Key Solution

HOME WORK # 6

by

Dr. Juruf R.S.

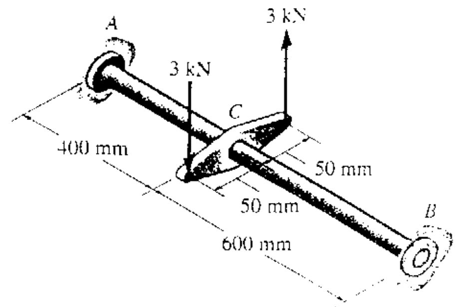
Civil Engineering Department

King Fahd University of Petroleum and Minerals

Dhahran

PROBLEM # 1 [5-75]: -

5–75. The steel shaft has a diameter of 40 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft. $G_{st} = 10.8(10^3)$ ksi.



Prob. 5–75

SOLUTION: -

Given: -

$G_{st} = 10.8 \times 10^3$ ksi

$c = \frac{40}{2} = 20\text{mm}$

$J = \frac{\pi}{2}(c)^4$

$= \frac{\pi}{2}\left(\frac{40}{2}\right)^4$

$\Rightarrow J = 251.33 \times 10^3 \text{ mm}^4$.

REQUIRED: -

$\tau_{AC\,max} = ?$ & $\tau_{CB\,max} = ?$

EQUILIBRIUM: -

first we will find the relation between T_{AC} & T_{CB}

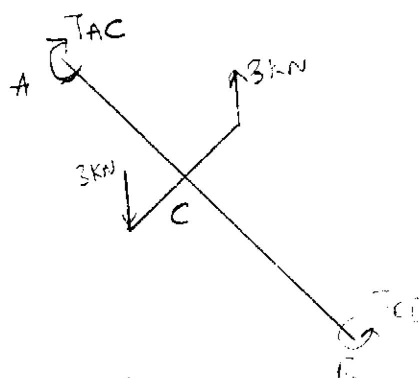


Fig (1)

from fig. (1).

$+\circlearrowleft \Sigma T = 0$

$-T_{AC} + 2 \times 3 \times 50 + T_{CB} = 0$

$\Rightarrow T_{CB} = T_{AC} - 300$ - - - (1)

compatibility: -

$\phi_{A/C} - \phi_{C/B} = 0$ ($\because$ the shaft is fixed).

$\Rightarrow \frac{-T_{AC} \cdot L_{AC}}{JG} - \frac{T_{CB} L_{CB}}{JG} = 0$

$\Rightarrow -T_{AC} \cdot L_{AC} - T_{CB} \cdot L_{CB} = 0$.

$\Rightarrow -T_{AC} \times (400) - T_{CB} \times 600 = 0$ - - - (2)

Subsituting ② in ③.

$$-T_{AC} \times 400 - (T_{AC} - 300) \times 600 = 0$$

$$\therefore T_{AC} = 180 \text{ KN-mm}.$$

$$\therefore \tau_{AC} = \frac{T_{AC} \cdot C}{J} = \frac{T_{AC} \times 20}{251.33 \times 10^3} = \frac{180 \times 10^3 \times 20}{251.33 \times 10^3}$$

$$\Rightarrow \tau_{AC} = \underline{\underline{14.32}} \text{ Mpa}.$$

$$\& \ \tau_{CB} = \frac{T_{CB} \cdot C}{J} = \frac{(T_{AC} - 300)\, 20}{251.33 \times 10^3}$$

$$\Rightarrow \tau_{CB} = \frac{(180 - 300) \times 10^3 \times 20}{251.33 \times 10^3} = -9.55 \text{ Mpa}.$$

$$\Rightarrow \tau_{CB} = \underline{\underline{9.55 \text{ Mpa}}}.$$

PROBLEM #2 [5-81].

SOLUTION:-

Given:-

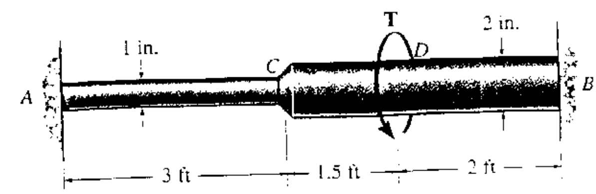
$G = 11 \times 10^3$ ksi. (A-36 steel)

$T = 500$ lb·ft

$= 6000$ lb-in.

REQUIRED:- $\tau_{max} = ?$

5–81. The A-36 steel shaft is made from two segments: AC has a diameter of 1 in. and CB has a diameter of 2 in. If it is fixed at its ends A and B and subjected to a torque of $T = 500$ lb · ft, determine the absolute maximum shear stress in the shaft.

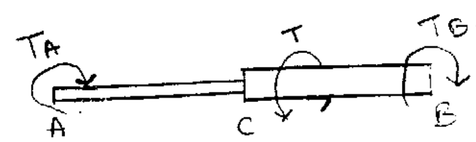


Probs. 5–81/5–82/5–83

EQUILIBRIUM:-

FROM FBD

$+ \circlearrowleft \Sigma T = 0$.



FBD

$\Rightarrow \quad -T_A + T - T_B = 0.$

$\Rightarrow \quad T_A + T_B = T$

$\Rightarrow \quad T_A + T_B = 6000$ — — — (1)

COMPATIBILITY:-

$\phi_{A/B} = 0$ ($\because$ Fixed Ends)

$$\frac{T_{AC} L_{AC}}{J_{AC} \cdot G} + \frac{T_{CD} \cdot L_{CD}}{J_{CD} \cdot G} + \frac{T_{DB} \cdot L_{DB}}{J_{DB} \cdot G} = 0$$

$$\Rightarrow \quad \frac{T_A \times 3 \times 12}{\frac{\pi}{2} \times (\frac{1}{2})^4} + \frac{T_A \times 1.5 \times 12}{\frac{\pi}{2} (\frac{2}{2})^4} + \frac{T_B \times 2 \times 12}{\frac{\pi}{2} (\frac{2}{2})^4} = 0$$

$378.17\, T_A + 15.28\, T_B = 0.$ — — — (2)

Solving (1) & (2)

$378.17\, T_A + 15.28\,(6000 - T_A) = 0$

$\Rightarrow T_A = 252.64$ lb·in.

& $T_B = 5747.36$ lb·in

$\therefore$

$$\tau_{AC} = \frac{T_A \times c}{J}$$

$$= \frac{252.64 \times 0.5}{\frac{\pi}{2} \times (0.5)^4}$$

$$= 1286.75 \text{ lbin}$$

$$\Rightarrow \tau_{AC} = 1.287 \text{ ksi}.$$

$$\tau_{CD} = \frac{T_A \times c}{J}$$

$$= \frac{252.64 \times 1}{\frac{\pi}{2}(1)^4}$$

$$= 160.84 \text{ lb.in}.$$

$$\tau_{DB} = \frac{T_B \times c}{J}$$

$$= \frac{5747.36 \times 1}{\frac{\pi}{2}(1)^4}$$

$$= 3659 \text{ lbin}$$

$$\Rightarrow \tau_{DB} = 3.66 \text{ ksi}.$$

$$\therefore \tau_{max} = \tau_{DB} = \underline{\underline{3}}.66 \text{ ksi}$$

PROBLEM # ③ (5-92)

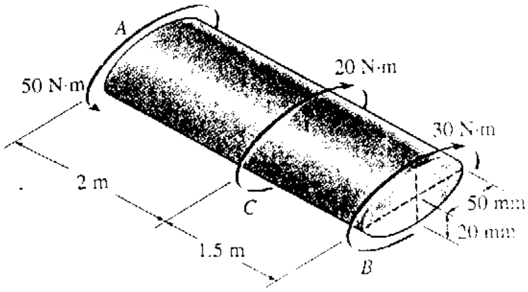
SOLUTION:-

Given:-

$G = 37$ GPa.

*5–92. The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading shown, determine the maximum shear stress within regions *AC* and *BC*, and the angle of twist ϕ of end *B* relative to end *A*.

5–93. Solve Prob. 5–92 for the maximum shear stress within regions *AC* and *BC*, and the angle of twist ϕ of end *B* relative to *C*.



Probs. 5–92/5–93

REQUIRED:-

$\tau_{max} = ?$

$\phi_{B/A} = ?$

from fig.

$T_{AC} = 50$ N-m.

$T_{CB} = 50 - 20 = 30$ N-m.

∴ Maximum Torque in section AC

∴ $T_{max} = T_{AC} = 50$ N-m.

∵ $\tau_{max_{AC}} = \dfrac{2T_{max}}{\pi a b^2}$ (For ellipse)

$= \dfrac{2 \times 50 \times 10^3}{\pi \times 50 \times 20^2} = 1.59$ Mpa.

$\Rightarrow \tau_{max_{AC}} = 1.59$ Mpa.

& Maximum Torque in CB = 30 N-m.

∴ $\tau_{max_{CB}} = \dfrac{2 T_{max}}{\pi a b^2} = \dfrac{2 \times 30 \times 10^3}{\pi \times 50 \times 20^2} = 0.955$

$\Rightarrow \tau_{max_{CB}} = 0.96$ Mpa.

P.T.O.

$$\phi_{B/A} = \phi_{B/C} + \phi_{C/A}.$$

$$\Rightarrow \phi_{B/A} = \frac{(a^2+b^2)\,T_{BC}.L_{BC}}{\pi a^3 b^3 G_2} + \frac{(a^2+b^2)\,T_{CA}.L_{CA}}{\pi a^3 b^3 G_2}$$

$$= \frac{(50^2+20^2)}{\pi(50^3)(20^3)\times 37\times 10^3}\left[\begin{array}{l}30\times 10^3\times 1.5\times 10^3 \\ \quad + 50\times 10^3\times 2\times 10^3\end{array}\right]$$

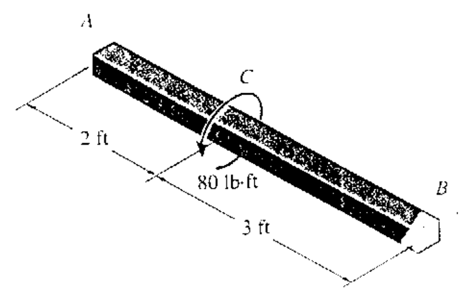
$$= 3.62\times 10^{-3} \text{ radians}.$$

$$\phi_{B/A} = 3.62\times 10^{-3}\times \frac{180}{\pi}$$

$$\therefore \phi_{B/A} = 0.207^\circ.$$

PROBLEM # 4 (5-97):-

5–97. The aluminum strut is fixed between the two walls at A and B. If it has a 2 in. by 2 in. square cross section, and it is subjected to the torque of 80 lb · ft at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 3.8(10^3)$ ksi.



Prob. 5–97

SOLUTION:-

GIVEN:-

$G_{al} = 3.8 \times 10^3$ ksi.

$T = 80$ lb·ft

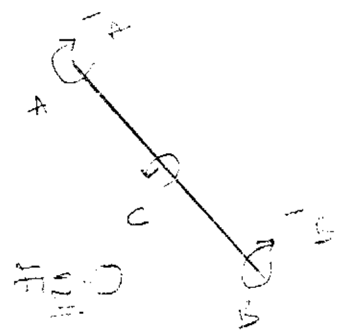
$= 80 \times 12 = 960$ lb·in.

REQUIRED:-

$T_A = ?$; $T_B = ?$

$\phi_C = ?$

EQUILIBRIUM:-



$+\circlearrowleft \Sigma T = 0$

$-T_A + 80 - T_B = 0$

$\Rightarrow T_A + T_B = 80$ - - - - (1)

COMPATIBILITY:-

$\phi_{A/B} = 0$

$\Rightarrow \phi_{C/A} + \phi_{C/B} = 0.$

$$\Rightarrow \frac{7.10\, T_{AC} \cdot L_{AC}}{a^4 \cdot G} + \frac{7.10\, T_{CB} L_{CB}}{a^4 \cdot G} = 0.$$

$\Rightarrow -T_A \times 2 + T_B \times 3 = 0$

$\Rightarrow -2\,T_A + 3\,T_B = 0$ - - - (2)

solving (1) & (2).

$-2\,T_A + 3\,(80 - T_A) = 0$

$\Rightarrow T_A = 48$ lb·ft.

& $T_B = 32$ lb·ft.

P.T.O.

$$\phi_C = \phi_{A/C} = \frac{7.10 \times T_{AC} \times L_{AC}}{a^4 \times G_2}$$

$$\Rightarrow \phi_C = \frac{7.10 \times 48 \times 12 \times 2 \times 12}{(2^4) \times 3.8 \times 10^6}$$

$$\Rightarrow \phi_C = 1.614 \times 10^{-3} \text{ rad.}$$

$$\Rightarrow \phi_C = 1.614 \times 10^{-3} \times \frac{180}{\pi}$$

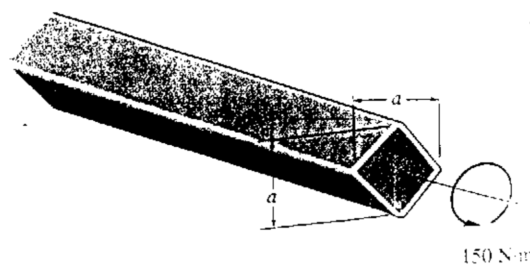
$$= 0.0925^\circ$$

$$\therefore \phi_C = \underline{\underline{0.0925^\circ}}.$$

PROBLEM # 5 (5-99):-

5–99. The plastic tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm. Neglect stress concentrations at the corners.

***5–100.** The plastic tube is subjected to a torque of 150 N · m. Determine the average shear stress in the tube if the mean dimension $a = 200$ mm. Each side has a thickness of $t = 3$ mm. Neglect stress concentrations at the corners.



Probs. 5–99/5–100

SOLUTION:-

Given:-

$\tau_{all} = 60\,MPa.$

$T = 150\,N\text{-}m.$

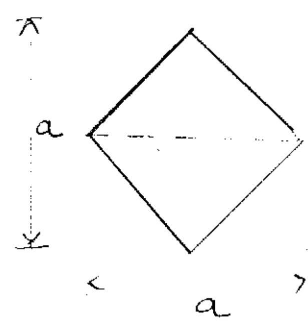
$t = 3\,mm.$

REQUIRED:-

$a = ?$

$\because \tau = \dfrac{T}{2t\,A_m}$

$A_m = 2 \times \left(\frac{1}{2} \times a \times \frac{a}{2}\right)$



$\Rightarrow A_m = \dfrac{a^2}{2}.$

$\because \tau = \dfrac{T}{2t\,A_m}$

$\Rightarrow A_m = \dfrac{T}{\tau \times 2t}$

$= \dfrac{150 \times 10^3}{60 \times 2 \times 3}$

$\Rightarrow A_m = 416.7\ mm^4$

$\Rightarrow \dfrac{a^2}{2} = 416.7$

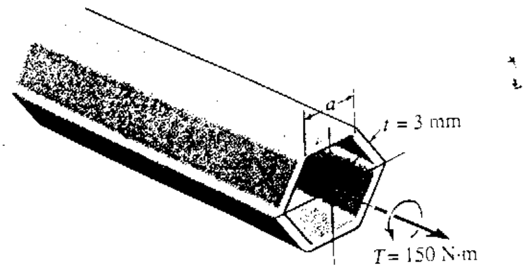
$\Rightarrow a = 28.9\ mm$

$\therefore a = \underline{28.9\ mm}.$

PROBLEM # 6 (5-110)

SOLUTION:-

5–110. The plastic hexagonal tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm.



Prob. 5–110

Given:-

$T = 150$ N-m.

$\tau_{all} = 60$ MPa.

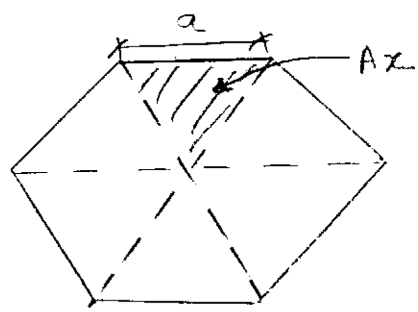
$t = 3$ mm.

REQUIRED:-

$a = ?$

$\because \tau = \dfrac{T}{2 t A_m}$

$\Rightarrow A_m = \dfrac{150 \times 10^3}{2 \times 3 \times 60} = 416.7 \text{ mm}^4.$



$A_m = 6 \times A_x$

$A_x = \frac{1}{2} \times a \times a \sin 60$

$\Rightarrow A_x = \frac{1}{2} \times a \times a \times \frac{\sqrt{3}}{2}$

$\Rightarrow A_x = \frac{\sqrt{3}}{4} a^2.$

$\therefore A_m = 6 \times \frac{\sqrt{3}}{4} \times a^2$

$\Rightarrow 416.7 = \frac{6 \times \sqrt{3}}{4} \times a^2$

$\Rightarrow a = \underline{12.7 \text{ mm}}$