King Fahd University of Petroleum & Minerals
Civil & Environmental Engineering Department

CE318 Numerical & Statistical Methods in Civil Engineering
Lab 2: Introduction to Mathematica

Objectives

To introduce the basic computational and graphical functions in Mathematica

1- Arithmetic Operations

x + y + z	Add
−x	Subtract
x / y	Divide
x ^ y	Raise to a power
x y z or **x * y * z**	Multiply
Order operations with parentheses :	
8 x^1 / 3	$= \frac{8x}{3}$
8 x^(1 / 3)	$= 8x^{1/3}$

Exercises (try different operations)

2 - Exact vs. Approximate Results

Mathematica can give exact results for numerical calculations, without round-off error. For example, it will compute the sum of two fractions exactly:

1 / 3 + 2 / 7

$$\frac{13}{21}$$

53 ^ 4 / 89 ^ 12

$$\frac{7\,890\,481}{246\,990\,403\,565\,262\,140\,303\,521}$$

You can tell *Mathematica* to give you an approximate numerical result in scientific notation by using N :

```
N[1 / 3 + 2 / 7]
0.619048
```

OR :

```
(1 / 3 + 2 / 7) // N
0.619048
```

If you mix an exact number like **2/7** with an inexact number like **0.1** in a calculation, *Mathematica* will round off the calculation to the machine precision, and furthermore will only display 6 significant figures of the result:

```
2 / 7 + 0.1
0.385714
```

You can perform the evaluation to arbitrary numerical precision. For example, to obtain a precision of 25 significant figures, one adds a second argument to the function **N** that specifies the precision of the evaluation:

```
Sqrt[2]
```

$\sqrt{2}$

```
Sqrt[2.]
1.41421
```

```
N[Sqrt[2], 20]
1.4142135623730950488
```

```
N[Sqrt[2], 500]
1.414213562373095048801688724209698078569671875376948073\
 176679737990732478462107038850387534327641572735013846\
 230912297024924836055850737212644121497099935831413222\
 665927505592755799950501152782060571470109559971605970\
 274534596862014728517418640889198609552329230484308714\
 321450839762603627995251407989687253396546331808829640\
 620615258352395054745750287759961729835575220337531857\
 011354374603408498847160386899970699004815030544027790\
 316454247823068492936918621580578463111596668713013015\
 61856898 72372
```

3- Some *Mathematica* Built-in Functions

Important : 1 – The arguments of all *Mathematica* functions (and your defined functions) must be enclo
2 – The names of all *Mathematica built – in* functions must begin with *capital letters*.

`Sqrt[x]`	$\sqrt{x}$
`Exp[x]`	e^x
`Log[x]`	$\ln x$ or $\log_e x$
`Log[b, x]`	$\log_b x$
`Sin[x], Cos[x], Tan[x]`	Trigonometric functions (arguments in r
`ArcSin[x], ArcCos[x], ArcTan[x]`	Inverse trigonometric functions
`n!`	Factorial function
`Abs[x]`	$\lvert x \rvert$
`Round[x]`	Closest integer to x

Exampples:

```
Sqrt[2]
```

$\sqrt{2}$

```
Sqrt[2.]
```

1.41421

```
Sqrt[a]
```

$\sqrt{a}$

```
ArcSin[1]
```

$\frac{\pi}{2}$

```
5!
```

120

```
100!
```

93 326 215 443 944 152 681 699 238 856 266 700 490 715 968 264 381 ⋱
621 468 592 963 895 217 599 993 229 915 608 941 463 976 156 518 ⋱
286 253 697 920 827 223 758 251 185 210 916 864 000 000 000 000 ⋱
000 000 000 000

4 - Some Special Numbers

Pi	π (3.14159...)
I	i ($\sqrt{-1}$)
Infinity	∞
Degree	$\pi/180$ (degree to radian conversion factor)

Exampples:

```
Sin[Pi / 2]
1

Cos[Pi]
-1

Exp[-Infinity]
0

Sin[45 Degree]
```
$$\frac{1}{\sqrt{2}}$$
```
Sin[45 * 1 Pi / 180]
```
$$\frac{1}{\sqrt{2}}$$
```
Sqrt[-2]
```
$i\sqrt{2}$
```
I ^ 2
-1
```

5- Defining Variables

```
x = 5
5

y = 12
12
```

```
x
5

x + y
17

x / y
5/12

N[%]
0.416667
```

To clear the value assigned to x, use Remove[x] or x = .

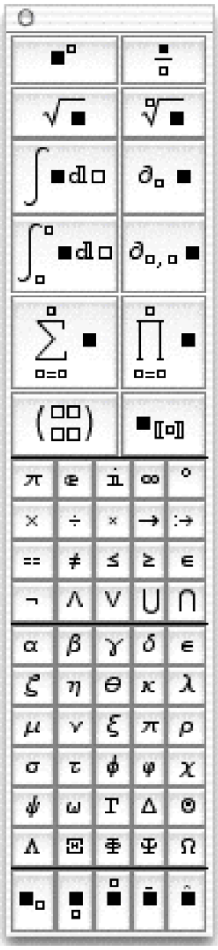
```
x =.

y =.

x
x

y
y

z = 6
6

z ^ 2
36

Remove[z]

z
z
```

If you do not want to have some of the output to be printed, you can end the input by ;

```
x = 1; y = 2;

f = (x + y) ^ 2
9

g = f ^ 2
81

f =.; g =.; x =.; y =.
```

6- Palettes and Keyboard Equivalents

Variables can also be Greek letters or other special characters, such as α or $\wp$. *Mathematica* supports a large collection of special characters that can be accessed using *palettes*. These palettes can also be used to implement various intrinsic functions. Palettes are available in the main menue. There are several palettes, each with different uses. The most useful one is the BasicInput palette. An image of this palette is shown below. To use a palette, simply click on the character or function desired.



Exercises : Try to use the BasicMathInput palette

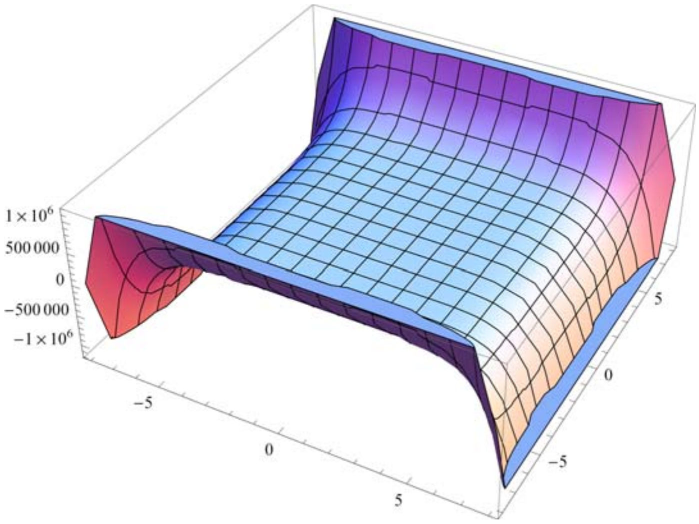
In[2]:= $\int_y^z x\text{^}7\, dx$

Out[2]= $-\frac{y^8}{8}+\frac{z^8}{8}$

In[4]:= `Plot3D[-y^8/8 + z^8/8, {y, -8, 8}, {z, -8, 8}]`

$$\text{Plot3D}\left[-\frac{y^8}{8}+\frac{z^8}{8},\ \{y,-8,8\},\ \{z,-8,8\}\right]$$

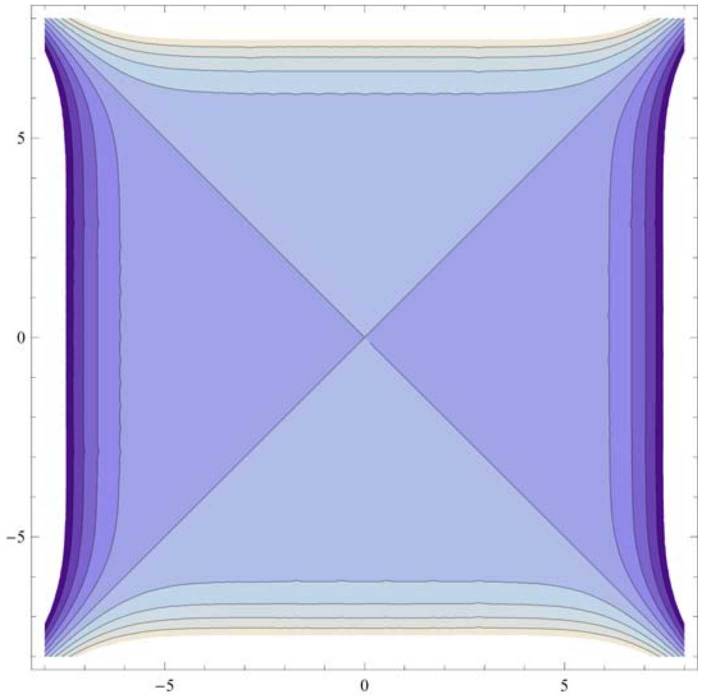
Out[4]=



In[3]:=

$$\text{ContourPlot}\left[-\frac{y^8}{8}+\frac{z^8}{8},\ \{y,-8,8\},\ \{z,-8,8\}\right]$$

Out[3]=



7- Symbolic Computations (Computer Algebra)

Mathematica can do symbolic as well as numerical mathematics:

f = 4 x + 5 – 3 x + 15

20 + x

However, *Mathematica* will not expand out expressions in brackets unless you ask it to do so using the **Expand** function:

```
f = 5 (x + 2 y) + 10 (x - 4 y^2 x - y)
```

$5\,(x+2\,y)+10\,(x-y-4\,x\,y^2)$

```
Expand[f]
```

$15\,x-40\,x\,y^2$

Another way to simplify an expression is to apply the **Factor** command, which tries to factor out all common elements in an expression:

```
Factor[f]
```

$-5\,x\,(-3+8\,y^2)$

There are also several command that are useful when dealing with complex expressions that include terms with numerators and denominators. For instance, the expression

```
f = x / (x^2 + 2) + 3 / (x - 1) + (x^2 + x - 1) / (x^2 + 1) + 5
```

$$5+\frac{3}{-1+x}+\frac{x}{2+x^2}+\frac{-1+x+x^2}{1+x^2}$$

```
Together[f]
```

$$\frac{-2+5\,x-4\,x^2+14\,x^3-x^4+6\,x^5}{(-1+x)\,(1+x^2)\,(2+x^2)}$$

The numerator and denominator of the expression can then be extracted using the functions **Numerator** and **Denominator**:

$$f=\frac{x^2-x+1}{(-1+x)\,(2+x^2)}$$

$$\frac{1-x+x^2}{(-1+x)\,(2+x^2)}$$

```
Numerator[f]
```

$1-x+x^2$

```
Denominator[f]
```

$(-1+x)\,(2+x^2)$

If you really want *Mathematica* to do all of the work (and after all, isn't that what computers are for?), the function **Simplify** will try various tricks to simplify a given expression . The function **FullSimplify** has an even larger bag of simplification tricks, but for this reason takes somewhat longer to run. For example, **Simplify** knows all the trigonometric identities:

$$f = \frac{1}{3\,(1+x)} - \frac{-1+2\,x}{6\,\left(1-x+x^2\right)} + \frac{2}{3\,\left(1+\frac{1}{3}\,(-1+2\,x)^2\right)}$$

$$\frac{1}{3\,(1+x)} - \frac{-1+2\,x}{6\,\left(1-x+x^2\right)} + \frac{2}{3\,\left(1+\frac{1}{3}\,(-1+2\,x)^2\right)}$$

```
Simplify[f]
```

$$\frac{1}{1+x^3}$$

All of the above manipulating and simpilfying commands can be inserted using the AlgebraicManipulation Pallete. Some of them are given below:

Some Useful Functions for Doing Computer Algebra	
Expand[expr]	Multiply out products and powers
Factor[expr]	Reduce to a product of factors
Together[expr]	Put all terms over a common denominator
Apart[expr]	Separate out into terms with simple denominators
Simplify[expr]	Try to simplify an expression
FullSimplify[expr]	Try a larger collection of tricks to simplify an expression
Numerator[expr]	Pick out the numerator of an expression
Denominator[expr]	Pick out the denominator of an expression

8- Replacement

In order to evaluate an expression at specific values of the variables that appear in the expression, one can perform a *replacement*. For example, to evaluate the expression x^2 at $x = 2$, we can replace the variable x by 2 using the notation

```
x^2 /. x → 2
```

4

```
y Cos[x] /. {x → 45 Degree, y → 2}
```

$\sqrt{2}$

$$f = \frac{\text{Sin}[x]}{1+x}$$

$$\frac{\text{Sin}[x]}{1+x}$$

```
f /. x → a
```

$$\frac{\text{Sin}[a]}{1 + a}$$

$$f\ /.\ x \to \frac{a}{b} + c$$

$$\frac{\text{Sin}\left[\frac{a}{b} + c\right]}{1 + \frac{a}{b} + c}$$

```
f = x^2 + y^2
```

$x^2 + y^2$

```
f /. {x → a, y → b}
```

$a^2 + b^2$

9- Defining Functions

`f[x_] = expr`	immediately evaluate **expr** and define **f[x]**
`?f`	Ask for the definition of a function

Let us clear the assignment given to f earlier:

```
f =.
```

To define a function:

```
f[x_] = 1 + x^2
```

$1 + x^2$

```
f[3]
```

10

```
f[a]
```

$1 + a^2$

```
f[a + 3]
```

$1 + (3 + a)^2$

```
f[a + b + c]
```

$1 + (a + b + c)^2$

```
?f
```

Info3537466739-3812581

```
Global`f
```

Info3537466739-3812581

```
f[x_] = 1 + x^2
```

```
g[x_, y_, a_] = x^2 + y^2 + a^2
```

$a^2 + x^2 + y^2$

```
g[1, 2, 3]
```

14

```
g[z, r1, r2]
```

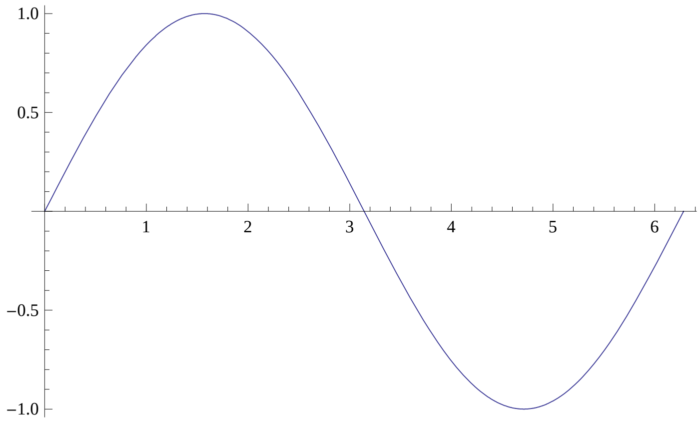
$r1^2 + r2^2 + z^2$

9- Graphics

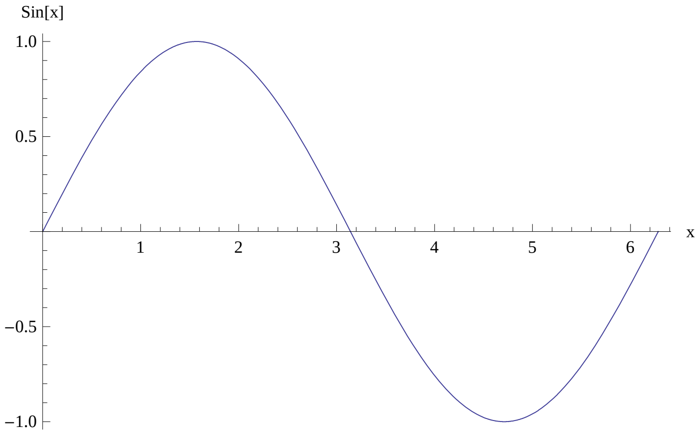
Plot[*f*, {*x, xmin, xmax*}, *option* –> *value*]	Plot *f* as a function of *x* from *xmin* to *xmax*, and add an *option*
Plot[{*f1, f2, , …*}, {*x, xmin, xmax*}]	Plot several functions on the same graph
Show[*plot1, plot2, …*]	Show several plots

To plot one function along x-axis:

```
Plot[Sin[x], {x, 0, 2 Pi}]
```

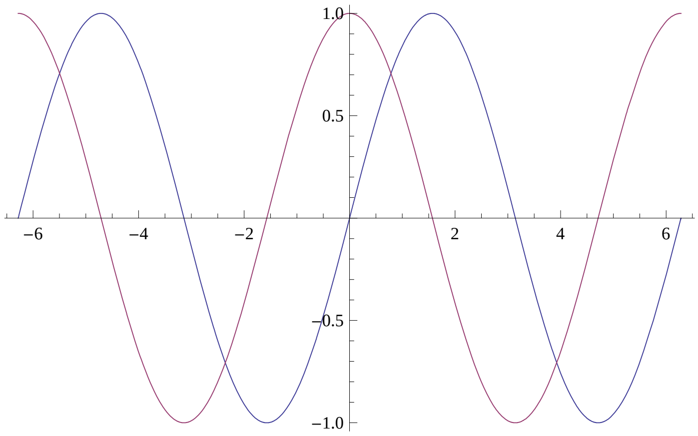


```
Plot[Sin[x], {x, 0, 2 Pi}, AxesLabel → {"x", "Sin[x]"}]
```

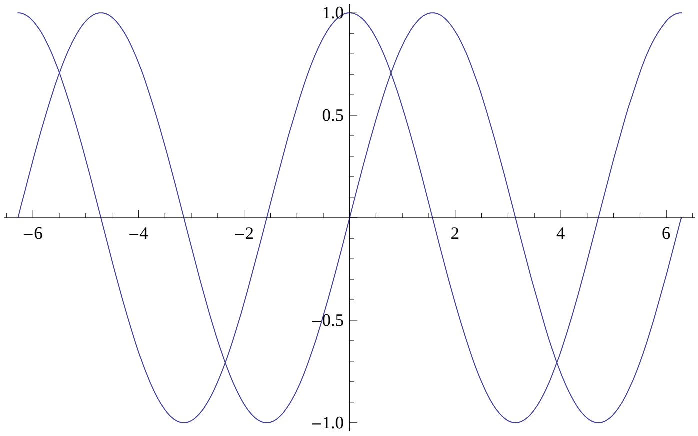


To plot more than one function:

```
Plot[{Sin[x], Sin[ x + Pi / 2]}, {x, -2 Pi, 2 Pi}]
```

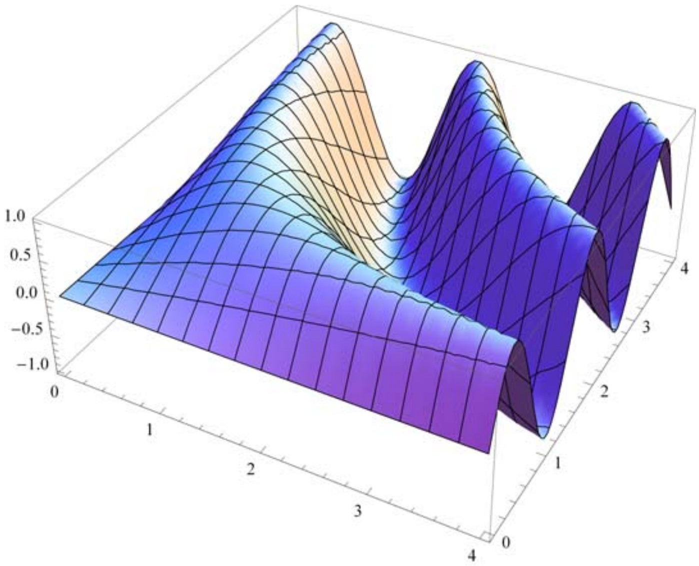



```
p1 = Plot[Sin[x], {x, -2 Pi, 2 Pi}];
p2 = Plot[Sin[ x + Pi / 2], {x, -2 Pi, 2 Pi}];
Show[p1, p2]
```



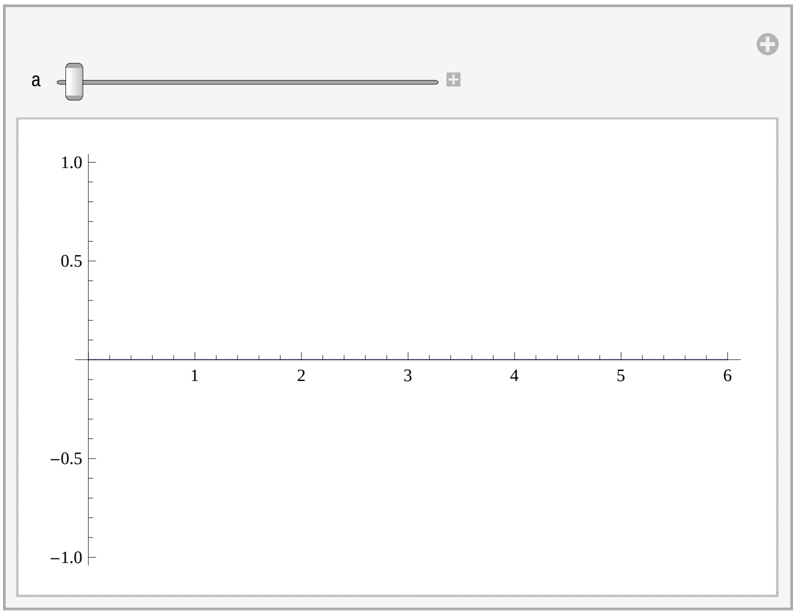
To do 3-D plot:

Plot3D[Sin[x * y], {x, 0, 4}, {y, 0, 4}]

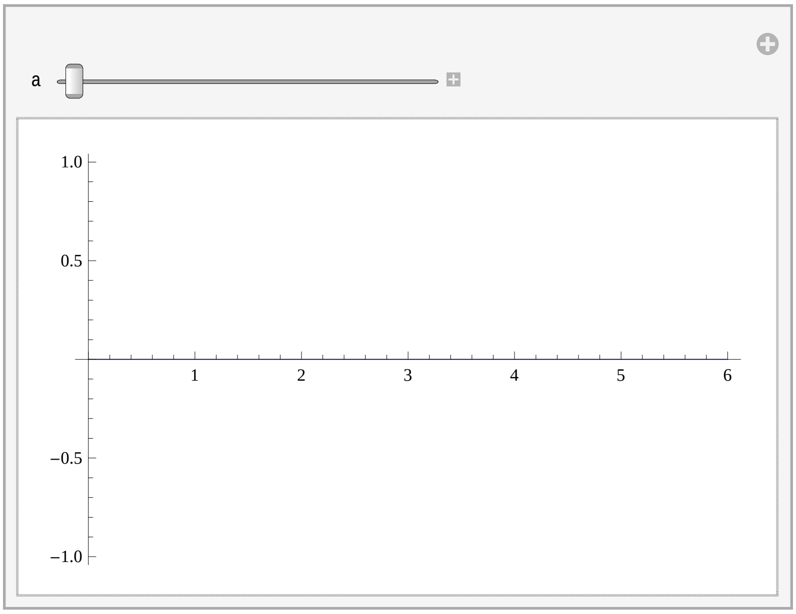


Manipulate[*expr*, {*u*, u_{min}, u_{max}}]	generates a version of *expr* with c
Manipulate[*expr*, {*u*, u_{min}, u_{max}, *du*}]	allows the value of *u* to vary betw
Manipulate[*expr*, {*u*, {u_1, u_2, …}}]	allows *u* to take on discrete valu
Animate[*expr*, {*u*, u_{min}, u_{max}}]	generates an animation of *expr* in
Animate[*expr*, {*u*, u_{min}, u_{max}, *du*}]	takes *u* to vary in steps *du*.
Animate[*expr*, {*u*, {u_1, u_2, …}}]	makes *u* take on discrete values *u*

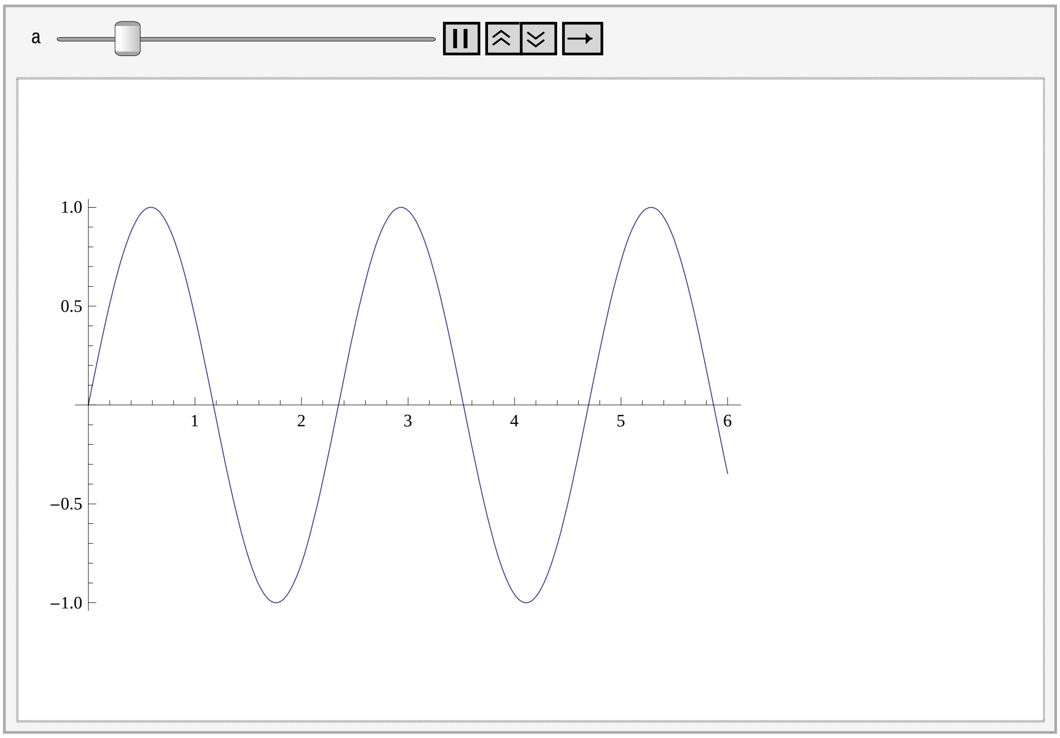
Manipulate[Plot[Sin[a x], {x, 0, 6}], {a, 0, 3}]



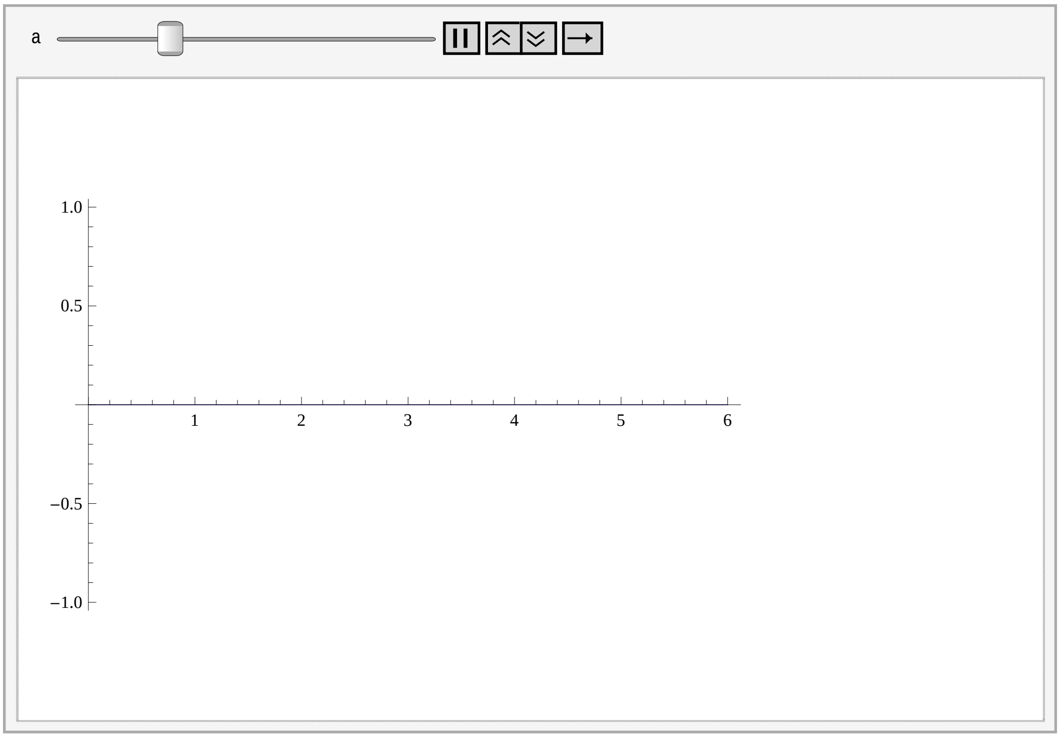
```
Manipulate[Plot[Sin[a x], {x, 0, 6}], {a, 0, 3, 0.2}]
```




```
Animate[Plot[Sin[a x], {x, 0, 6}], {a, 0, 3}]
```



```
Animate[Plot[Sin[a x], {x, 0, 6}], {a, 0, 3, 0.1}]
```

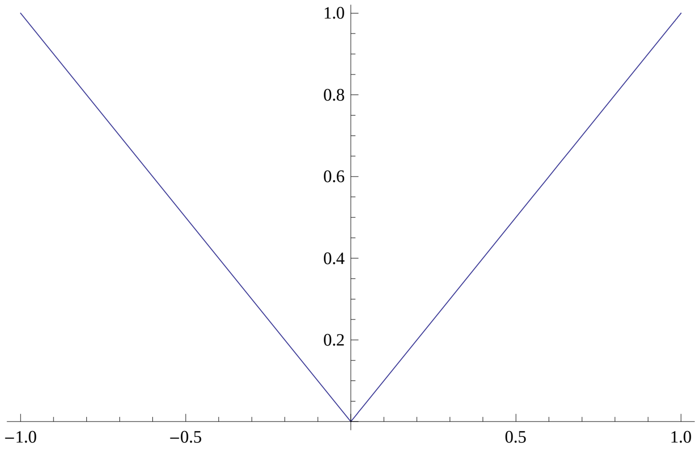


11 - Conditional Statements

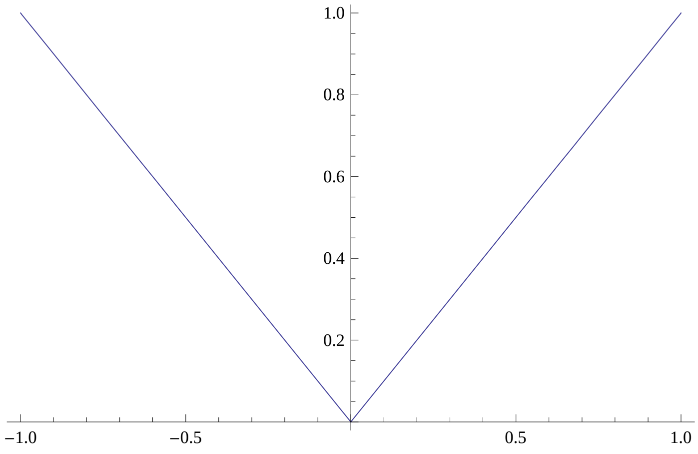
Equal ==	is the logical equal func
And[condition1, condition2, ...] && condition1 && condition2	is the logical AND functi giving False immediately
Or[condition1, condition2, ...] || condition1 || condition2	is the logical OR functio giving True immediately
If[*condition*, *t*, *f*]	gives *t* if *condition* evaluates to Tru
Piecewise[{{val_1, $cond_1$}, {val_2, $cond_2$}, ...}]	represents a piecewise function wi

```
Equal[1, 2]  (*   Is 1 equal 2?   *)
False
```

```
1 == 2 (*    Is 1 equal 2?    *)
False

1 == 1 (*    Is 1 equal 1?    *)
True

And[1 < 2, 1 > 0]
(*    Is 1 less than 2 and greater than 0?    *)
True

f1 = If[x < 0, -x, x] (*    If x<0 ,
 f1=-x, otherwise f1=x    *)
If[x < 0, -x, x]

Plot[f1, {x, -1, 1}]
```



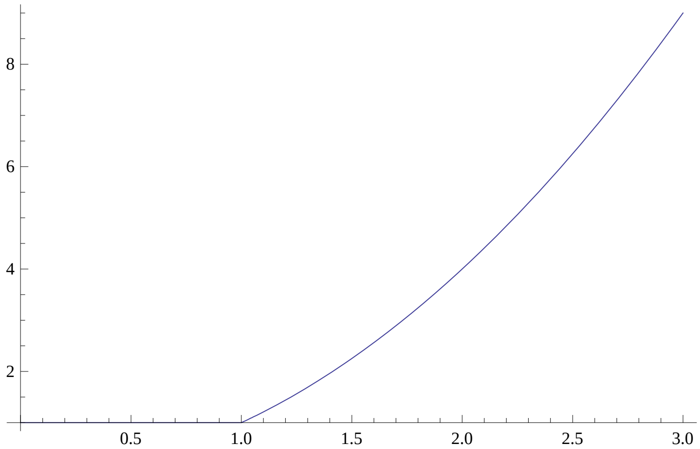

```
f2[x_] = If[x < 0, -x, x]
If[x < 0, -x, x]

Plot[f2[x], {x, -1, 1}]
```



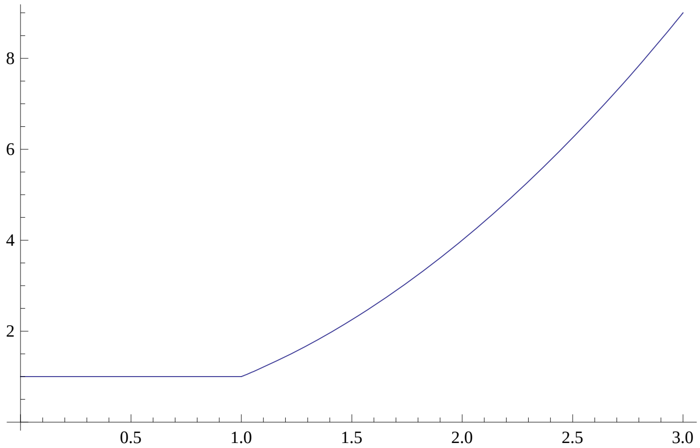
```
u[x_] = If[x > 0 && x < 1, 1, x^2]
 (*    If  0<x<1  ,u[x]=1,  otherwise  u[x]=x^2     *)
```

$If[x > 0 \&\& x < 1, 1, x^2]$

```
Plot[u[x], {x, 0, 3}]
```



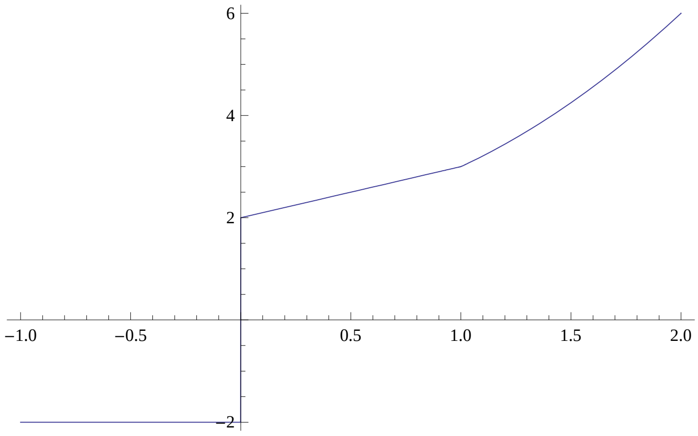
```
Plot[u[x], {x, 0, 3}, AxesOrigin → {0, 0}]
```



```
u[x_] = If[x < 0, -2, If[x > 0 && x < 1, x + 2, x^2 + 2]]
              x<0        then  u[x]=-2
 (*  If      0<x<1      then  u[x]=x+2        *)
          otherwise        u[x]=x^2+2
```

$If[x < 0, -2, If[x > 0 \&\& x < 1, x + 2, x^2 + 2]]$

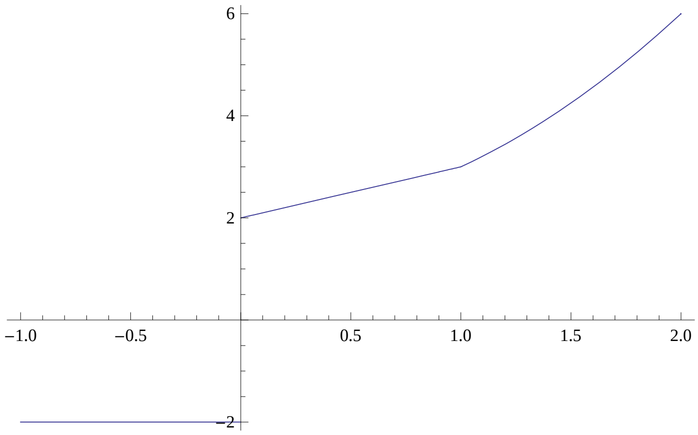
```
Plot[u[x], {x, -1, 2}]
```




```
w[x_] = Piecewise[
  {{-2, x < 0}, {x + 2, x > 0 && x < 1}, {x^2 + 2, x > 1}}]
```

$$\begin{cases} -2 & x < 0 \\ 2 + x & x > 0 \&\& x < 1 \\ 2 + x^2 & x > 1 \\ 0 & \text{True} \end{cases}$$

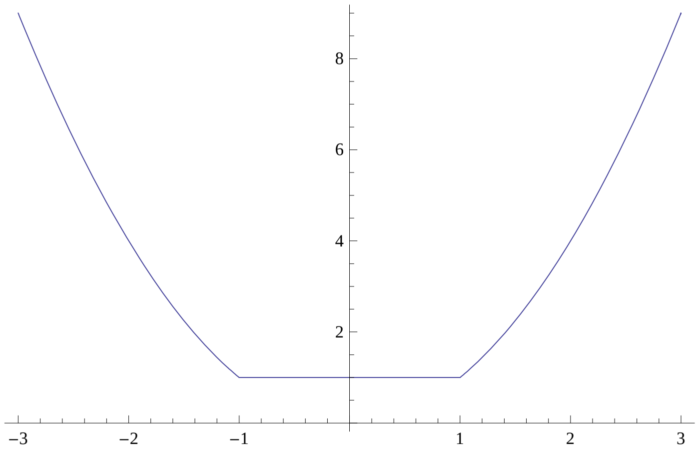
```
Plot[w[x], {x, -1, 2}]
```




```
f[x_] = If[x < -1 || x > 1, x^2, 1]
```

$\text{If}\left[x < -1 \;||\; x > 1,\, x^2,\, 1\right]$

```
Plot[f[x], {x, -3, 3}]
```



Exercise : Redo Problem 2 of Lab Assignment 1 (Textbook problem 2.22)

2.22 A simply supported beam is loaded as shown in Fig. P2.22. Using singularity functions, the displacement along the beam can be expressed by the equation:

$$u_y(x) = \frac{-5}{6}[\langle x-0\rangle^4 - \langle x-5\rangle^4] + \frac{15}{6}\langle x-8\rangle^3 + 75\langle x-7\rangle^2 + \frac{57}{6}x^3 - 238.25x$$

By definition, the singularity function can be expressed as follows:

$$\langle x-a\rangle^n = \begin{Bmatrix} (x-a)^n & \text{when } x > a \\ 0 & \text{when } x \le a \end{Bmatrix}$$

Develop a program that creates a plot of displacement versus distance along the beam x. Note that $x = 0$ at the left end of the beam.

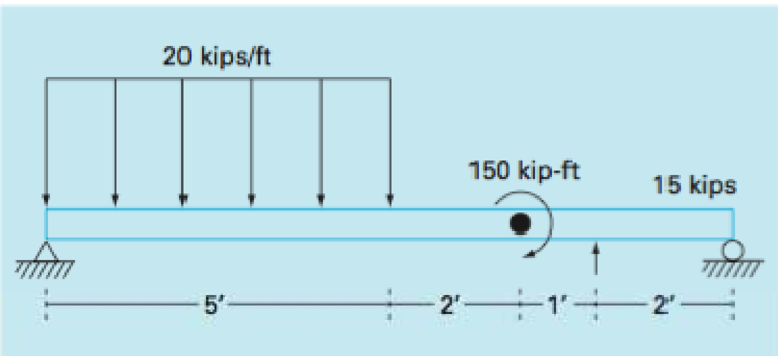


Figure P2.22

Lab Assignment #2: Check the blackboard page of the lab