بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING
Second Semester 2011-2012 (112)

CE 305
Structural Analysis I

First Major Examination

Date: Tuesday, March 06, 2012 Avenue: 4-150 Time: 7:00 p.m. Time allowed: 2 hours

Family First
Name: .... Key Solution Number: ....
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Summary of Performance

Problem	Full Mark		Score
1	*a*	5	
	b	5	
	c	10	
2	*a*	15	
	b	5	
3	*a*	10	
	b	5	
	c	10	
4	*a*	10	
	b	10	
	c	15	
Total		**100**	

Notes: ....

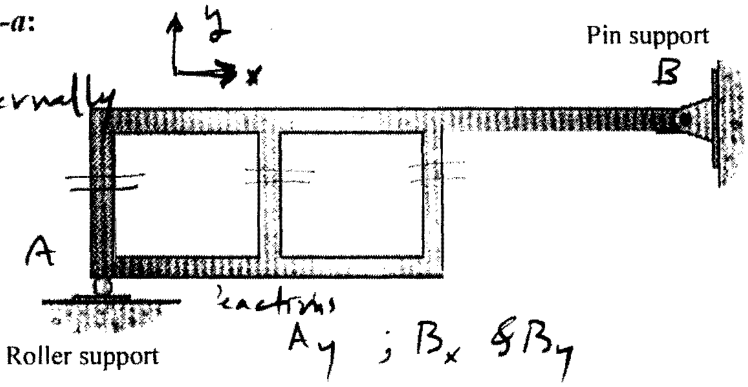
Instructor: Dr. Saeid A. Alghamdi

Problem 1:

1-*a*: Study the *frame* structure shown in Fig P1-*a* and *classify* it completely according to the statical *determinacy* (internal and external), and *its stability*.

Fig. P1-*a*:

S. Determinate externally
S. Indet. internally
Stable structure.



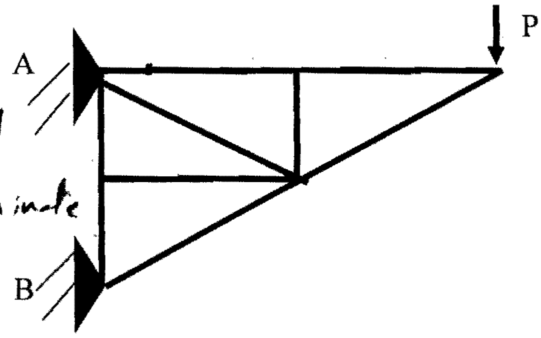
reactions A_y ; B_x & B_y

Externally : $N_r = N_{equ's}$
Internally : using two sub structures $\Rightarrow \Sigma N_{if} + \Sigma N_r - 2\times 3 = 6$

1-*b*: Study the *truss* structure shown in Fig P1-*b* and classify it completely according to the definitions of determinacy (internal and external), if supports A and B are both of pin-type.

Fig. P1-*b*:

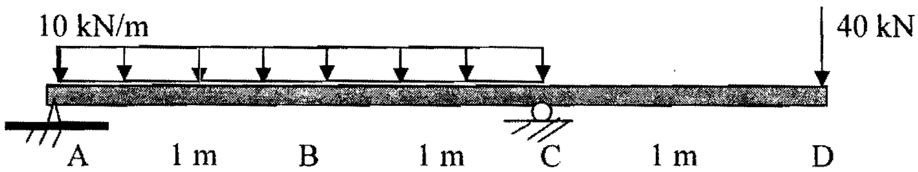
S. Indeterminate externally
S. Indeterminate indeterminate to degree 1



1-*c*: Study the beam ABCD structure shown in Fig P1-*c* and show (only with duagrams) how the *principle of superposition* should be used to simplify computations of internal moment at a cross section at point *B* at center of span AC.

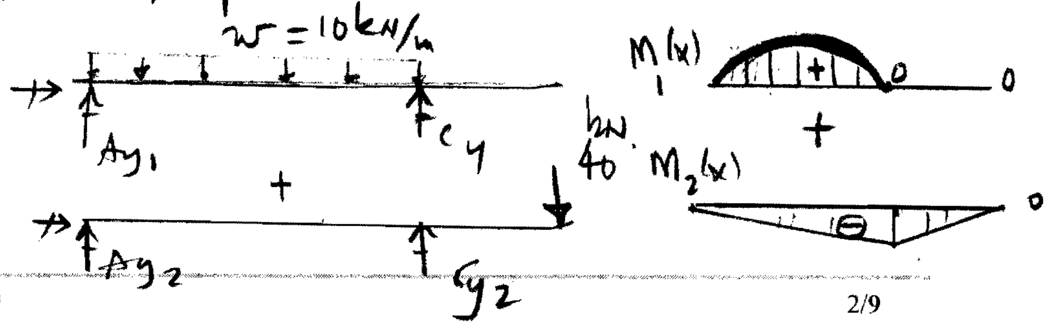
Fig. P1-*c*:

$M_{tot} = M_1(x) + M_2(x)$



consider the separate load cases

$A_y = A_{y1} + A_{y2}$
$C_y = C_{y1} + C_{y2}$



Problem 2:

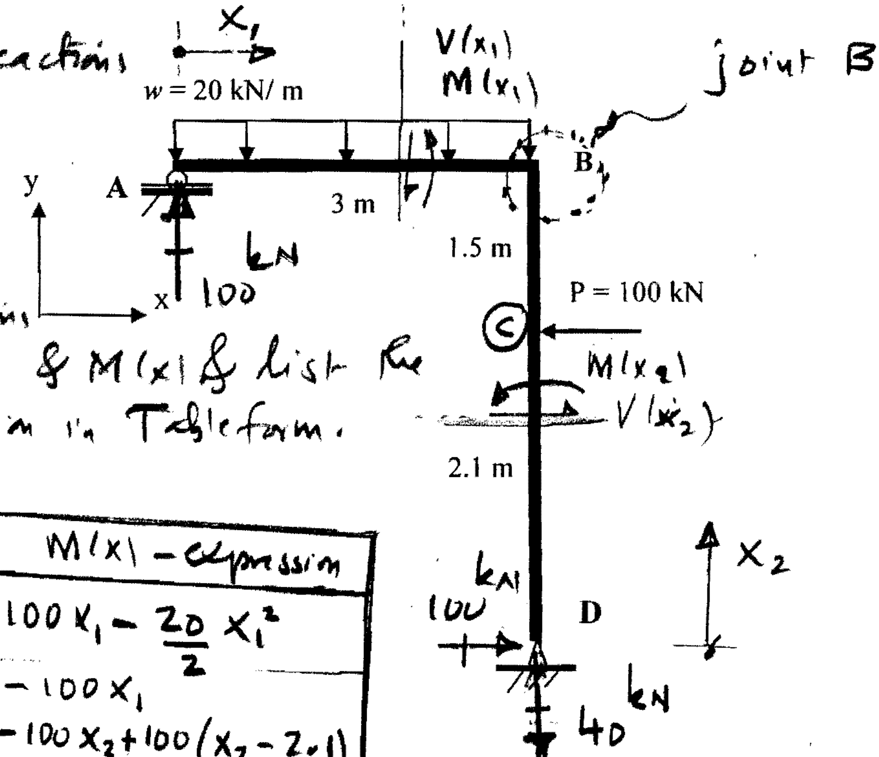
Study the rigid *frame* structure ABCD shown in Fig. P-2, (with roller support at A and pin-support at D) then:

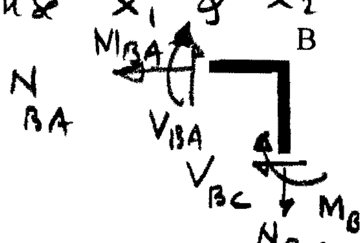
a. draw the SFD and BMD.

b. show clearly the FBD of joint **B** equilibrium and the points of *maximum* and *minimum* shear force and bending moments on the two members AB and BCD.

Fig. P-2: Rigid 2D frame ABCD with given reaction forces A_y= 100 kN and D_y = 40 kN (↓)

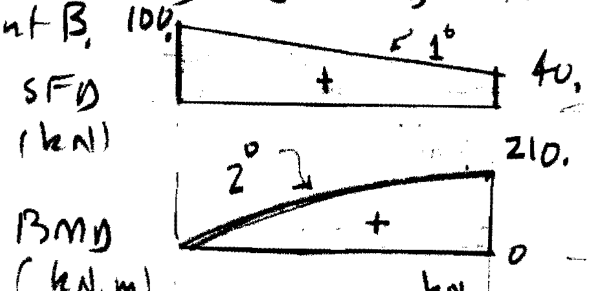
After verifying that reactions given are correct and $D_x = 100$ kN (⟶), use x_1 & x_2 to get expressions for $V(x)$ & $M(x)$ & list the expression in Table form.



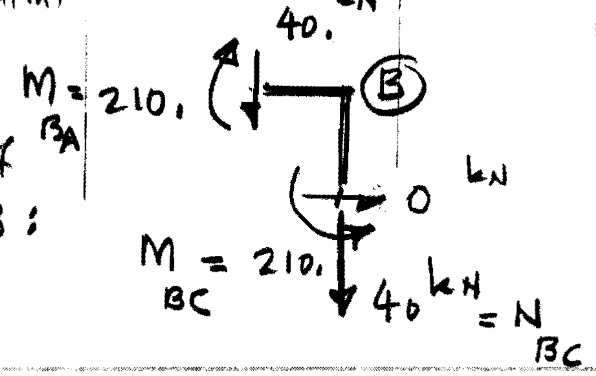


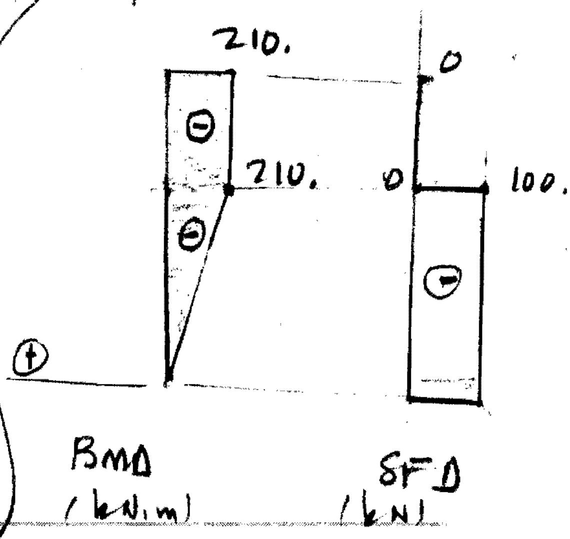
member	$V(x)$ - Expression	$M(x)$ - expression
AB $0 \le x_1 \le 3$ m	$100 - 20x_1$	$100x_1 - \frac{20}{2}x_1^2$
BD $0 \le x_2 \le 2.1$ m	$V_1(x_2) = -100$	$-100x_1$
$2.1 \le x_2 \le 3.6$ m	$V_2(x_2) = 0$	$-100x_2 + 100(x_2 - 2.1)$ $\therefore M(x_2) = -210.$

Draw SFD & BMD for AB & BCD separated from joint B.



Equilibrium of joint B:





Problem 3:

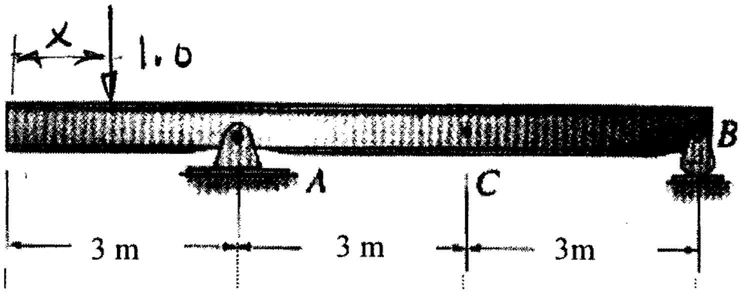
3-*a*: Study the beam structure shown in Fig. P-3*a* and use *Müller Breslau* Principle to draw only *qualitative* Influence Lines (IL-s) for:

a. bending moment at support A.

b. support reaction at B,

c. shear force on a cross section at C.

Fig. P-3*a*:



For unit a load at any x:

IL's

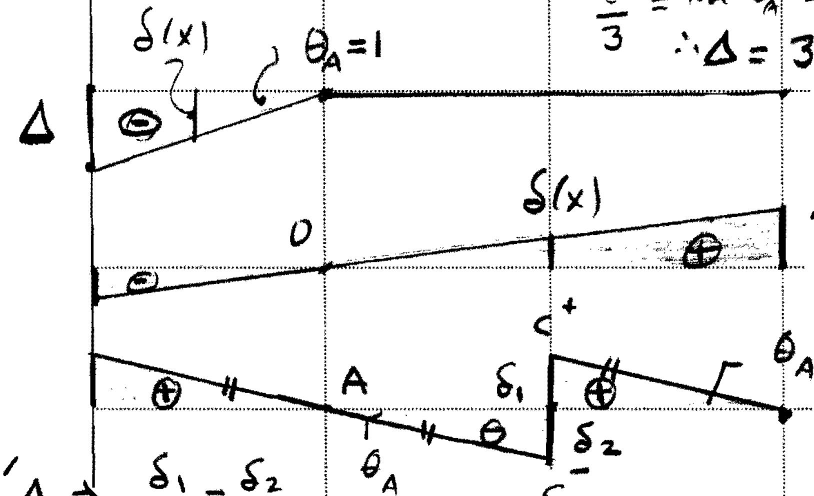
$M_A \propto \delta(x)$

$\frac{\Delta}{3} = \tan\theta_A \simeq 1.0_{(max)}$ $\therefore \Delta = 3 = M_A$

M_A :

B_y :

V_c ;



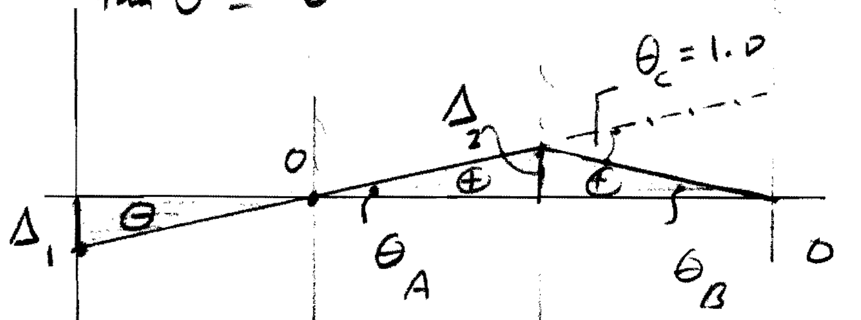
Note $c^+B \,//\, c^-A \Rightarrow \frac{\delta_1}{3} = \frac{\delta_2}{3} \Rightarrow \delta_1 = \delta_2 \Rightarrow \delta_1 = \delta_2 = \Delta = 1/2$

$\delta_1 + \delta_2 = 1$

3-b: Compute the ***ordinates*** of the IL-diagram of bending moment at C for the beam given in **Fig. P3-*a*** (shown above).

$\tan\theta \simeq \theta$

IL - M_c



$\theta_c = \theta_A + \theta_B$

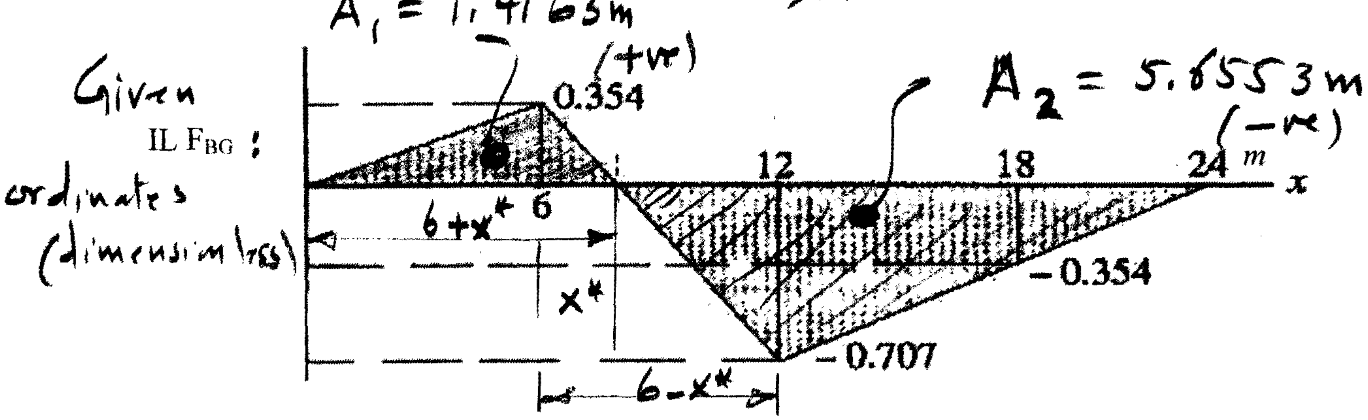
$1.0 = \frac{\Delta_2}{3} + \frac{\Delta_2}{3} \Rightarrow 1.0 = \frac{2\Delta_2}{3} \Rightarrow \Delta_2 = 3/2$

$\therefore \frac{\Delta_1}{3} = \frac{\Delta_2}{3} \Rightarrow \Delta_1 = \Delta_2 = 3/2.$

3-c: The IL-diagram shown in **Fig. P3-*c*** is for memebr normal force F_{BG} in a given truss structure that has a span of 24 meters. If the truss is to be subjected to Uniform Dead Load UDL=100 kN/m, and Live *Moving* Loads of ULML w = 200 kN/m, and Concentarted (LL) P= 150 kN, *compute the maximum **possible negative** force in this truss mamber for the combination of given loads.*

$$\frac{0.354}{x^*} = \frac{0.707}{6-x^*} \Rightarrow x^* = 2.002\text{ m}$$

Fig. P3-*c*:



To compute a max. effect place the loads on the areas that will add to give the same effects.

∴ Place UDL loads every where on areas A_1 & A_2. But place ULL & CLL only on area A_2.

$$\therefore \text{Max } F = UDL*(A_1 + A_2) + CLL*(-0.707) + ULL*(A_2)$$

$$= 100(1.4163 - 5.6553) + 150(-0.707) + 200(-5.6553)$$

$$(F_{BG})_{max} = -1661.\ \text{kN (C).}$$

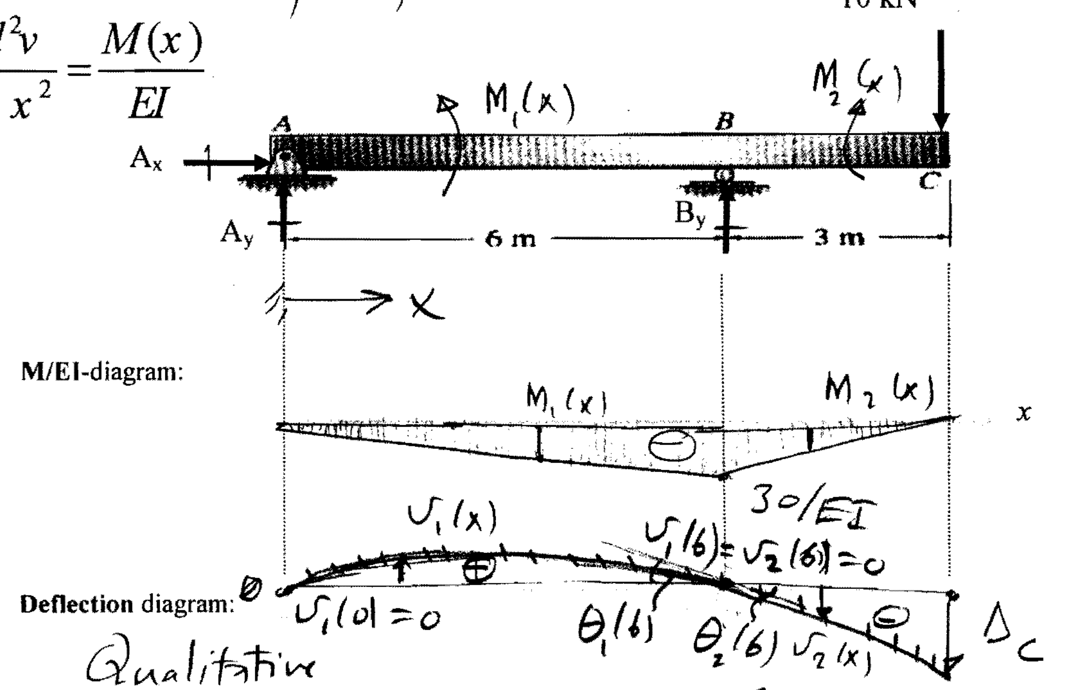
Problem 4:

The bending of a uniform beam (with constant flexure-rigidity EI) structure ABC shown in **Fig. P-4**, is governed by the *moment-curvature differential equation* to determine the dispalcement *v(x)* and the slope *θ(x)*.

Fig. P-4:

$A_x = 0$
$A_y(6) + 10(3) = 0 \Rightarrow A_y = -5.\ kN\ (\downarrow)$
$B_y + A_y - 10 = 0 \Rightarrow B_y = 15.\ kN\ (\uparrow)$

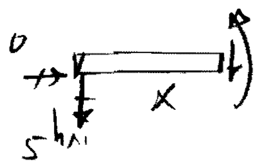
$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI}$$



Qualitative satisfying B.C.'s at A & B.
$\Delta_A = \Delta_B = 0$; $\theta_A \neq 0$ & $\theta_B \neq 0$.

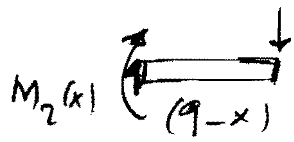
4-*a*: Compute the supports reactions and draw the *curvature* diagram (M/EI-diagram) and a qualitative *deflected shape. Then write the differential* equations for the two-segments (namely: AB and BC) and specify *clearly* the *boundary conditions (BCs)* and the *continuity* onditions (CCs).

$\overset{+}{\rightarrow} \sum F_x = 0$: $A_x = 0$; $(\circlearrowleft \sum M_B = 0 \Rightarrow A_y = -5\ kN (\downarrow)$
$+\uparrow \sum F_y = 0 \Rightarrow B_y = 15.\ kN\ (\uparrow)$



$\Rightarrow M_1(x) + 5x = 0$

$\therefore M_1(x) = -5x$ (for: $0 \le x \le 6$ m)



$M_2(x) + 10(9-x) = 0$

$M_2(x) = -10(9-x)$ (for: $6 \le x \le 9$ m)

Note : $M_1(x) = M_2(x)$ (at $x = 6$ m) $= -30$ kN.m

$EI\dfrac{d^2v_1(x)}{dx^2} = -5x$ (for $0 \le x \le 6$ m) & $EI\dfrac{d^2v_2(x)}{dx^2} = -10(9-x)$ (for $6 \le x \le 9$ m).

B.C.'s: $v_1(0) = 0$ & $v_2(6) = 0$

C.C.'s: $v_1(6) = v_2(6)$ & $\theta_1(6) = \theta_2(6)$

4-*b*: Based on the beam and loading shown in **Fig. P-4** (above) with constant E =210 GPa and moment of inertia I = 80(10^6) mm^4, *use the method of integration* to compute the *slope* θ(*x*) at point A of the beam..

$EI = 210\times10^9 \times 80(10)^6 \times 10^{-12}$ N.m^2 $= 1.68\times10^7$ N.m^2

$0 \le x \le 6$ m | $6 \le x \le 9$ m

$EI\,\theta_1(x) = -5\frac{x^2}{2} + C_1$ | $EI\,\theta_2(x) = \frac{10}{2}(9-x)^2 + C_3$

$EI\,v_1(x) = -5\frac{x^3}{6} + C_1 x + C_2$ | $EI\,v_2(x) = -\frac{10}{6}(9-x)^3 + C_3 x + C_4$

To Evaluate constants C_1, C_2, C_3 & C_4 use the given B.C. & C.C.'s:

B.C.'s:

$v_1(0) = v_A = 0 \Rightarrow 0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$

$v_2(6) = v_B = 0 \Rightarrow 0 = -\frac{10}{6}(3)^3 + C_3(6) + C_4$

$6C_3 + C_4 = \frac{27}{6}(10)$ --- (1)

C.C.'s:

$v_1(6) = v_2(6) \Rightarrow -5\frac{(6)^3}{6} + C_1(6) = -\frac{10}{6}(3)^3 + C_3(6) + C_4$

$\theta_1(6) = \theta_2(6) \Rightarrow -5\frac{(6)^2}{2} + C_1 = \frac{10}{2}(3)^2 + C_3$

$-135 = -6C_1 + 6C_3 + C_4$ --- (2)

$-135 = -C_1 + C_3$ --- (3)

$45 = 6C_3 + C_4$ --- (1)

Using Eqn (1) with (2) $\Rightarrow$ $-135 = -6C_1 + 45 \Rightarrow C_1 = 30.$

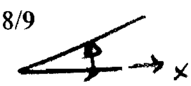
From Eqn (3) $\Rightarrow$ $C_3 = -135 + C_1 \Rightarrow C_3 = -105$

From Eqn (1) $\Rightarrow$ $C_4 = 45 - 6C_3 \Rightarrow C_4 = 675$

$\therefore\ \theta_1(x) = \frac{1}{EI}\left[-\frac{5x^2}{2} + C_1\right] \Rightarrow \theta_1(x) = \frac{1}{EI}\left[-\frac{5x^2}{2} + 30\right]$

$\theta_A = \theta_1(0) = \frac{1}{EI}\left[-\frac{5(0)^2}{2} + 30\right] = \frac{30}{EI} = \frac{30 \text{ kN.m}^2}{1.68\times10^4 \text{ kN.m}^2}$

$\therefore\ \theta_A = +0.001786$ rad (ccw)



4-c: Use the *moment area-theorem* to compute the *displacement v(x)* at point C of the beam given in **Fig. P-4**.

Note: the moment area theorems used to compute the ***changes* in slope and deflection** between any two points 2 and 1 (based on the tangents at the two points) are:

Units : $\frac{M(x)\,dx}{EI} = \frac{(kN.m)\,m}{\frac{kN}{m^2}\cdot m^4} = xx$ (rad)

$\frac{\bar{x}\,M(x)\,dx}{EI} = \frac{m\,(kN.m)\,m}{kN.m^2} = xx\,(m)$

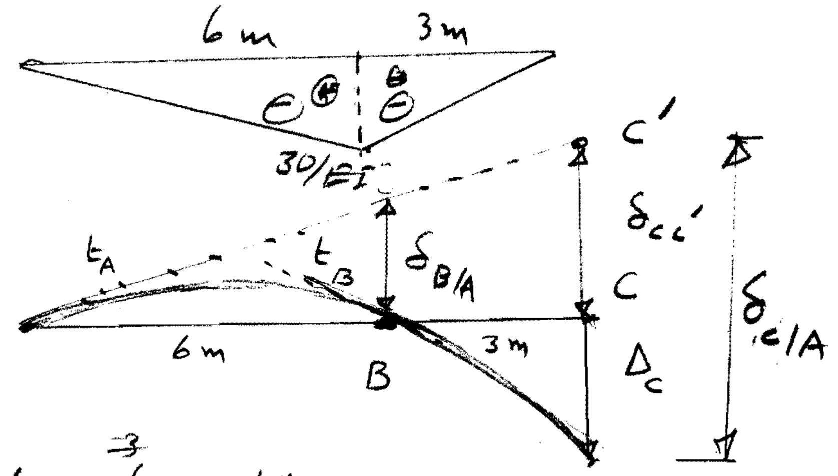
$$\theta_{2/1} = \int \frac{M(x)}{EI}dx\,;$$

$$v_{2/1} = \int \bar{x}\,\frac{M(x)}{EI}\,dx\,.$$

Draw the curvature diagram.

M / EI.

$\Delta_C \equiv v_2(9)$



$$\delta_{B/A} = \frac{30}{EI} * \frac{6}{2} \times \frac{6}{3} = 180/EI$$

$$\frac{\delta_{cc'}}{9} = \frac{\delta_{B/A}}{6} \Rightarrow \delta_{cc'} = \frac{9}{6}\,\delta_{B/A} = \frac{3}{2}\left(\frac{180}{EI}\right) = \frac{270}{EI}$$

$$\delta_{c/A} = \frac{30}{EI}\left(\frac{3}{2}\right)\left(3\times\frac{2}{3}\right) + \frac{30}{EI}\left(\frac{6}{2}\right)\left(3+\frac{6}{3}\right)$$

$$= \frac{90}{EI} + \frac{90}{EI}\left(\frac{15}{3}\right) = 540/EI$$

$$\therefore\ \Delta_c = \delta_{cc'} - \delta_{c/A} = \frac{270}{EI} - \frac{540}{EI}$$

$$= -\frac{270}{EI}\ (\downarrow)$$

Note : Using previous results from integration $\Rightarrow$

$$\Delta_c = v_2(9) = \left[-\frac{10}{6}(9-9)^3 - 105(9) + 675\right]\frac{1}{EI}$$

$$= -\frac{270}{EI}\,kN.m^3 = \frac{-270\ kN.m^3}{1.68\times10^4\ kN.m^2}$$

$$\therefore\ \Delta_c \cong 16.1\ mm\ (\downarrow)$$