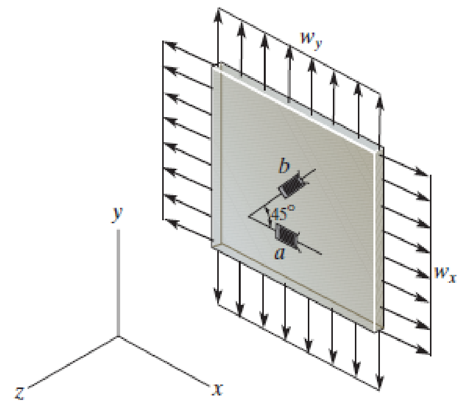
10–51. Two strain gauges a and b are attached to the surface of the plate which is subjected to the uniform distributed load $w_x = 700$ kN/m and $w_y = -175$ kN/m. If the gauges give a reading of $\epsilon_a = 450(10^{-6})$ and $\epsilon_b = 100(10^{-6})$, determine the modulus of elasticity E, shear modulus G, and Poisson's ratio ν for the material.



Normal Stress and Strain: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{700(10^3)}{0.025} = 28(10^6)\,\text{N/m}^2$$

$$\sigma_y = -\frac{175(10^3)}{0.025} = -7(10^6)\,\text{N/m}^2$$

$$\sigma_z = 0 \text{ (plane stress)}$$

Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0°$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a \cos\theta_a$$

$$450(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450(10^{-6})$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b \cos\theta_b$$

$$100(10^{-6}) = 450(10^{-6})\cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250(10^{-6})$$

Generalized Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$450(10^{-6}) = \frac{1}{E}\left[28(10^6) - \nu\left[-7(10^6) + 0\right]\right]$$

$$450(10^{-6})E - 7(10^6)\nu = 28(10^6) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$-250(10^{-6}) = \frac{1}{E}\left\{-7(10^6) - \nu\left[28(10^6) + 0\right]\right\}$$

$$250(10^{-6})E - 28(10^6)\nu = 7(10^6) \qquad (2)$$

Solving Eqs. (1) and (2),

$$E = 67.74(10^9)\,\text{N/m}^2 = 67.7\text{ GPa} \qquad \textbf{Ans.}$$

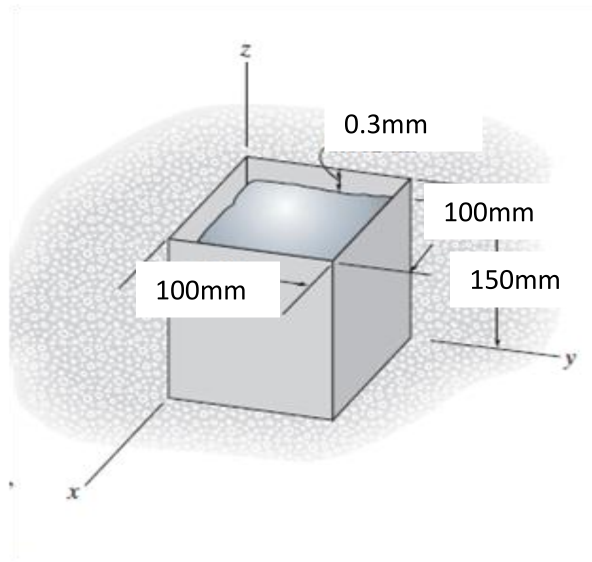
$$\nu = 0.3548 = 0.355 \qquad \textbf{Ans.}$$

Using the above results,

$$G = \frac{E}{2(1+\nu)} = \frac{67.74(10^9)}{2(1+0.3548)}$$

$$= 25.0(10^9)\,\text{N/m}^2 = 25.0\text{ GPa} \qquad \textbf{Ans.}$$

10-54. The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.3 mm from the top of the cavity. If the top of the cavity is not covered and the temperature is increased by 110˚C, determine the strain components ε_x, ε_y, and ε_z in the aluminum. Hint: Use Eqs. 10-18 with an additional strain term of $\alpha\Delta T$ (Eq.4-4).



Solution:

Normal strains: Since the aluminum is confined at its sides by a rigid container, then $\boldsymbol{\varepsilon_x = \varepsilon_y = 0}$ **Ans.**

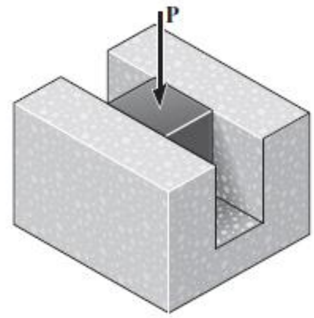
In z direction, it is not restrained, then σ_z=0.
From Page 864 (text book),
E_{al} = 68.9 (10^3) MPa., υ_{al} =0.35,
α=24*10^-6/˚C.

$$\varepsilon_x = \frac{1}{E_{al}}\left[\sigma_x - \upsilon\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$
$$0 = \frac{1}{68.9 * 10\text{^}3}\left[\sigma_x - 0.35 * \left(\sigma_y + 0\right)\right] + 24 * 10^{-6} * 110$$
$$0 = \sigma_x - 0.35\sigma_y + 181.9 \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$
$$\varepsilon_y = \frac{1}{E_{al}}\left[\sigma_y - \upsilon(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$
$$0 = \frac{1}{68.9 * 10\text{^}3}\left[\sigma_y - 0.35 * (\sigma_x + 0)\right] + 24 * 10^{-6} * 110$$
$$0 = \sigma_y - 0.35\sigma_x + 181.9 \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Solving Both Equations (1) and (2), $\boldsymbol{\sigma_y = \sigma_x = -279.85\ MPa.}$

$$\varepsilon_z = \frac{1}{E_{al}}\left[\sigma_z - \upsilon\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$
$$\varepsilon_z = \frac{1}{68.9 * 10\text{^}3}[0 - 0.35 * (-279.85 + (-279.85))] + 24 * 10^{-6} * 110$$
$$\boldsymbol{\varepsilon_z = 5.48\ (10^{-3})} \qquad \boldsymbol{Ans.}$$

10–57. The rubber block is confined in the U-shape smooth rigid block. If the rubber has a modulus of elasticity E and Poisson's ratio ν, determine the effective modulus of elasticity of the rubber under the confined condition.



Generalized Hooke's Law: Under this confined condition, $\varepsilon_x = 0$ and $\sigma_y = 0$. We have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$0 = \frac{1}{E}(\sigma_x - \nu\sigma_z)$$

$$\sigma_x = \nu\sigma_z \qquad (1)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + 0)]$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x) \qquad (2)$$

Substituting Eq. (1) into Eq. (2),

$$\varepsilon_z = \frac{\sigma_z}{E}\left(1 - \nu^2\right)$$

The effective modulus of elasticity of the rubber block under the confined condition can be determined by considering the rubber block as unconfined but rather undergoing the same normal strain of ε_z when it is subjected to the same normal stress σ_z, Thus,

$$\sigma_z = E_{\text{eff}}\,\varepsilon_z$$

$$E_{\text{eff}} = \frac{\sigma_z}{\varepsilon_z} = \frac{\sigma_z}{\frac{\sigma_z}{E}\left(1 - \nu^2\right)} = \frac{E}{1 - \nu^2} \qquad \textbf{Ans.}$$