1. Use the Newton-Raphson Method to determine the roots for the following two problems with errors in the computed roots $_{not}$ more than 0.5%.

   i) Solve part (b) of problem 6.10 [textbook page 158].

   ii) Solve problem **6.13** after re-writing (re-arranging) the nonlinear equations as

$$u(x,y) = 0.$$
$$v(x,y) = 0.$$

Solution:-

i) Required the roots of $f(x) = 8\sin(x)\,e^{-x} - 1;$

$using\ Newton\ Raphson\ Method\ (three\ iterations\ x_i = 0.3)$

Using the iteration with $x_i = 0.3$ and using the following equation; $\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$

$$f'(x) = 8cos(x)e^{-x} + 8\sin(x)\,e^{-x}$$

$$\boldsymbol{x_1 = 0.3 - \frac{0.7514}{7.4132} = 0.1986}$$

$$\boldsymbol{x_2 = 0.1986 - \frac{0.294}{7.7242} = 0.1606}$$

$$\boldsymbol{x_3 = 0.1606 - \frac{0.0895}{7.815} = 0.1492}$$

$$\boldsymbol{Error = \frac{0.1606 - 0.1492}{0.1606} = 7.1}\ \%;$$

$$\boldsymbol{then\ continue\ until\ the\ error\ as\ mentioned\ above\ in\ the\ problem.}$$

ii) Required the roots of $(x-4)^2 + (y-4)^2 = 5$

$$x^2 + y^2 = 16;$$

To get the initial guess, plot the function together and the intersection points will be the initial guess. Fro graphical,x=1.8 and y=3.6.

Rearranging the equations as $u(x,y) = 5 - (x-4)^2 - (y-4)^2$

$$v(x,y) = 16 - x^2 - y^2;$$

$$\frac{\partial u}{\partial x} = -2(x-4) = 4.4; \frac{\partial u}{\partial y} = -2(y-4) = 0.8; \frac{\partial v}{\partial x} = -2x = -3.6; \frac{\partial v}{\partial y} = -2y = -7.2$$

The determinant of Jacobian is $4.4(-7.2) - 0.8 * -3.6 = -28.8$

The functions at the initial guess are: $u(1.8,3.6) = 0$

$$v(1.8,3.6) = -0.2$$

Using the equation 6.21 in the textbook ;

$$x = 1.8 - \frac{0 - (-0.2) * 0.8}{-28.8} = 1.80556; \quad y = 3.6 — \frac{0.2 * 4.4 - 0}{-28.8} = 3.56944$$

And continue the same way up to the number of iteration required to get the error less than 0.5%.

$\boldsymbol{x = 1.805829; y = 3.569171}$

2. Use the method of $_{Gauss\ Elimination}$ to solve textbook Problem 9.11 [textbook page272]. Also compute the determinant of the coefficient matrix and check the accuracy of your results x^* by substitution in $\mathbf{A}\,x^* = b^*$ and computing the ratio of norms of vectors Δb and b ($_{namely}$: $\frac{\|\Delta \mathbf{b}\|}{\|\mathbf{b}\|}$).

From the textbook, Problem # 9.11,

$$-3x_2 + 7x_3 = 2; \quad x_1 + 2x_2 - x_3 = 3; \quad 5x_1 - 2x_2 = 2$$

$$[A][x] = [b]$$

⇒

$$\begin{bmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$\begin{bmatrix} 0 & -3 & 7 & 2 \\ 1 & 2 & -1 & 3 \\ 5 & -2 & 0 & 2 \end{bmatrix}$ ⇒ switch the row 2 and one ⇒ $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -3 & 7 & 2 \\ 5 & -2 & 0 & 2 \end{bmatrix}$

Switch row 2 and 3; $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 5 & -2 & 0 & 2 \\ 0 & -3 & 7 & 2 \end{bmatrix}$

Multiply row 1 by 5/1 ;

$\begin{bmatrix} 5 & 10 & -5 & 15 \\ 5 & -2 & 0 & 2 \\ 0 & -3 & 7 & 2 \end{bmatrix}$ then subtract row 1 from 2 ; $\begin{bmatrix} 5 & 10 & -5 & 15 \\ 0 & -12 & 5 & -13 \\ 0 & -3 & 7 & 2 \end{bmatrix}$;

Multiply row 2 by -3/-12 ;

$\begin{bmatrix} 5 & 10 & -5 & 15 \\ 0 & -3 & 15/12 & -39/12 \\ 0 & -3 & 7 & 2 \end{bmatrix}$;

and then subtract row 2 from row 3;

$\begin{bmatrix} 5 & 10 & -5 & 15 \\ 0 & -3 & 15/12 & -39/12 \\ 0 & 0 & 5.75 & 5.25 \end{bmatrix}$;

Now

Use back substitution to get all $x_1,\ x_2\ and\ x_3$ as follows:

$\boldsymbol{x_3 = 0.913}$; $\boldsymbol{x_2 = 1.464}$; $\boldsymbol{and\ x_1 = 0.985}$. then check the solution by substitution in the equations.

To get the determinant of the $\begin{bmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{bmatrix}$ $matrix; 0\big(2*0-(-1*-2)\big)-(-3)*(1*0-5*-1)+7*(1*-2-2*5) = 0+15-84 = \mathbf{69}$

3. Determine the inverse of matrix A (given in textbook Problem 10.12 page 294, and calculate its three matrix norms $\|A\|_1, \|A\|_e, \|A\|_\infty$.

Solution:-

From the problem ; $[A] = \begin{bmatrix} -6 & -2 & 5 \\ 8 & 1.1 & -2.5 \\ -3 & -1 & 10.3 \end{bmatrix}$; $the\ \boldsymbol{inverse\ is}$

0.120431	0.212766	-0.00682
-1.02155	-0.6383	0.340971
-0.0641	0	0.128205

To scale the matrix, Divide first row by -6, and second row by 8 and third row by 10.3

The scaled matrix will be; $\begin{bmatrix} 1 & 0.3333 & -0.83333 \\ 1 & 0.1375 & -0.3125 \\ -0.291 & -0.0971 & 1 \end{bmatrix}$

$$\|A\|_e = \sqrt{\sum_{i=1,j=1} a_{ij}{}^2} = \sqrt{a_{11}^2 + a_{12}^2 + a_{13}^2 + ..)} = \mathbf{2.004}\ \boldsymbol{Ans}.$$

$$\|A\|_1 = maximum\ of\ the\ summation\ of absolute\ value\ of\ each\ column\ elements$$
$$= \mathbf{2.291}\quad \boldsymbol{Ans}.$$

$\|A\|_\infty = maximum\ of\ the\ summation\ of\ each\, row\ elements = \mathbf{2.1666}$ **Ans.**

4. Compute the condition number for the matrix given in text book problem 10.11. Also, check if the matrix is ill-conditioned or not. Then if it is ill-conditioned specify the number of significant digits that will be lost due to ill-conditioning.

Solution:-

$$[A] = \begin{bmatrix} 0.125 & 0.25 & 0.5 & 1 \\ 0.015625 & 0.625 & 0.25 & 1 \\ 0.00463 & 0.02777 & 0.16667 & 1 \\ 0.001953 & 0.015625 & 0.125 & 1 \end{bmatrix}$$

Using the row sum,

First row = 1.875; second row=1.890625; third row =1.19907; and fourth row=1.142578

$$\|A\|_\infty = 1.890625$$

The matrix inverse is computed using excel as follows:

$$[A]^{-1} = \begin{bmatrix} 10.233 & -2.2339 & -85.3872 & 77.388 \\ -0.1008 & 1.767 & -4.3949 & 2.7283 \\ -0.6280 & -0.3716 & 30.7645 & -29.765 \\ 0.0601 & 0.0232 & -3.6101 & 4.5268 \end{bmatrix}$$

$$\|A^{-1}\|_\infty = 175.242$$

Then the condition is computed as follows:

Cond(A)= $\|A\|_\infty * \|A^{-1}\|_\infty = 331.32$

Because of the inverse has elements more than 1, it **is ill conditioned**.

To get the digits lost by the ill condition,

Log(331.32)=2.52

So, three digits will be lost.