This is a copy of key solution for a typical FINAL Examination for CE 201

King Fahd University of Petroleum & Minerals
CIVIL ENGINEERING DEPARTMENT

STATICS: CE 201

Final Examination

Date: January 21, 1992 [7:30 a.m.] Time Allowed: 2½ hours. 346

STUDENT NAME: Key Solution	NO:	SECTION:

Problem	Possible Points	Score
1	15	
2	20	
3	25	
4	20	
5	20	
TOTAL	100%	

Instructor: Dr. Saeid A. Alghamdi

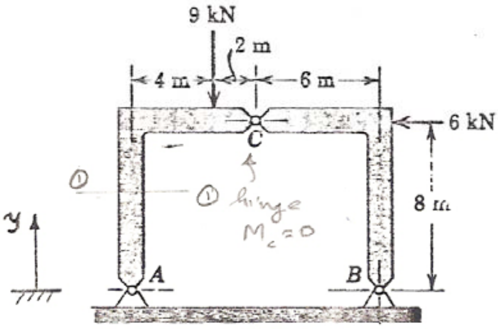
Typical old Exam

Problem [1]:

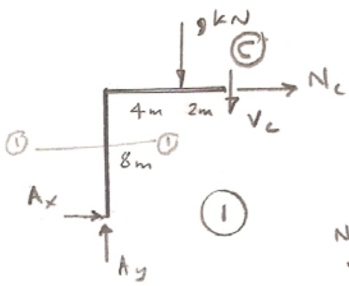
The frame shown is pin-supported at A and B and has a hinge at C. For the loads shown:

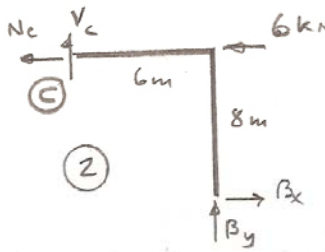
9% **a:** determine the equations of internal forces N(y), V(y), M(y);

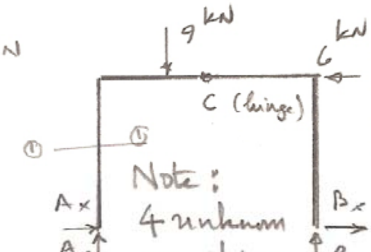
6% **b:** evaluate the equations at y = 4 m. (at section 1-1). 345



FBD's :







Note: 4 unknown reactions ∴ dismantle.

No. of unknowns: 6.

Scheme of solution:

Writing three equations of equilibrium for the frame and supplementing this by a moment equation about C for half the frame.

Solution:

$$A_x + B_x = 6$$

$$A_y + B_y = 9$$

$$12A_y - 9(8) - 6(8) = 0$$

$$\therefore A_y = \frac{48 + 72}{12} = 10 \text{ kN} \uparrow$$

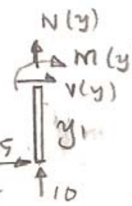
$$A_x = (6A_y - 18)/8 = 5.25 \text{ kN} \rightarrow$$

and from ①: $\circlearrowleft + \Sigma M_c = 0$

$$-A_y(6) + A_x(8) + 9(2) = 0$$

$$\therefore 6A_y = 18 + 8A_x$$

Equations of Internal Forces:



$$N(y) + 10 = 0 \Rightarrow N(y) = -10 \text{ kN}$$

$$V(y) + 5.25 = 0 \Rightarrow V(y) = -5.25 \text{ kN}$$

$$5.25\,y - M(y) = 0 \Rightarrow M(y) = 5.25\,y$$

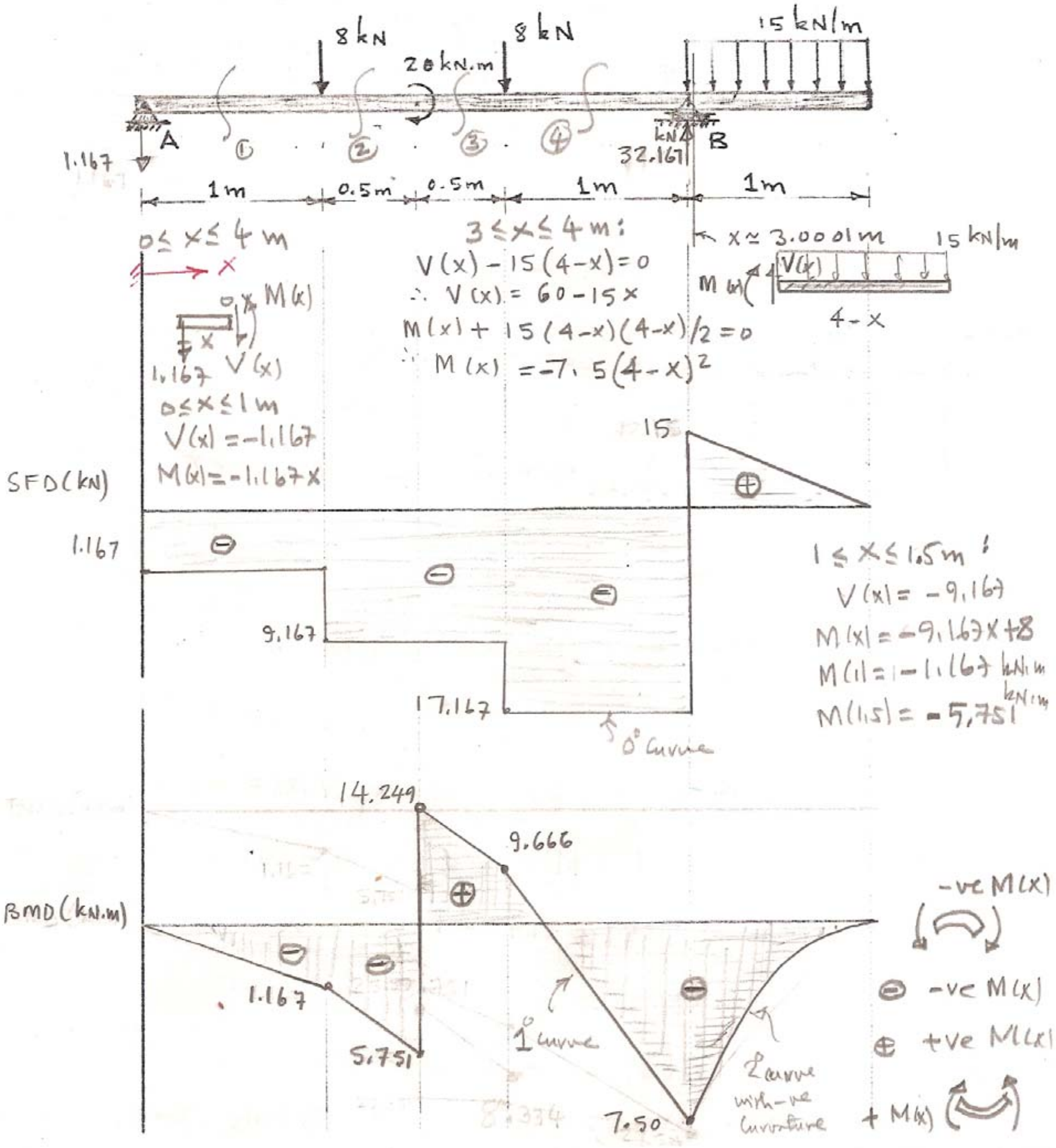
at $y = 4$ m: $N = -10$ kN; $V = -5.25$ kN; $M(4) = 21.$ kN.m

Problem [3]:

The beam shown supports two concentrated loads, a concentrated moment and a uniform load. For this loading the reaction R_A = 1.167 kN (↓).

5% **a**: Write the equations of V(x) and M(x) at x ≥ 3.0001 m [assuming the reaction is exactly at x = 3 m].

20% **b**: Plot the shear force diagram SFD, and the bending moment diagram BMD.



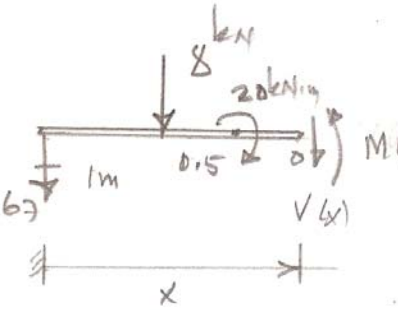
Details of 3rd & 4th Cuts

For $V(x)$ & $M(x)$ expressions.

For the 3rd cut at

$1.5 \le x \le 2$ m :

(use a FBD & Equilibrium Eqns as follows)



$+\downarrow \Sigma F_y = 0$ & $\circlearrowleft + \Sigma M_{@o} = 0$

$1.167 + V(x) + 8 = 0 \quad \Rightarrow \quad V(x) = -9.167$ kN

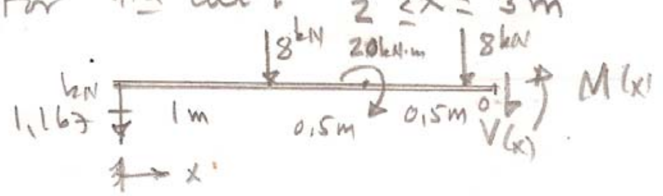
$1.167\,x + 8(x-1) - 20 + M(x) = 0$

$\therefore \; M(x) = 20 - 9.167\,x + 8$

$= 28 - 9.167\,x$

$\therefore \; M(1.5) = +14.25$ kN.m ; $M(2) = 9.667$ kN.m

For 4th cut : $2 \le x \le 3$ m



$V(x) + 16 + 1.167 = 0 \Rightarrow V(x) = -17.167$ kN

$\circlearrowleft + \Sigma M_o = 0 : \quad M(x) + 8(x-2) - 20 + 8(x-1) + 1.167x = 0$

$\therefore \; M(x) = 44 - 17.167\,x$

$M(2) = +9.667$ kN.m

$M(3) = -7.50$ kN.m

Note

All expressions are plotted on SFD & BMD (previous page).

Problem [4]:

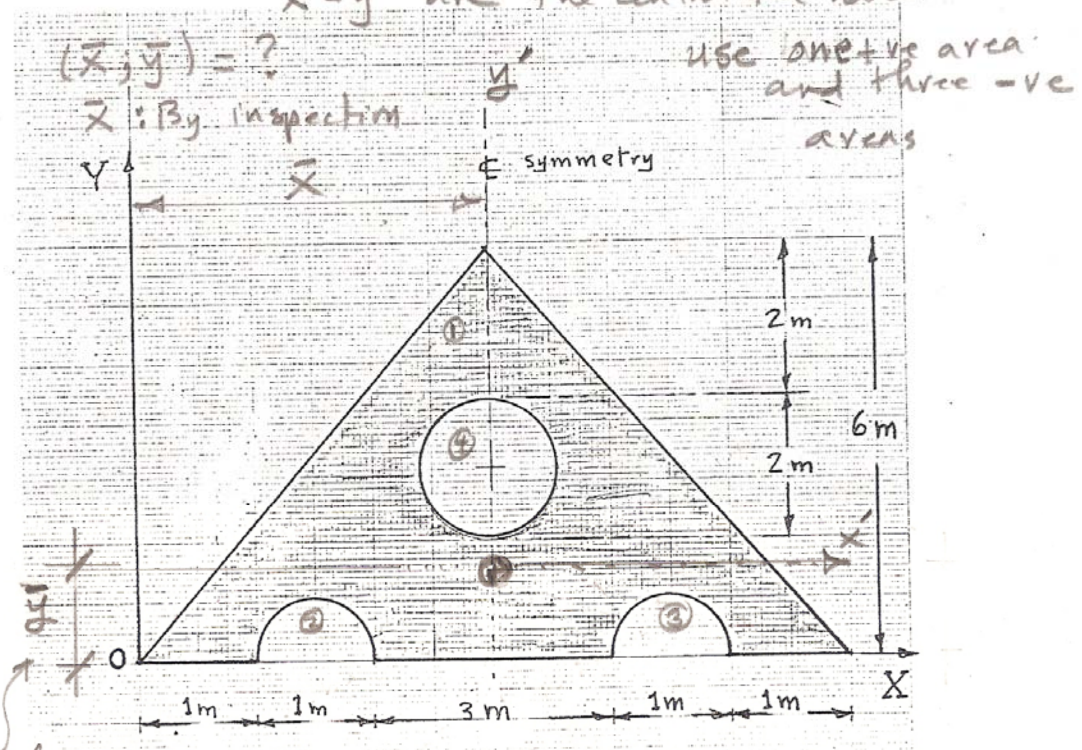
The triangle area shown has a circular hole and two semi-circular holes. If it has one axis of symmetry then:

15% a: Use the Parallel-axis theorem to determine the centroidal moments of inertia $\bar{I}_{x'}$;

5% b: Compute the radius of gyration k_{x}; explain its meaning.

Hint: You may use the Table attached here with this exam.

$x'-y'$ are the centroidal axes.



location of centroid ⊕ Computed $\bar{y} = 1.8983$ m

Segment i		A_i	$\tilde{x}_i$	$\tilde{y}_i$	$A_i \tilde{y}_i$	d_y	d_x	$\bar{I}_{x'}$	$A_i d_y^2$
△	1	21	3.5	2	42	-.10173	Not needed because of symmetry	42.	0.21733
⌓	2	-0.3927	1.5	0.2122	-0.08331	1.6861		-0.0245	-1.1164
⌓	3	-0.3927	5.5	0.2122	-0.08331	1.6861		-.0245	-1.1164
O	4	-3.142	3.5	3	-9.425	-1.1017		-0.785	-3.8138
Σ		17.073	-	-	32.41	-	-	41.166	-5.8293

Computations

$\bar{y} = \frac{\Sigma A_i \tilde{y}_i}{\Sigma A_i} = 1.8983 \quad \therefore I_{x'} = \Sigma \bar{I}_{x'} + \Sigma A_i d_y^2$

$\therefore \bar{I}_{x'} = 41.166 - 5.8293 = 35.34 \text{ m}^4$

$I_x = I_{x'} + A d_y^2 = 35.34 + 17.073 (1.8983)^2 = 96.86 \text{ m}^4$

$k_x = \sqrt{I_x / A} = \sqrt{96.86 / 17.073} = 2.38 \text{ m}$

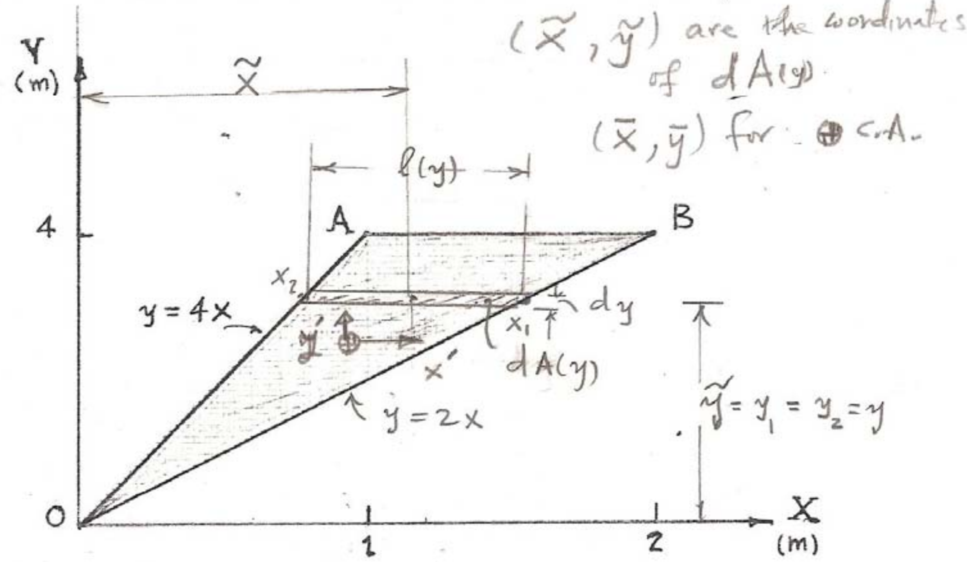
Problem [2]:

The triangular area is defined by the intersection of three lines OA, AB and BO. For this area:

8% a: determine the coordinates $\bar{x}$ and $\bar{y}$ of the centroid [express your results with respect to the axes X & Y];

6% b: determine the moment of inertia I_x;

6% c: determine the centroidal moment of inertia $\bar{I}_{x'}$.



a: $dA(y) = \ell(y)\, dy \quad ; \quad \tilde{x} = x_2 + \frac{\ell(y)}{2} = \frac{(2)y}{4_{(2)}} + \frac{y}{8} = \frac{3y}{8}$

$$\ell(y) = x_1 - x_2 = \frac{y}{2} - \frac{y}{4} = \frac{y}{4} \Rightarrow dA(y) = \frac{y}{4}\, dy$$

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA} = \frac{\int \frac{3y}{8}\left(\frac{y}{4}\right) dy}{\int (y/4)\, dy} = \frac{(3/32)\int y^2 dy}{(1/4)\int y\, dy} = \frac{3}{8}\,\frac{\left[y^3/3\right]_0^4}{\left[y^2/2\right]_0^4}$$

$$\therefore \bar{x} = \frac{3}{8}\left(\frac{2}{3}\right)\left(\frac{(4)^3}{(4)^2}\right) = \frac{1}{4}(4) = 1 \text{ m}$$

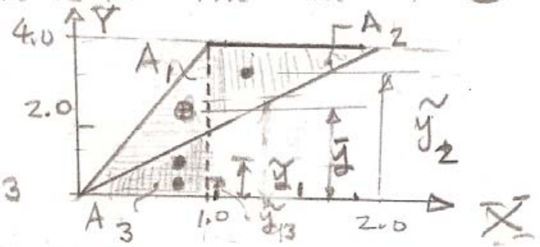
$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA} = \frac{\int y (y/4)\, dy}{\int (y/4)\, dy} = \frac{\int y^2 dy}{\int y\, dy} = \frac{\left[y^3/3\right]_0^4}{\left[y^2/2\right]_0^4} = \left[\frac{2}{3}y\right]_0^4$$

$$\therefore \bar{y} = \frac{2}{3}(4) = \frac{8}{3} = 2.67 \text{ m}$$

b: $I_x = \int_A \tilde{y}^2\, dA = \int_0^4 y^2 \left(\frac{y}{4}\right) dy = \frac{1}{4}\left[\left(y^4/4\right)\right]_0^4 = \frac{1}{16}(4)^4 = 16 \text{ m}^4$

c: $\bar{I}_{x'} = I_x - A d_y^2 = 16 - [1(4)/2]\,(2.67)^2 = 1.742 \text{ m}^4$

Using the parallel axis theorem since x' is // to $\bar{X}$.



And since

$$A_{tot} = A_1 + A_2 - A_3$$

$$= \frac{1\times 4}{2} + \frac{2\times 1}{2} - \frac{1\times 2}{2} = 2\ m^2$$

$$\bar{y} = \frac{\sum A_i \tilde{y}_i}{A_T}$$ (using three areas A_1 and A_2 are +ve and A_3 is -ve)

$$= \frac{(1\times 4/2)\tilde{y}_1 + (1\times 2/2)\tilde{y}_2 - (1\times 2/2)\tilde{y}_3}{2\ m^2}$$

where $\tilde{y}_1 = 4/3\ m$; $\tilde{y}_2 = (2 + 2\times 2/3)m$; $\tilde{y}_3 = 2\times 1/3\ m$

$$\therefore \bar{y} = \frac{4/2(4/3) + 1.\times(10/3) - 1(2/3)}{2\ m^2} = 2.67\ m$$

Also, based on the // - axes theorem. where: $d_{iy} = \bar{y} - \tilde{y}_i$

$$I_{x'} = \sum \bar{I}_{x'} + \sum A_i d_{iy}^2$$

$$= \frac{1}{36}\left[1\times 4^3 + 1.\times 2^3 - 1.0\times 2^3\right] + \left(1\times\frac{4}{2}\right)(2.67 - 4/3)^2 + (1\times 2/2)\left(2.67 - \frac{10}{3}\right)^2 - (1\times 2/2)(2.67 - 2/3)^2$$

$$= 1.7778 + 2.2226\times 10^{-5}$$

$$\cong 1.78\ m^4$$

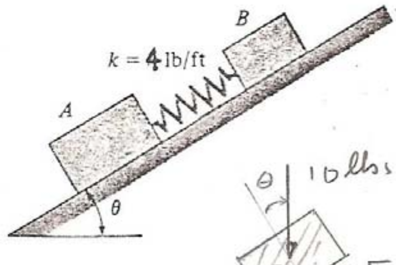
Note:
Work is best done in a Table form to keep track of $\tilde{y}_i$, A_i, $\bar{I}_{x'i}$ and $d_{iy} = \bar{y} - \tilde{y}_i$; etc.

Problem [5]:

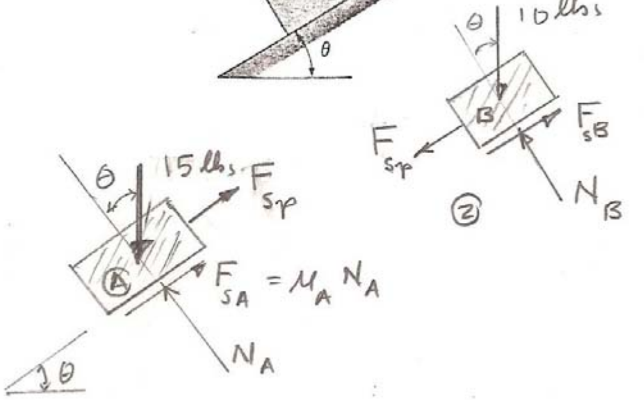
Two blocks A and B have a weight of 15 lbs and 10 lbs, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.25$ and $\mu_B = 0.15$. If the two blocks are connected by a spring with k = 4 lbs/ft, then:

10% a: determine the angle θ for which both blocks begin to slide;

10% b: determine the change in length of the spring. Is it elongation or shortening?



F.B.D:



From ①: $N_A = 15 \cos\theta$ ...(1) & $F_{sp} + 0.25 N_A - 15 \sin\theta = 0$ ...(2)

From ②: $N_B = 10 \cos\theta$ ...(3) & $-F_{sp} + 0.15 N_B - 10 \sin\theta = 0$ (4)

Adding ② & ④: $0.25 N_A + 0.15 N_B = 25 \sin\theta$

And using eqn. (3): $0.25 N_A + 0.15(10\cos\theta) = 25 \sin\theta$

Also using eqn. (1) $0.25(15\cos\theta) + 0.15(10\cos\theta) = 25\sin\theta$

$\therefore$ $5.25 \cos\theta = 25 \sin\theta$

or $\tan\theta = \frac{5.25}{25} = 0.21 \Rightarrow \theta = 11.9^\circ$

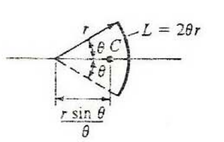
Therefore from eqn.② : $F_{sp} = 15 \sin 11.9 - 0.25(15 \cos 11.9^\circ)$

$= -0.5871$ lbs (compression)

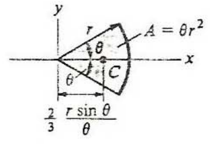
$\therefore$ Change in spring length $\Delta l = \frac{F_{sp}}{k} = -0.1468 \text{ft} \equiv 1.76$ ins. (short.)

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia



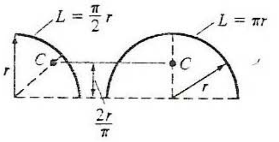
Circular arc segment



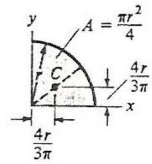
Circular sector area

$I_x = \frac{1}{4} r^4(\theta - \frac{1}{2}\sin 2\theta)$
$I_y = \frac{1}{4} r^4(\theta + \frac{1}{2}\sin 2\theta)$

$\bar{y} = 0$; $\bar{x} = \frac{2}{3} r \frac{\sin\theta}{\theta}$



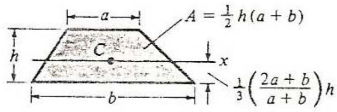
Quarter and semicircular arcs



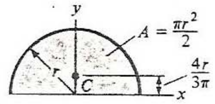
Quarter circular area

$I_x = \frac{1}{16}\pi r^4$
$I_y = \frac{1}{16}\pi r^4$

$\bar{x} = \bar{y} = 4r/3\pi$



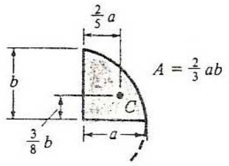
Trapezoidal area



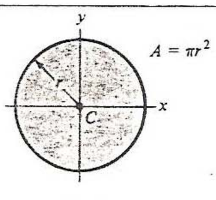
Semicircular area

$I_x = \frac{1}{8}\pi r^4$
$I_y = \frac{1}{8}\pi r^4$

$\bar{x} = 0$; $\bar{y} = \frac{4r}{3\pi}$

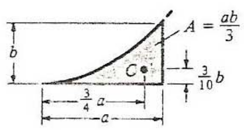


Semiparabolic area

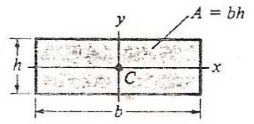


Circular area

$I_x = \frac{1}{4}\pi r^4$
$I_y = \frac{1}{4}\pi r^4$

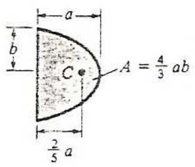


Exparabolic area

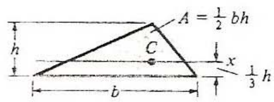


Rectangular area

$I_x = \frac{1}{12} bh^3$
$I_y = \frac{1}{12} hb^3$



Parabolic area



Triangular area

$I_x = \frac{1}{36} bh^3$

Notes: 1) For each case C is the centroid point.
2) Centroidal axes x and y must both