1. Solve item a, b, and c of problem 18.9 (**textbook page 506**) and compare the per cent *relative* errors.

a. Analytical solution as follows:

$$\frac{x^2}{1+x^2} = 0.85 \;; \quad \Rightarrow \quad x = 2.381$$

b. Cubic interpolation:-

x	y
1	0.5
2	0.8
3	0.9
4	0.9411

Using the following law;

$$F(x_1, x_0) = \frac{x_1 - x_0}{y_1 - y_0}$$

From which the value of **x** can be determined; x= 2.291

The relative error will be

$$\frac{2.381 - 2.291}{2.381} = \mathbf{0.038}$$

c. Quadratic interpolation

x	F(x)
2	0.8
3	0.9
4	0.9411

From this one, x = 2.428

The relative error is

$$\left|\frac{2.381 - 2.428}{2.381}\right| = \mathbf{0.020}$$

OK.

2. Use *quadratic interpolation* to estimate *logarithm of* 7.5 *to base* 10 [i.e.: log_{10} 7.5], *then*

(a) interpolate between *log* 7 and *log* 8;
(b) interpolate between *log* 7.2 and *log* 7.8;
(c) compute *per cent relative* error ε_t for the results in (*a*) and (*b*) *relative to the true value.*
(d) plot (using Excel) and compare the *interpolating*-function versus the exact *log*-function within the range from 5 to 8.
(e) Comment on the suitability of the interpolating function.

Solution:-

Using this formula, $f_1(x) = f(x_0) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x - x_0)$, true value log 7.5 =0.8750612

∴ a) between log7=0.8950980 and log8=0.90308998 ⇒ log7.5 ;

$X = 7.5$, and $x_1 = 8, and\ x_0 = 7$

$$f(7.5) = 0.895098 + \frac{0.90308998 - 0.895098}{8-7}(7.5-7)$$

$$\boldsymbol{f(7.5) = 0.8740939}$$

b. Repeat the same way between log7.2 and log7.8=0.7403627 ⇒ log5 ;

$$\boldsymbol{f(7.5) = 0.8747135611}$$

c) the calculation of the relative errors in both a and b as follows:

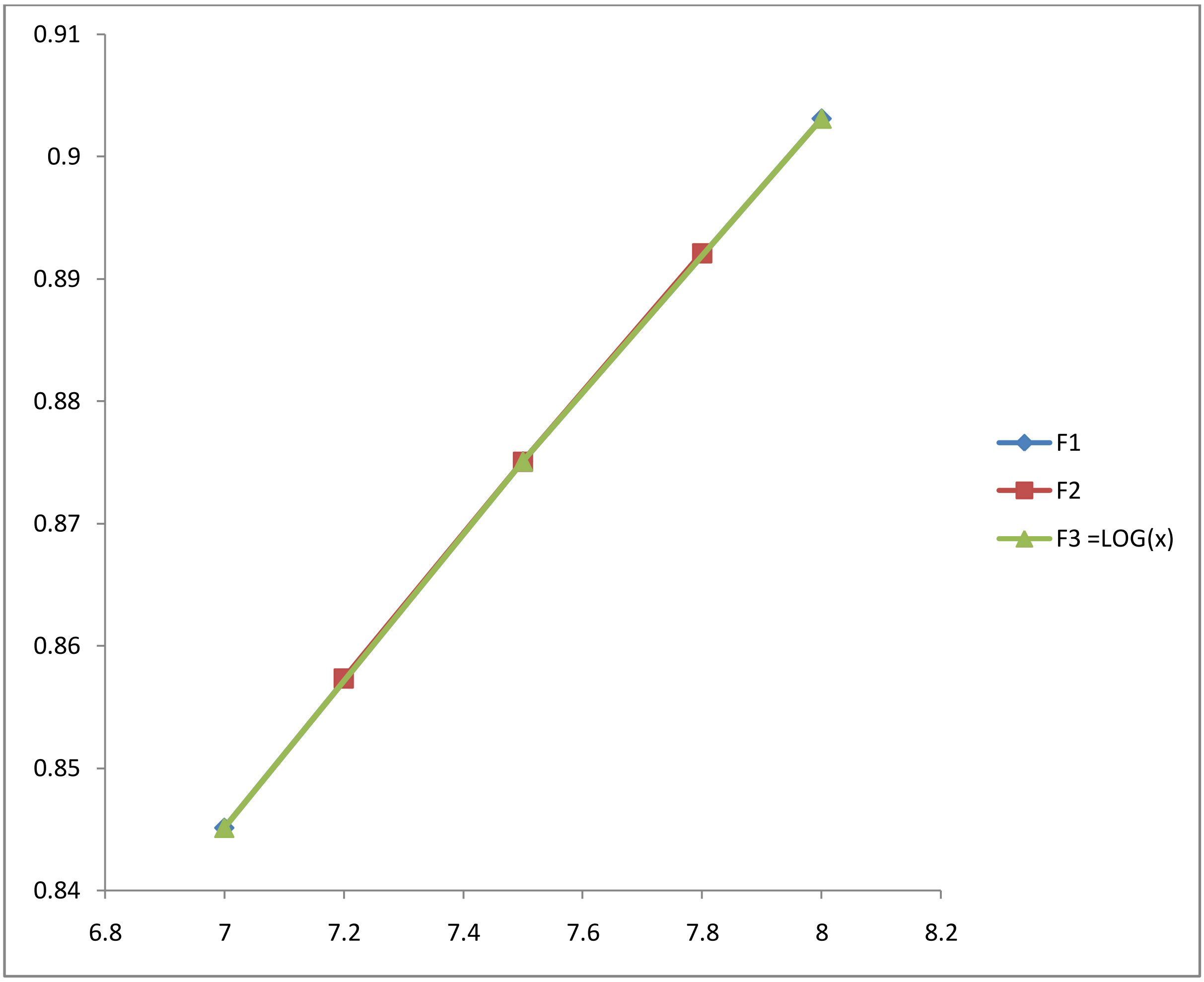
$$\Rightarrow a)\boldsymbol{\varepsilon_t} = \frac{0.8750612 - \mathbf{0.8740939}}{0.8750612} * \mathbf{100} = \mathbf{0.1105}\%$$

$$b)\boldsymbol{\varepsilon_t} = \frac{0.8750612 - \mathbf{0.8747135611}}{0.8750612} * \mathbf{100} = \mathbf{0.0397}\%$$

d***) see the next page***

e) the relative error in case b is smaller than the relative error in case a which indicates that when the two points are more close to the calculation point the relative is becoming more smaller which is logically correct.

F1		F2		F3	
x	**f(x)**	**x**	**f(x)**	**x**	**f(x)**
7	0.84509804	7.2	0.857332	7	0.845098
7.5	0.875061263	7.5	0.875061	7.5	0.875061
8	0.903089987	7.8	0.892095	8	0.90309



3. Develop the *cubic spline* for the following given three data points:

x	1.	2.5	4.0
$f(x)$	-2.5	5.5	7.5

Then use the derived *spline to: (a) predict* $f(2.5)$; *(b) verify* $f(3)$.

Solution:-

$x_0 = 1, \quad f(x_0) = -2.5$

$x_1 = 2.5, \quad f(x_1) = 5.5$

$x_2 = 4, \quad f(x_2) = 7.5$

Using the following formula;

$$(x_i - x_{i-1})f''(x_{i-1}) + 2(x_{i+1} - x_{i-1})f''(x_i) + (x_{i+1} - x_i)f''(x_{i+1}) =$$
$$\frac{6}{(x_{i+1} - x_i)}[f(x_{i+1}) - f(x_i)] + \frac{6}{(x_i - x_{i-1})}[f(x_{i-1}) - f(x_i)]$$

Now substitute the given values above;

$$(2.5 - 1)f''(1) + 2(4 - 1)f''(2.5) + (4 - 2.5)f''(3) =$$
$$\frac{6}{(4 - 2.5)}[7.5 - 5.5] + \frac{6}{(2.5 - 1)}[-2.5 - 5.5]$$

$\because f''(1) = f''(4); \textit{first and end points}$, from the above equation we can find the corresponding value of $f''(2.5) = -4$

Now we can use this formula to get the required equations as follows:

$$f_i(x) = \frac{f_i''(x_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{f_i''(x_i)}{6(x_i - x_{i-1})}(x - x_{i-1})^3 +$$
$$\left[\frac{f(x_{i-1})}{(x_i - x_{i-1})} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x)$$
$$+ \left[\frac{f(x_i)}{(x_i - x_{i-1})} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})$$

By substitution in that formula at one and two, the following results will be provided.

$f_1(x) = -0.444(x - 1)^3 - 1.6667(2.5 - x) + 4.667(x - 1) - - - - (1) \qquad 1 \le x \le 2.5$

$f_2(x) = -0.444(4 - x)^3 + 4.667(4 - x) + 5(x - 2.5) - - - - (2) \qquad 2.5 \le x \le 4$

$\Rightarrow \boldsymbol{f_2(2.5) = 5.5}$

$\boldsymbol{f_2(3) = 4.223 = f(3) \quad o.k}$

4. Use *least-squares regression* to fit a straight line to the following data:

x	1	2	4	6	3	5	7	9
y	-1	3.5	5.7	2.1	8.1	3.2	5.5	-5

Then use the *linear fit* to:

(a) *determine slope and intercept;*
(b) *determine the standard error; and*
(c) *compare the linear fit to the plot of data.*

Solution:-

a) The slope is a_1 and a_0 is the intercept. n=8;

$$a_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2};\ a_0 = \bar{y} - a_1\bar{x}$$

$$\sum x_i = 37; \sum y_i = 22.1; \sum x_i y_i = 75.2;\ \sum x_i^2 = 221;$$

$$\bar{x} = \frac{\sum x_i}{n} = 4.625; \bar{y} = \frac{\sum y_i}{n} = 2.7625$$

$$\mathbf{a_1} = \frac{8 * 75.2 - 37 * 22.1}{8 * 221 - (37)^2} = \mathbf{-0.5416}, \boldsymbol{the\ slope};$$

$$\mathbf{a_0} = 2.7625 + 0.5416 * 4.625 = \mathbf{5.2674}, \mathbf{the\ intercept}$$

b) The standard errors:-

$$S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

$$S_{y/x} = \sqrt{\frac{S_r}{n-2}}\quad ; by\ substitution\ with\ all\ values, \Rightarrow S_r = 105.6$$

$$S_{y/x} = 4.19\quad , high\ value.$$