Key Solution

HOME WORK # 5

by

Dr. Al-Juruf

Civil Engineering Department

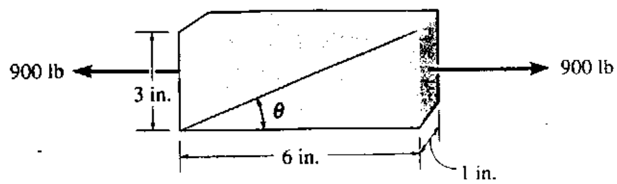
King Fahd University of Petroleum and Minerals

Dhahran

KEY TO HOMEWORK # 5:-

PROBLEM NO:- ① [10-37]:-

10–37. The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown, determine the value of Poisson's ratio if the angle θ decreases by $\Delta\theta = 0.01°$ after the load is applied. $E_{pvc} = 800(10^3)$ psi.



Prob. 10–37

GIVEN:-

$\Delta\theta = 0.01°$.

$E_{PVC} = 800 \times 10^3$ psi

REQUIRED:-

$\nu_{pvc} = ?$

From figure ①.

$\theta = \tan^{-1}(3/6) = 26.565°$.

& $\theta' = \theta - \Delta\theta$

$= 26.565 - 0.01$

$\Rightarrow \theta' = 26.555°$.

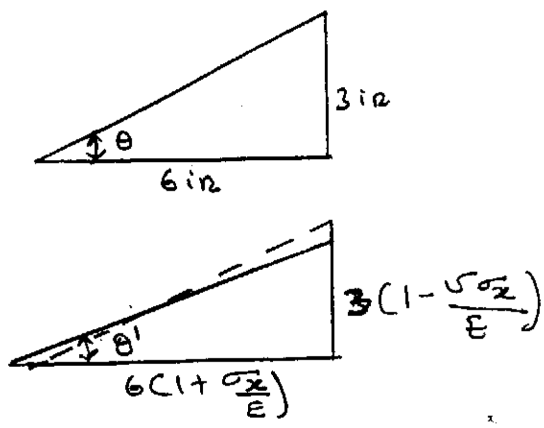


Fig ①

∵ we know the final length after deformation is

$L_f = L_o(1+\epsilon)$.

here; $L_o \rightarrow$ initial length.

$\epsilon \rightarrow$ Strain.

$$\epsilon_x = \frac{\sigma_x}{E} - \cancel{\frac{\nu\sigma_y}{E}}^{\,0} - \cancel{\frac{\nu\sigma_z}{E}}^{\,0}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \cancel{\frac{\sigma_y}{E}}^{\,0} - \cancel{\frac{\nu\sigma_z}{E}}^{\,0}$$

∵ $\sigma_y = \sigma_z = 0$

& $\sigma_x = \frac{P}{A} = \frac{900}{3 \times 1} = 300$ psi.

∴ The horizontal length 6 in. becomes

$L_f = 6\left(1 + \frac{\sigma_x}{E}\right)$

& the vertical length 3 in. becomes

$L_f = 3\left(1 - \frac{\nu\sigma_x}{E}\right)$

P.T.O

$$\therefore \sigma_x = \frac{P}{A} = \frac{900}{3 \times 1} = 300 \text{ psi.}$$

$$\& \quad \sigma_y = \sigma_z = 0.$$

now from figure ①

$$\tan\theta' = \frac{3\left(1 - \frac{\nu \sigma_x}{E}\right)}{6\left(1 + \frac{\sigma_x}{E}\right)}$$

$$\Rightarrow \tan 26.555^\circ = \frac{3\left(1 - \frac{\nu \times 300}{800 \times 10^3}\right)}{6\left(1 + \frac{300}{800 \times 10^3}\right)}$$

$$\Rightarrow \quad 0.4997 = \frac{3 - 1.125 \times 10^{-3} \nu}{6 + 2.25 \times 10^{-3}}$$

$$\Rightarrow 1.125 \times 10^{-3} \nu = 6.757 \times 10^{-4}$$

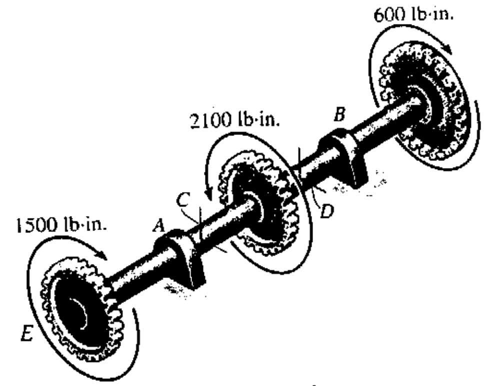
$$\Rightarrow \nu = 0.60$$

$$\therefore \nu = \nu_{pvc} = 0.60$$

PROBLEM NO:- ② (5-3)

5–3. The shaft has an outer diameter of 1.25 in. and an inner diameter of 1 in. If it is subjected to the applied torques as shown, determine the absolute maximum shear stress developed in the shaft. The smooth bearings at A and B do not resist torque.

*5–4. The shaft has an outer diameter of 1.25 in. and an inner diameter of 1 in. If it is subjected to the applied torques as shown, plot the shear-stress distribution acting along a radial line lying within region EA of the shaft. The smooth bearings at A and B do not resist torque.



Probs. 5–3/5–4

SOLUTION:-

GIVEN:-

outer diameter = 1.25 in.

inner diameter = 1.0 in.

REQUIRED:-

$\tau_{max} = ?$

$\because \tau_{max} = \frac{Tc}{J}$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}\left(\left(\frac{1.25}{2}\right)^4 - \left(\frac{1.0}{2}\right)^4\right)$$

$\Rightarrow J = 0.1415 \text{ in}^4.$

Finding Shear Stress at C & D.

τ at C:-

$$\tau = \frac{Tc}{J} = \frac{1500 \times \frac{1.25}{2}}{0.1415} = 6625 \text{ psi}$$

τ at D:-

$$\tau = \frac{Tc}{J} = \frac{(2100-1500) \times \frac{1.25}{2}}{0.1415} = 2650 \text{ psi}$$

$\therefore \tau_{max} = \tau_{at\,C} = 6625 \text{ psi}.$

$\Rightarrow \tau_{max} = 6625 \text{ psi} = 6.625 \text{ ksi}.$

PROBLEM NO:- 3 (5-13)

SOLUTION:-

GIVEN:-

$C_o = \frac{2.50}{2} in = 1.25 in$

$C_i = \frac{2.30}{2} in = 1.15 in.$

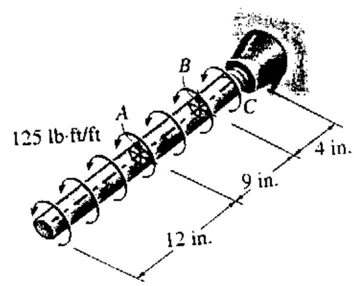
REQUIRED:-

$\tau_A = ?$

& $\tau_B = ?$

5–13. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and a uniformly distributed torque is applied to it as shown, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.

5–14. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.



Probs. 5–13/5–14

$\because \tau = \frac{TC}{J}$

$J = \frac{\pi}{2}(C_o^4 - C_i^4) = \frac{\pi}{2}\left(\left(\frac{2.5}{2}\right)^4 - \left(\frac{2.3}{2}\right)^4\right)$

$\Rightarrow J = 1.088 \ in^4.$

Now finding shear stresses.

$\tau_A = \frac{Tc}{J} = \frac{125 \times 12 \times \frac{2.5}{2}}{1.088}$

$\Rightarrow \tau_A = 1724 \ Psi$

$\Rightarrow \tau_A = 1.724 \ ksi$

$\tau_B = \frac{Tc}{J} = \frac{125 \times 21 \times \frac{2.5}{2}}{1.088}$

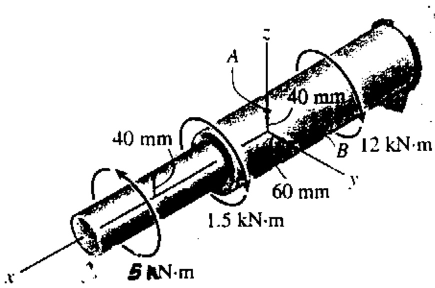
$\Rightarrow \tau_B = 3017 \ Psi$

$\Rightarrow \tau_B = 3.017 \ ksi$

PROBLEM # 4 (5-19).

SOLUTION:-

5–19. The steel shaft is subjected to the torsional loading shown. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line where it is maximum.



Probs. 5–18/5–19

REQUIRED:-

$\tau_{max} = ?$

$\because \tau = \frac{Tc}{J}$

Considering Section CD:-

$T = 5\ KN\text{-}M$

$J = \frac{\pi}{2}(40)^4 = 4021238\ mm^4.$

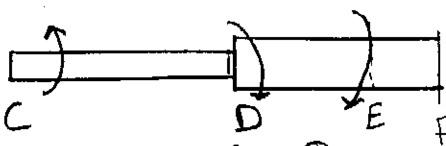


Fig. 1

$\therefore \tau_{CD} = \frac{Tc}{J} = \frac{5 \times 10^6 \times 40}{4021238} = 49.73\ Mpa.$

Shear Stress Distribution along two arbitrary radial lines.

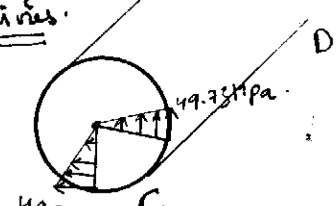


Fig. 2

Considering section DE:-

$T = 5 - 1.5 = 3.5\ KN\text{-}m.$

$J = \frac{\pi}{2}(60)^4 = 20357520\ mm^4$

$\therefore \tau_{DE} = \frac{Tc}{J} = \frac{3.5 \times 10^6 \times 60}{20357520} = 10.32\ Mpa.$

Considering Section EF:-

$T = 5 - 1.5 - 12 = -8.5\ KN\text{-}m.$

$\& \ J = \frac{\pi}{2}(60)^4 = 20357520\ mm^4.$

$\therefore \tau_{EF} = \frac{Tc}{J} = \frac{8.5 \times 10^6 \times 60}{20357520} = 25\ Mpa.$

$\therefore \tau_{max} = \tau_{CD} = 49.73\ Mpa.$

& the shear stress distribution along two arbitrary radial lines are shown in figure 2.

PROBLEM NO:- (5) (5-42)

5–42. The propellers of a ship are connected to a solid A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

SOLUTION:-

Given:-

$G = 75 \times 10^3$ Mpa for A-36 steel.

$L = 60m = 60\,000\,mm$.

$D = 340\,mm$.

$d = 260\,mm$

$P = 4.5\,MW$

$\omega = 20\,rad/sec$.

REQUIRED:-

$\tau_{max} = ?$ & $\phi = ?$

$$\because T = \frac{P}{\omega} = \frac{4.5 \times 10^6}{20} = 225\,KNm.$$

Finding τ_{max} & ϕ.

$$\tau_{max} = \frac{Tc}{J} = \frac{225 \times 10^6 \times 340/2}{\frac{\pi}{2}\left[\left(\frac{340}{2}\right)^4 - \left(\frac{260}{2}\right)^4\right]}$$

$$\Rightarrow \tau_{max} = 44.3\,Mpa.$$

$$\& \ \phi = \frac{TL}{JG} = \frac{225 \times 10^6 \times 60{,}000}{\frac{\pi}{2}\left[\left(\frac{340}{2}\right)^4 - \left(\frac{260}{2}\right)^4\right] \times 75 \times 10^3}$$

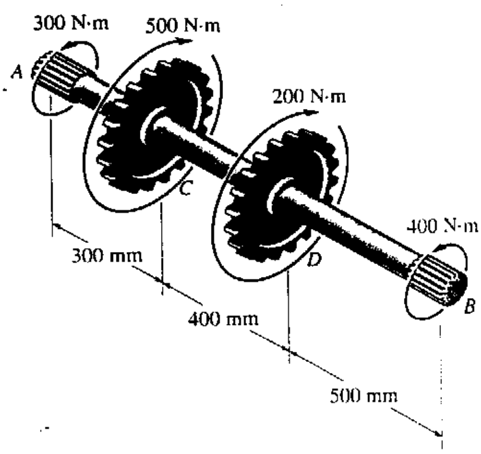
$$\Rightarrow \phi = 0.208\,rad.$$

$$\Rightarrow \phi = 0.208 \times \frac{180}{\pi} = 11.94^\circ$$

$$\Rightarrow \phi = 11.94^\circ$$

PROBLEM NO:- (6) (5-45).

5–45. The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm.



Prob. 5–45

SOLUTION:-

$G = 75$ GPa for A-36 steel

$J = \frac{\pi}{2}(c)^4$

$= \frac{\pi}{2}\left(\frac{40}{2}\right)^4$

$\Rightarrow J = 251.33 \times 10^3 \; mm^4.$

REQUIRED:-

$\phi_{B/A} = ?$

Internal Torques:-

$T_{AC} = 300 \; N\cdot m.$

$T_{CD} = -300 + 500 = 200 \; N\cdot m.$

$T_{DB} = -300 + 500 + 200 = 400 \; N\cdot m.$

ANGLE OF TWIST:-

$$\phi_{B/A} = \phi_{C/A} + \phi_{D/C} + \phi_{B/D} = \Sigma \frac{TL}{JG}.$$

$$= \frac{-300 \times 10^3 \times 300}{251.33 \times 10^3 \times 75 \times 10^3} + \frac{200 \times 10^3 \times 400}{251.33 \times 10^3 \times 75 \times 10^3}$$

$$+ \frac{400 \times 10^3 \times 500}{251.33 \times 10^3 \times 75 \times 10^3}$$

$\Rightarrow \phi_{B/A} = 0.0101$ radians.

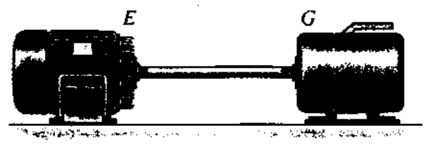
$= 0.0101 \times \frac{180}{\pi} = 0.578°$

$\therefore \phi_{B/A} = \underline{0.578°}$

PROBLEM # 7 (5-53)

5–53. The A-36 steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{allow} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad.

5–54. The A-36 solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Determine the smallest angular velocity the shaft can have if it is restricted not to twist more than 1°.



Probs. 5–53/5–54

SOLUTION:-

GIVEN:-

$G = 75\times10^3$ Mpa for A36 Steel.

$L = 2m = 2000mm$.

$D = 40mm$.

$\omega = 80$ rad/sec.

$P = 32$ kW.

$\tau_{all} = 140$ Mpa.

$\phi = 0.05$ rad.

REQUIRED:-

$t_{shaft} = ?$

$$\because T = \frac{P}{\omega} = \frac{32\times10^3}{80} = 400 \text{ N-m}$$

$$\& \quad \tau_{all} = \frac{Tc}{J} \Rightarrow 140 = \frac{400\times10^3\times(40/2)}{\frac{\pi}{2}\left(\left(\frac{40}{2}\right)^4 - \left(\frac{d}{2}\right)^4\right)}$$

$$\Rightarrow d = 37.5 \text{ mm}.$$

$$\& \quad \phi = \frac{TL}{JG} \Rightarrow 0.05 = \frac{400\times10^3\times2000}{\frac{\pi}{2}\left[\left(\frac{40}{2}\right)^4 - \left(\frac{d}{2}\right)^4\right]\times75\times10^3}$$

$$\Rightarrow d = 24.94 \text{ mm}.$$

Finding tshaft:-

By considering τ_{all}.

$$t_{shaft} = \frac{D-d}{2} = \frac{40-37.5}{2} = 1.25 \text{ mm}.$$

By considering ϕ.

$$t_{shaft} = \frac{D-d}{2} = \frac{40-24.94}{2} = 7.53 \text{ mm}.$$

$\therefore$ tshaft = greater of above two = 7.53 mm.