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DEPARTMENT OF CIVIL & ENVIRONMENTAL ENGINEERING – KFUPM
Numerical and Statistical Methods in Civil Engineering
CE 318- 53-'15 (151)

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Comp. Lab. Session   NO. 10

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Subj.: Solution of a system of linear equations A x = b, where matrix A is a symmetric matrix

DATE: Nov. 25, '15

Objectives are to:

To Use IMSL subroutine, and Mathematica software to find the solution of an ordinary differential equation (s) with specified conditions.

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1. Use **IMSL**-subroutine (IVPRK) to the following system of ODEs (Ref. textbook pages 808-827):

$$\frac{dx}{dt} = 1.2x - 0.6xy$$

$$\frac{dy}{dt} = -0.8y + 0.3xy$$

With the initital conditions (ICs): $x = 2$ and $y = 1$ at $t = 0$,.
Hint: 1) create a **Function subprogram** to define the Differential Equations (DEs), and then 2) call the **IMSL IVPRK** *subprogram* through a main driver program.

2. Use Mathematica software to solve the following ordinary differential equation with two different mesh sizes (namely: using different number of grid points).

$$x'' + \frac{1}{t}x' + \left(1 - \frac{1}{4\,t^2}\right) x = \sqrt{t}\,\cos(t)$$

over $[1, 6]$ with $x(1) = 1.0$ and $x(6) = -0.5$.

Provide a print-out of comparison of the numerical results obtained with 26 and 51 grid points.

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Procedure for Lab.-Report Evaluation:
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1. Start working in the assigned session, then complete your computer works *preferably* within the session *or* shortly afterwards using the same computing machine on which you may save your work for future use (if necessary).
2. Submit for evaluation your summary of organized computer work assignment in the beginning of the next lab.
3. Your report *should* include: i)**Introduction** expalining the work undertaken and its main objectives; ii) Clear outline of the numerical procedure(s) used; iii) **Print-out** of the work completed; and iv) **Summary** and conclusions.

The Driver Program:

```fortran
      PROGRAM PREDPREY
      USE MSIMSL
      INTEGER::MXPARM,N
      PARAMETER(MXPARM=50,N=2)
      INTEGER ::IDO,ISTEP,NOUT
      REAL  ::PARAM(MXPARM),T,TEND,TOL,Y(N)
      EXTERNAL FCN
      OPEN(UNIT=6,FILE='OUTPUT.TXT')
      CALL UMACH (2,NOUT)
      T=0.0
      Y(1)=2.0
      Y(2)=1.0
      TOL=0.0005
      CALL SSET(MXPARM,0.0,PARAM,1)
      PARAM(10)=1.0
      PRINT'(4X,"ISTEP",5X,"TIME",9X,"Y1",11X,"Y2")'
      WRITE(6,1)
    1 FORMAT(4X,"ISTEP",10X,"TIME",16X,"Y1",16X,"Y2")
      IDO=1
      ISTEP=0
      DO
      ISTEP=ISTEP+1
      TEND=ISTEP
C     CALL IVPRK (IDO,N,FCN,T,TEND,TOL,PARAM,Y)
C
      IF(ISTEP.GT.10) EXIT
      WRITE(6,10) ISTEP,T,Y
   10 FORMAT(3X,I6,5X,F12.3,5X,F12.3,5X,F12.3)
      IF(ISTEP.EQ.10) IDO=3
      END DO
      END PROGRAM
C
C        This is for bringing the function
      SUBROUTINE FCN(N,T,Y,YPRIME)
      IMPLICIT NONE
      INTEGER::N
C
      REAL::T,Y(N),YPRIME(N)
      YPRIME(1)=1.2*Y(1)-0.6*Y(1)*Y(2)
      YPRIME(2)=-0.8*Y(2)+0.3*Y(1)*Y(2)
C
      END SUBROUTINE
```

Supporting Notes:
Using *Mathematica* to determine the numerical solution of a differential equation.

Differential equations (DE) are of mainly of two types: Ordinary Differential Equations (ODE) or Partial Differential Equations (PDE). These equations are derived and used to describe processes in various fields of study. For example, in Civil Engineering these equations are used to describe behavior of a beam under loads, the consolidation of a soil specimen under sustained pressure load, etc. In engineering applications, these problems are often referred to as boundary value problems (BVs) and their solutions require the specification of boundary conditions (BCs) and initial conditions (ICs).

The solutions of these DE depend on their types. And the solutions are either analytical solutions (for simple problems) and/or numerical solutions that are applicable to simple and more challenging problems.

Numerical solutions include the use of the method of finite differences, and the method of finite elements. These methods are frequently programmed to be used by computer packages such as IMSL-subroutines (FORTRAN sub-programs), Excel, MATLAB, and Mathematica.

This method of finite differences is introduced and used in this computer-laboratory session to solve ordinary differential equations using the IMSL-subroutine library (namely: subroutine IVPRK) and the Mathematica software package.

Module for the Finite Difference Method for Boundary Value Problems

Theorem (boundary value problem). Assume that $f(t, x, y)$ is continuous on the region $R = \{(t, x, y) : a \le t \le b, -\infty < x < \infty, -\infty < y < \infty\}$ and that $\dfrac{\partial f}{\partial x} = f_x(t, x, y)$ and $\dfrac{\partial f}{\partial y} = f_y(t, x, y)$ are continuous on R. If there exists a constant $M > 0$ for which f_x and f_y satisfy

$$f_x(t, x, y) > 0 \text{ for all } (t, x, y) \in R$$

and

$$|f_y(t, x, y)| < M \text{ for all } (t, x, y) \in R,$$

then the **boundary value problem**

$$x'' = f(t, x, x') \text{ with } x(a) = \alpha \text{ and } x(b) = \beta$$

has a unique solution $x = x(t)$ for $a \le t \le b$.

Program (Finite-Difference Method). To approximate the solution of the boundary value problem $x'' = p(t)x'(t) + q(t)x(t) + r(t)$ with $x(a) = \alpha$ and $x(b) = \beta$ over the interval $[a, b]$ by using the finite difference method of order $O(h^2)$. The mesh we use is $a = t_1 < t_2 < \ldots < t_{n+1} = b$ and the solution points are $\{(t_j, x_j)\}_{j=1}^{n+1}$.

Procedures to obtain a numerical solution for an ODE:

    (i) Construct the tri-diagonal matrix and vector.
    (ii) Solve the system in (i).
    (iii) Join the mesh points and vector from (ii) to form the solution points.

Using Mathematica software the following two examples outline the programming step to obtain a numerical solution of one ODE.

Example 1:

Solve $x'' + \dfrac{1}{t} x' + \left(1 - \dfrac{1}{4 t^2}\right) x = \sqrt{t} \cos(t)$ over $[1, 6]$ with $x(1) = 1.0$ and $x(6) = -0.5$.

Use the finite difference method with 25 subintervals (total of 26 points).

Solution:

```
p[t_] = - 1/t ;

q[t_] = -(1 - 1/(4 t^2) );

r[t_] = √t Cos[t];
points1 = FineDDE[1.0, 6.0, 1.0, -0.5, 25];
Needs["Graphics`Colors`"];
ListPlot[points1, PlotJoined → True, PlotRange → {{1.0, 6.2}, {-3.2, 1.2}}, PlotStyle → Magenta, Ticks
Print["The solution to"];
Print["x'' + (", -p[t], ")x' + (", -q[t], ")x = ", r[t]];
Print["x(", a, ") = ", α];
Print["x(", b, ") = ", β];
Print["The finite difference method was used with with n = 26 points"];
```
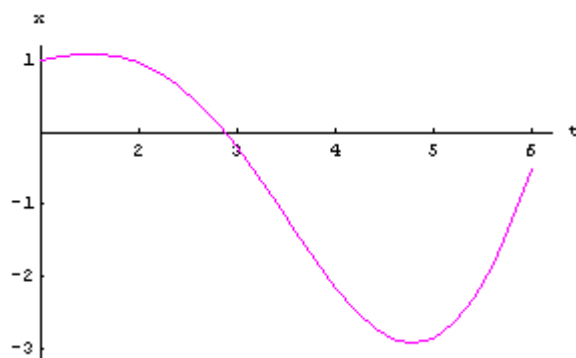


```
The solution to

x'' + ( 1/t )x' + (1 - 1/(4 t^2) )x = √t Cos[t]

x(1.) = 1.
x(6.) = -0.5
```
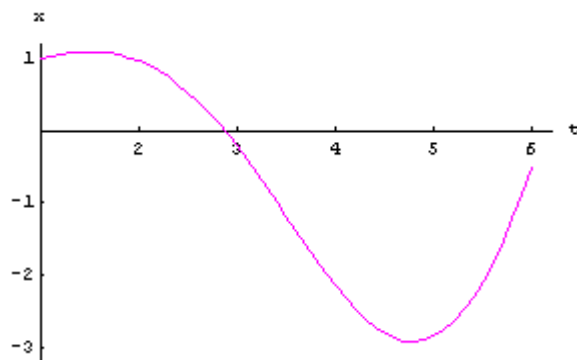
Example 2:

Solve $x'' + \dfrac{1}{t} x' + \left(1 - \dfrac{1}{4t^2}\right) x = \sqrt{t}\,\cos(t)$ over $[1, 6]$ with $x(1) = 1.0$ and $x(6) = -0.5$.

Use the finite difference method with 50 subintervals (total of 51 points).

Solution:

$p[t\_] = -\dfrac{1}{t};$

$q[t\_] = -\left(1 - \dfrac{1}{4 t^2}\right);$

$r[t\_] = \sqrt{t}\ Cos[t];$

points2 = FineDDE[1.0, 6.0, 1.0, -0.5, 50];

Needs["Graphics`Colors`"];

ListPlot[points2, PlotJoined→True, PlotRange→{{1.0, 6.2}, {-3.2, 1.2}}, PlotStyle→Magenta, Ticks→{Range[1, 6, 1], Range[-3, 2, 1]}, AxesLabel→{"t", "x"}];

Print["The solution to"];

Print["x'' + (", -p[t], ")x' + (", -q[t], ")x = ", r[t]];

Print["x(", a, ") = ", α];

Print["x(", b, ") = ", β];

Print["The finite difference method was used with with n = 51 points"];

The solution to

$x'' + \left(\dfrac{1}{t}\right)x' + \left(1 - \dfrac{1}{4 t^2}\right)x = \sqrt{t}\ Cos[t]$

$x(1.) = 1.$
$x(6.) = -0.5$