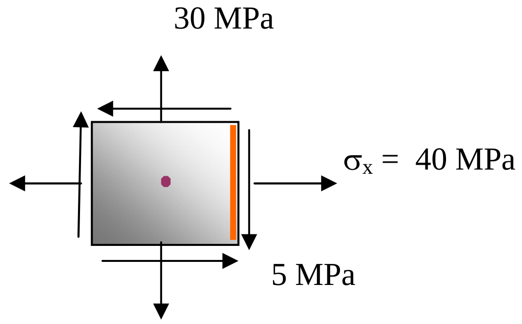
Analysis of Plane State of Stress Using Mohr's Circle Method

The method is a fast graphical procedure to determine the state of stress on a given plane passing through a stressed-material point. **It is particularly noted that:**

1. **All angles on the circle are twice the real angles.**
2. **Every point (of the ∞ number of points on the perimeter of the circle represents the state of stress on an inclined plane at the point under-stress.**
3. **To enter the circle use 2θ and to obtain an angle from the circle divide the angle by 2.**

Use the above points to analyze the following state of compound stresses (at a point) and to determine the state of stress on an element inclined with angle θ= 10 degrees counter-clockwise from x-axis.



Solution: Given that $\sigma_{x=}$ 40 MPa; $\sigma_{y=}$ 40 MPa; $\tau_{x=}$ - 40 MPa, the circle center and radius are C (σ_{avg}; 0) at (35, 0) MPa, and r = R = $\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$ = 5$\sqrt{2}$ MPa (≅ 7.1 MPa).

Angle AOB = $\sin^{-1}$(5/7.1) = 44.8 º.

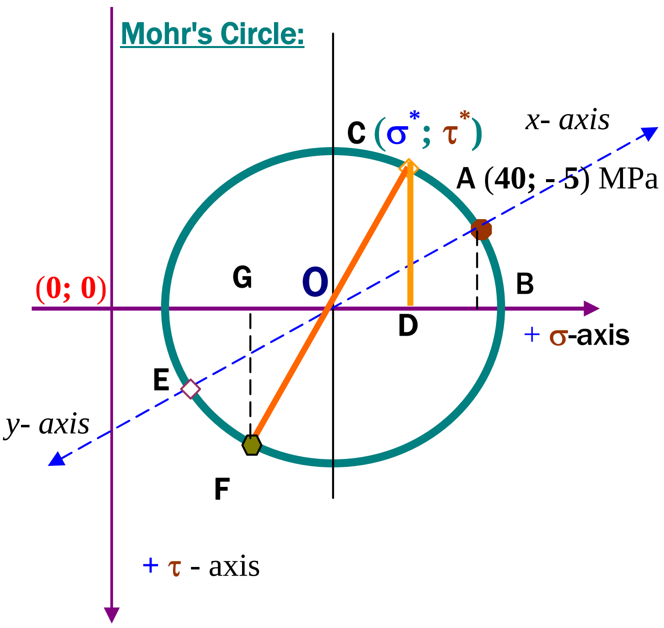
The given angle θ = 10º is shown on the circle as point C with (σ^*; τ^*) yet to be determined from analysis of geometry of angle given noting that angle COA is 20º. Therefore since angle COB is = 64.8º, the height **CD** of triangle COD gives shear stress τ^*, and the base **OD** of the triangle is added to $\sigma_{avg.}$ to give σ^*.

The values are: σ^* = 35 + R *cos* 64.8º = 38.02 MPa; and τ^* = R *sin* 64.8º = 6.42 MPa (-ve).

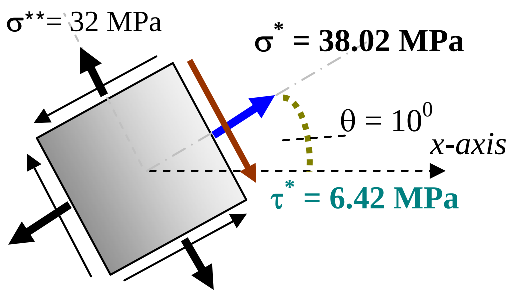
Its noted that on the circle the angle between points A and E is 180º and the real angle is only 90º. Therefore the stresses at point E are for a plane normal to the plane of point A. **The same note applies to the two planes for points C and F. It is also noted that due to symmetry OG = OD, and this makes the determination for state of stress on the plane for point F straightforward with the values as (σ^{**}= 35 - R *cos* 64.8º ≅ 32.0 MPa; τ^{**} = + 6.42 MPa).**

The properly oriented element for points C and F is given below showing the angle θ= 10º .

Quiz 8: Compute the maximum shear stresses and show them on a properly oriented element.



Properly oriented element for state defined by angle θ= 10º (CCW from x-axis).



Further Note:

1. Mohr's circle is used to determine the extreme-values of stresses resulting from applied loads to any structure for specified loading conditions. So if service loads are applied to a given structure, the resulting extreme values of stresses (namely: tensile; compressive or shear) and for a specified level of structural safety using Factor of Safety (**FS** $\rangle\, 1.0$), the required strengths (tensile; compressive and shear strength) are easily determined from the extreme values obtained from Mohr's Circle as follows:

Required Strength S_r for a material is:

S_r = FS x Extreme Stress Value from Mohr's Circle