Key Solution

HOME WORK # 8

by

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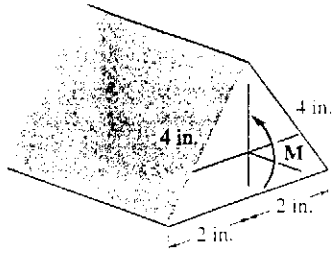
KEY TO HOMEWORK # 8.

PROBLEM # 1 (641).

6–41. A member has the triangular cross section shown. If a moment of $M = 800$ lb · ft is applied to the cross section, determine the maximum tensile and compressive bending stresses in the member. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



Probs. 6–40/6–41

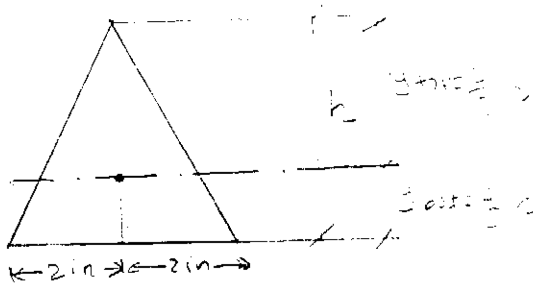
Solution:-

REQUIRED:-

Maximum tensile and compressive bending stresses in the member.

SECTION PROPERTIES:-

$$h = \sqrt{4^2 - 2^2} = 3.465 \text{ in}$$



$$\therefore y_t = \frac{2}{3}h = \frac{2}{3} \times 3.465$$

$$y_t = 2.31 \text{ in}.$$

$$y_{bot} = \frac{1}{3}h = \frac{1}{3} \times 3.465$$

$$y_{bot} = 1.155 \text{ in}.$$

$$I = bh^3/36.$$

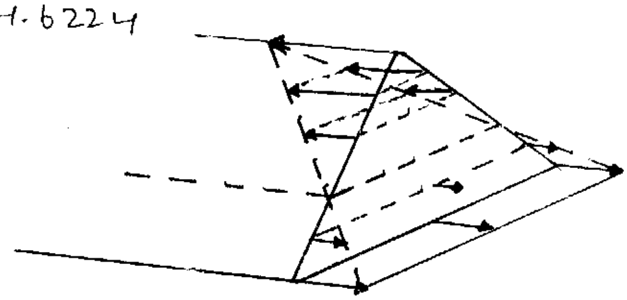
$$\Rightarrow I = \frac{4 \times (3.465)^3}{36}$$

$$\Rightarrow I = 4.6224 \text{ in}^4.$$

Stresses:-

$$(\sigma_{max})_c = \frac{M y_{top}}{I} = \frac{800 \times 12 \times 2.31}{4.6224} = 4.8 \text{ ksi}$$

$$(\sigma_{max})_t = \frac{M y_{bot}}{I} = \frac{800 \times 12 \times 1.155}{4.6224} = 2.4 \text{ ksi}$$



PROBLEM # ② (6 - 47) :-

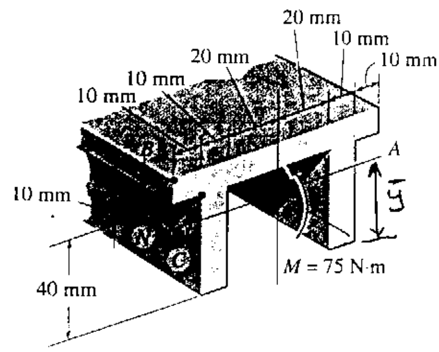
SOLUTION :-

REQUIRED :-

Bending stresses created at points B & C.

6–47. The aluminum machine part is subjected to a moment of $M = 75$ N · m. Determine the bending stress created at points B and C on the cross section. Sketch the results on a volume element located at each of these points.

*6–48. The aluminum machine part is subjected to a moment of $M = 75$ N · m. Determine the maximum tensile and compressive bending stresses in the part.



Probs. 6–47/6–48

SECTION PROPERTIES :-

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}.$$

$$I = \Sigma(\bar{I} + Ad^2)$$

$$\bar{y} = \frac{2[(10 \times 40) \times 20] + [(80 \times 10)(40+5)]}{2[10 \times 40] + [80 \times 10]} = 32.5 \text{ mm}.$$

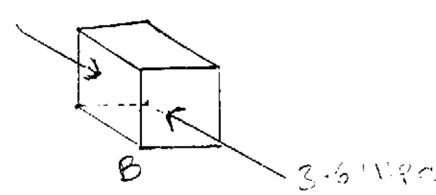
$$I = 2\left[\frac{10 \times 40^3}{12} + (10 \times 40)(32.5-20)^2\right] + \left[\frac{80 \times 10^3}{12} + (80 \times 10)(40+5-32.5)^2\right]$$

$$\Rightarrow I = 363.34 \times 10^3 \text{ mm}^4$$

Bending Stresses :-

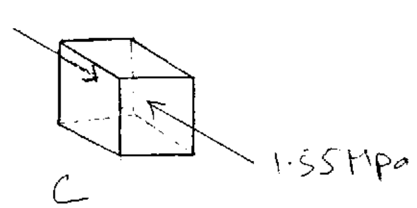
$$\sigma_B = \frac{Mc}{I} = \frac{(75 \times 10^3)(40+10-32.5)}{363.34 \times 10^3} = 3.61 \text{ Mpa}.$$

$$\Rightarrow \sigma_B = 3.61 \text{ Mpa}.$$



$$\sigma_C = \frac{Mc}{I} = \frac{(75 \times 10^3)(40-32.5)}{363.34 \times 10^3} = 1.55 \text{ Mpa}.$$

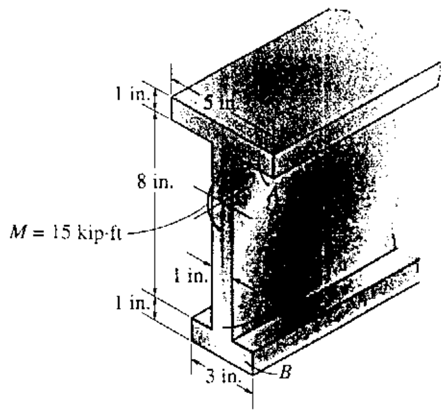
$$\Rightarrow \sigma_B = 1.55 \text{ Mpa}.$$



PROBLEM # 3 (6 - 55)! -

6–55. The beam is subjected to a moment of 15 kip · ft. Determine the resultant force the stress produces on the top flange *A* and bottom flange *B*. Also compute the maximum stress developed in the beam.

*6–56. The beam is subjected to a moment of 15 kip · ft. Determine the percentage of this moment that is resisted by the web *D* of the beam.



Probs. 6–55/6–56

Solution! -

REQUIRED! -

Resultant forces in beam at top flange A and bottom flange B.

Maximum stresses = ?

section properties! -

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$I = \Sigma(\bar{I} + A d^2).$$

$$\bar{y}_{Top} = \frac{1\times5\times0.5 + 8\times1\times5 + 1\times3\times(0.5+9)}{1\times5 + 8\times1 + 3\times1}$$

$\bar{y}_{Top} = 4.4375''$ & $\bar{y}_{bottom} = 5.5625''$.

$$I = \frac{5\times1^3}{12} + 5\times1\times(4.4375-0.5)^2 + \frac{1\times8^3}{12} + 1\times8\times(5.5625-5)^2$$

$$+ \frac{3\times1^3}{12} + 3\times1\times(5.5625-0.5)^2$$

$$= 200.2708 \text{ in}^4$$

stresses! -

AT Top flange A! -

$$(\sigma_A)_{max} = \frac{M y_{top}}{I} = \frac{15\times10^3\times12\times4.4375}{200.2708} = 3.99 \text{ ksi}$$

AT Bottom flange B! -

$$(\sigma_B)_{max} = \frac{M \bar{y}_{bottom}}{I} = \frac{15\times10^3\times12\times5.5625}{200.2708} = 5.0 \text{ ksi}.$$

At centre of top flange A :-

$$\sigma_A = \frac{My}{I} = \frac{15\times10^3 \times 12 \times (4.24375 - 0.5)}{200.2708} = 3.54 \text{ ksi}$$

At centre of bottom flange B :-

$$\sigma_B = \frac{My}{I} = \frac{15\times10^3 \times 12 \times (5.5625 - 0.5)}{200.2708} = 4.55 \text{ ksi}$$

RESULTANT FORCES :-

At Top flange A

$F_A = \sigma_A \times$ Area of top flange

$= 3.54 \times 5 \times 1 = 17.7$ kip.

$\therefore F_A = 17.7$ kip.

At Bottom flange B :-

$F_B = \sigma_B \times$ Area of bottom flange

$= 4.55 \times 3 \times 1 = 13.65$ kip

$\therefore F_B = 13.65$ kip.

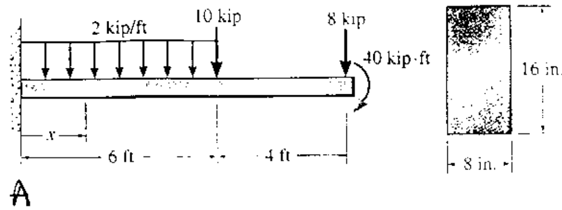
Maximum stress developed in the beam is at bottom flange.

$\sigma_{max} = (\sigma_B)_{max} = 5.0$ ksi

PROBLEM # (4) (6 − 61) :−

Solution:−

6–61. If the beam has a rectangular cross section with a width of 8 in. and a height of 16 in., determine the absolute maximum bending stress in the beam.



Prob. 6–61

REQUIRED:−

Absolute maximum bending stress

∵ The maximum bending occurs at fixed End

$$\therefore M_{max} = 40 + (8 \times 10) + (10 \times 6) + \left(2 \times 6 \times \frac{6}{2}\right)$$

$$= 216 \text{ kip.ft}$$

∴ Absolute Maximum Bending Stress in beam:−

$$\sigma_{max} = \frac{M_{max} \times y}{I}$$

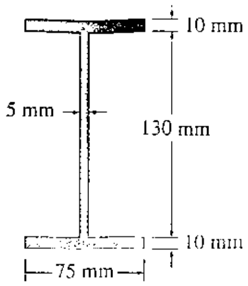
$$= \frac{216 \times 10^3 \times 12 \times 16/2}{(8 \times 16^3/12)} = 7593 \text{ Psi}$$

$$\Rightarrow \sigma_{max} = 7.593 \text{ Ksi}$$

PROBLEM # 5 (6-67):-

Let M = 0.691 kN m.

6–67. Determine the absolute maximum bending stress in the beam. The cross section of the beam is as shown.



Prob. 6–67

Solution:-

Required:-

Absolute maximum bending stress = ?

SECTION PROPERTIES:-

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$I = \Sigma (\bar{I} + Ad^2).$$

$$\bar{y}_{top} = \frac{10 \times 75 \times 5 + 130 \times 5 \times 75 + 10 \times 75 \times (145)}{10 \times 75 + 130 \times 5 + 10 \times 75}$$

$$\Rightarrow \bar{y}_{top} = 75 \text{ mm}.$$

$$\bar{y}_{bottom} = 150 - 75 = 75 \text{ mm}.$$

$$I = \frac{75 \times 10^3}{12} + 75 \times 10 \times (75-5)^2 + \frac{5 \times 130^3}{12} + \frac{75 \times 10^3}{12} + 75 \times 10 \times (75-5)^2$$

$$I = 8.28 \times 10^6 \text{ mm}^4$$

$$\sigma_{max} = \frac{My}{I}$$

$$= \frac{0.691 \times 10^6 \times 75}{8.28 \times 10^6}$$

$$\Rightarrow \sigma_{max} = 6.26 \text{ Mpa}.$$

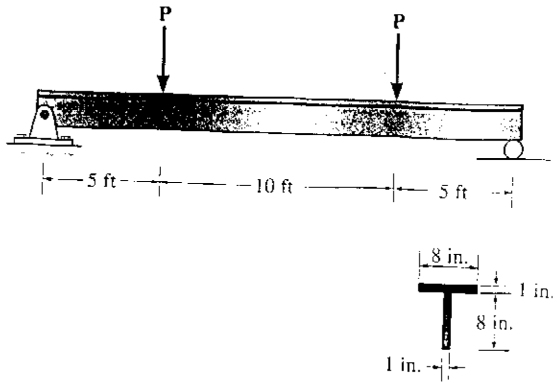
PROBLEM # ⑥ (6-79):-

Solution:-

REQUIRED:-

Maximum load $P = ?$

6–79. Determine the magnitude of the maximum load **P** that can be applied to the beam if the beam is made of a material having an allowable bending stress of $(\sigma_{allow})_c = 16$ ksi in compression and $(\sigma_{allow})_t = 18$ ksi in tension.



Probs. 6–78/6–79

Given:-

$(\sigma_{allow})_c = 16 \text{ ksi}.$

$(\sigma_{allow})_t = 18 \text{ ksi}.$

$M_{max} = P \times 5 = 5P \text{ kip}\cdot\text{ft}.$

$$y_{top} = \frac{1 \times 8 \times 0.5 + 1 \times 8 \times 5}{1 \times 8 + 1 \times 8}$$

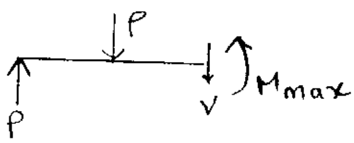


Fig. ①

$= 2.75 \text{ in}.$

$y_{bottom} = 6.25 \text{ in}.$

$$\therefore I = \frac{8 \times 1^3}{12} + 8 \times 1 \times (2.75 - 0.5)^2 + \frac{1 \times 8^3}{12} + 1 \times 8 \times (6.25 - 4)^2$$

$= 124.34 \text{ in}^4.$

Calculating 'P':-

From $(\sigma_{all})_c$:-

$$M_{max} = 5P = \frac{\sigma_{max} \times I}{y_{top}}$$

$$\Rightarrow 5 \times 12 \times P = \frac{16 \times 124.34}{2.75}$$

$\Rightarrow P = 12.05 \text{ kip}.$

From $(\sigma_{all})_t$:-

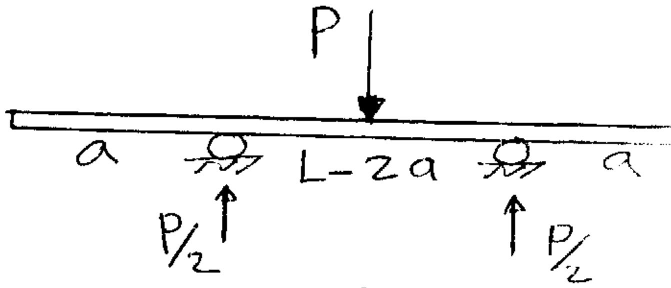
$$M_{max} = 5 \times 12 \times P = \frac{\sigma_{max} \times I}{y_{bot}} = \frac{18 \times 124.34}{6.25}$$

$\Rightarrow P = 5.97 \text{ kip}.$

$\therefore P = 5.97 \text{ kip (controls)}.$

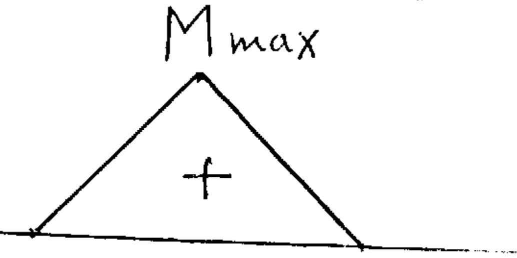
Problem # 7 (6-83)

Solution



$$\sigma_{max} = \frac{M_{max}\, C}{I}$$

C and I are constant



$\therefore \sigma_{max} = \text{constant} \times M_{max}$

$$M_{max} = \frac{P}{2}\left(\frac{L-2a}{2}\right)$$

for the moment to be maximum

$a = 0$

$$\therefore M_{max} = \frac{P}{2}\,\frac{L}{2} = \frac{PL}{4}$$

$C = d/2$

$$I = \frac{bd^3}{12}$$

$$\therefore \sigma_{max} = \frac{\frac{PL}{4} \times \frac{d}{2}}{\frac{bd^3}{12}}$$

$$\sigma_{max} = \frac{3PL}{2bd^2}$$