.5\ sec} = 4.905\frac{m}{sec}$. By repeating this operation until we get $V_{3\ sec} = 23.744\frac{m}{sec}$.

The same thing has been done using $\Delta t = 1\ sec.$ and the corresponding $V_{3\ sec} = 24.624\frac{m}{sec}$.

The all results are in the following table:-

Δt = 0.5 sec.		**Δt = 1 sec.**	
t (sec)	**V (m/sec)**	**t (sec)**	**V (m/sec)**
0.5	4.905	1.0	9.81
1	9.385	2.0	17.920
1.5	13.477	3.0	24.624
2	17.214		
2.5	20.627		
3	23.744		

Conclusion:-

From the values of the velocity at 3 sec., when **Δt decreases, the value of the velocity becomes more accurate and closes to exact solution value.**

2. Convert the *decimal* number *-47.3* into its equivalent *binary* form.

Solution:-

Start to convert the positive number +47.3, and then take the complement.

For 47, {47-32= 15, 15-8=7, 7-4=3, 3-2=1}.

0	0	1	0	1	1	1	1
128	64	32	16	8	4	2	1

Hence, $(47)_{10} = (00101111)_2$, for the part after the decimal point 0.7 do the following:

		After (.)	**Before (.)**
0.3*2	0.6	0.6	0
0.6*2	1.2	0.2	1
0.2*2	0.4	0.4	0
0.4*2	0.8	0.8	0
0.8*2	1.6	0.6	1
0.6*2	1.2	0.2	1
0.2*2	0.4	0.4	0
0.4*2	0.8	0.8	0
0.8*2	1.6	0.6	1
0.6*2	1.2	0.2	1
.	.	.	.

Hence, $(0.3)_{10} = (0100110011\ldots.)_2$

That gives the following result:

$(+47.3)_{10} = (00101111.\ 0100110011\ldots)_2$

For the negative value, just take the complement as the following:

$(-47.3)_{10} = (1$00101111. 0100110011$\ldots)_2$**

Sign position

3. Study the composition of the *binary* number 011.10101, then convert it into its *decimal* form.

Solution:

- First, the digits $(011)_2 = 2^0 * 1 + 2^1 * 1 + 2^2 * 0 = 1 + 2 = 3$
- Second, the fraction part $(0.10100)_2 = 2^{-1} * 1 + 2^{-2} * 0 + 2^{-3} * 1 + 2^{-4} * 0 + 2^{-5} * 1 = 0.65625$
- The value of the whole **number $(011.10100)_2$ = 3+0.65625 = $(3.65625)_{10}$**

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4. Assume a *four significant figures* machine representation [at all steps] to compute the value of *y*, where:
$y = [(574 - 5\,x^2)/\,6]^{1/2}$; $x = 7.63/7$.

Compare the *errors* in both *x* and *y* due to the *round-off errors*.

Solution:

From the given data and for a four significant figures, **X= 1.090** (direct calculation)

Based on that, $y = \sqrt{[\{574 - 5 * (1.090)^2\}/6]} = \sqrt{[\{574 - 5 * 1.188\}/6]}$

$$y = \sqrt{[\{574 - 5.940\}/6]} = \sqrt{[\{568.1\}/6]} = \sqrt{[94.68]} = \mathbf{9.730}$$

To get the value of X,

$$(9.730)^2 = \frac{(574-5*X^2)}{6} \rightarrow 94.67 = \frac{(574-5*X^2)}{6} \rightarrow 568.0 = (574 - 5 * X^2)$$

$$568.0 + 574 = 5 * X^2 \rightarrow 6.000 = 5 * X^2 \rightarrow 1.200 = X^2$$

$$\mathbf{X = 1.096}$$

Using this value for X → the round off error

$$= \frac{\mathbf{1.096-1.090}}{\mathbf{1.090}} * \mathbf{100 = 0.551}\ \%$$

The same thing can be repeated for y using the value of X =1.096, then get the round off error.

5. Draw two flowcharts that may be utilized to build a computer-code for solving problem 3.5 and 3.10 of your text-book page 72. Also use the flowcharts to provide your solutions.

Solution:

Regarding the flow charts, you can develop them similar the explained one in the lab session.

Problem 3.5.

- First, using this approach $e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \ ....;$

$e^{-5} = 1\ ; e^{-5} = 1 - 5;\ e^{-5} = 1 - 5 + \frac{5^2}{2}\ ; e^{-5} = 1 - 5 + \frac{5^2}{2} - \frac{5^3}{3!}\ ;$

and continue the same way up to 20 terms.

- Second, using this approach $e^{-x} = \frac{1}{e^x} = \frac{1}{1+x+\frac{x^2}{2}+\frac{x^3}{3!}+ \ ...};$

$e^{-5} = 1\ ; e^{-5} = \frac{1}{1+5};\ e^{-5} = \frac{1}{1+5+\frac{5^2}{2}}\ ; e^{-5} = \frac{1}{1+5+\frac{5^2}{2}+\frac{5^3}{3!}}\ ;$ and again

continue the same way up to 20 terms

These all calculations are in the tables below:

Series 1			
term	**$e^{(-5)}$**	**True error %**	**Relative Error %**
0	1	14741.31591	-
1	-4	59465.26363	300
2	8.5	126051.1852	112.5
3	-12.3333	183142.8962	45.09803922
4	13.70833	203349.7056	11.14864865
5	-12.3333	183142.8962	10.03039514
6	9.368056	138934.2719	24.04279279
7	-6.13294	91120.84816	34.53351689
8	3.555184	52663.60191	42.03130055
9	-1.82711	27216.64813	48.6072839
10	0.864039	12723.47689	52.70994842
11	-0.35921	5431.125393	58.42683354
12	0.150478	2133.292224	58.10842815
13	-0.04556	776.099167	69.72634572
14	0.024457	262.969187	46.31420882
15	0.001119	83.38693102	95.42300846
16	0.008412	24.84935585	651.5128964
17	0.006267	6.98484617	25.49809072

18	0.006863	1.857987725	9.506874451
19	0.006706	0.469073826	2.284613709
20	0.006746	0.112691562	0.584507158

Series 2				
term	**$e^{(5)}$**	**$e^{-5} = 1/e^{(5)}$**	**True error %**	**Relative Error %**
0	1	1	14741.31591	-
1	6	0.166666667	2373.552651	83.33333333
2	18.5	0.054054054	702.2332923	67.56756757
3	39.33333	0.025423729	277.3215909	52.96610169
4	65.375	0.015296367	127.0182166	39.83428936
5	91.41667	0.010938924	62.34803181	28.48678213
6	113.1181	0.008840322	31.20200692	19.18472589
7	128.619	0.007774898	15.38972013	12.05186351
8	138.3072	0.007230283	7.306918069	7.004785221
9	143.6895	0.006959453	3.287438498	3.745778597
10	146.3806	0.006831506	1.38854332	1.838457034
11	147.6038	0.006774891	0.548299103	0.828736847
12	148.1135	0.006751577	0.20229355	0.344118753
13	148.3096	0.006742653	0.069847737	0.132178424
14	148.3796	0.006739472	0.022630474	0.047184306
15	148.4029	0.006738412	0.006901287	0.015725629
16	148.4102	0.006738081	0.00198693	0.004914017
17	148.4124	0.006737983	0.000541623	0.001445278
18	148.413	0.006737956	0.000140156	0.000401465
19	148.4131	0.006737949	3.45078E-05	0.000105648
20	148.4131	0.006737948	8.09568E-06	2.64121E-05