

## Modeling of jointed rock

(1), (2)

o Strength as a function of joint orientation:

From before:  $\tau_{\max} = S_J + \sigma_d \tan \phi_J$

$$\sigma_d = \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos 2\beta$$

$$\tau_d = \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\beta$$

Combining  $\Rightarrow \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\beta = S_J + \left[ \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos 2\beta \right] \tan \phi_J$

after Algebraic .....

$$\frac{\sigma_1}{S_J} = \frac{2 + 2 \left( \frac{\sigma_3}{S_J} \right) \tan \phi_J}{(1 - \cos \beta \tan \tau_J) \sin 2\beta} + \frac{\sigma_3}{S_J}$$

$\sigma_1$  &  $\sigma_3$  : stresses at failure

- For what range of  $\beta$ , failure will be by sliding along joint??

\* Conditions for sliding Along a Discontinuity:

Failure criteria for intact rock:  $\tau_{\max} = S_R + \sigma \tan \phi_R$

Failure criteria for discontinuous rock:  $\tau_{\max} = S_J + \sigma \tan \phi_J$  if  $\sigma > \sigma_T$

Patton's law:  $\tau_{\max} = \sigma \tan (\phi_u + i)$  if  $\sigma < \sigma_T$

⇓

$$\text{If } \left. \begin{array}{l} \beta > \frac{2\beta_1}{2} \\ < \frac{2\beta_2}{2} \end{array} \right\} \text{if sliding}$$

if not  $\Rightarrow$  new crack

Failure will occur along the discontinuity of  $2\beta_1 < 2\beta < 2\beta_2$   
or  $\beta_1 < \beta < \beta_2$ .

alternatively  $\psi_1 + \psi > \psi_2$

Note: The value of  $\beta_1$  &  $\beta_2$  are function of

$$\beta_1 \text{ \& } \beta_2 = f(\sigma_1, \sigma_3, \phi_R, S_R, S_J, \phi_u, i, \sigma_T)$$

Failure is due to sliding along discontinuity

Failure through intact rock.

H.W. # 7

Water Pressure affects in Discontinuous Rock:

$$\sigma' + \sigma_T - P_w$$

Pore water pressure required to cause failure must be considered separately for each of the three modes of failure.

For a particular  $\psi$

a) Failure by riding over asperities

$$P_w = \sigma_3 + (\sigma_1 - \sigma_3) \left\{ \sin^2 \psi - \frac{\sin \psi \cos \psi}{\tan (\phi_u + 2)} \right\}$$

b) Failure by shearing through asperities:

$$P_w = \frac{S_J}{\tan \phi_J} + \sigma_3 + (\sigma_1 - \sigma_3) \left\{ \sin^2 \psi - \frac{\sin \psi \cos \psi}{\tan \phi_J} \right\}$$

c) Failure at intact rock:

$$P_w = \sigma_3 - \frac{[(\sigma_1 - \sigma_3) - 2 S_R \tan (45 + \frac{\phi_R}{2})]}{\tan^2 (45 + \frac{\phi_R}{2}) - 1}$$

Compute all three to find  $P_w$  minimum required to cause failure.

11. Deformability of rock masses (rather than intact rock) see Ch. 6 text.

11.1 Def.:

Modulus of permanent deformation.

M

11.2 In-situ Tests for determining rock mass modulus:

11.2.1 Plate Bearing test:-

i) Reaction Against Anchor

ii) Reaction Against an opposite wall

Theory: 
$$\varpi = \frac{Gp(1 - \nu^2)a}{E}$$

where  $\varpi$  = displacement measured (average of three readings)  
p = plate pressure (applied)  
a = plate radius  
G =  $\pi/2$  if plate is rigid or 1.7 if plate is flexible.

Typical Results:-

$$E = C a (1 - \nu^2) \frac{P}{\bar{\omega}_{\text{elastic component}}}$$

mass modulus

$$M = C a (1 - \nu^2) \frac{P}{\bar{\omega} - \bar{\omega}_{\text{elastic}}}$$

If extensometers are used:

$$E = \frac{(1 + \nu) P}{\bar{\omega}} \left[ \frac{-Z^2}{\sqrt{Z^2 + a^2}} + 2(1 - \nu) \sqrt{Z^2 + a^2} - (1 - 2\nu) \right]$$

$\bar{\omega}$  = displacement at depth Z.

See fig. pp.58

11.2.2 Borehole Dilatometer:- pressure a borehole  $\rightarrow \Delta u$

See fig. pp. 59

$$E = (1 - \nu) \Delta P \frac{a}{\Delta u}$$

easy test

### 11.3 Modulus of Fractured Rock (Rock mass) from intact Rock & Joint properties

A: Joint Testing

B: Intact Rock  
stress-strain test

elastic compression of intact rock

$$= \frac{\sigma_n}{E_{\text{rock}}} \cdot S \quad (\text{assuming } S \text{ much larger than joint width})$$

$$\text{compression of joint } \delta = \frac{\sigma_n}{K_n}$$

$$\text{Total Deformation} = \frac{\sigma_n}{E_{\text{rock}}} S + \frac{\sigma_n}{K_n}$$

$$\text{Strain in the rock mass} = \epsilon_{\text{rock mass}} = \frac{\text{Total Deformation}}{S}$$

$$= \frac{\sigma_n}{E_{\text{rock}}} + \frac{\sigma_n}{K_n S}$$

$$E_{\text{rock mass}} = \frac{\sigma_n}{\epsilon_{\text{rock mass}}} = \frac{\sigma_n}{\frac{\sigma_n}{E_{\text{rock}}} + \frac{\sigma_n}{K_n S}} = \frac{E_{\text{rock}} \cdot K_n S}{E_{\text{rock}} + K_n S}$$

Derivation for  $G_{\text{rock mass}}$  follows in the same manner.

$$G_{\text{rock mass}} = \frac{G_{\text{rock}} \cdot K_s \cdot S}{G_{\text{rock}} + K_s \cdot S}$$

→ Do both tests, to to field to measure S

→ Do test on rock, go to field & do tests with small  $\Rightarrow$  Extrapolate for larger S.

11.4 Rock Indices for estimating  $E_{\text{rock mass}}$ : (approximate)

a. Bieniawski (1978):  $E \text{ (GPa)} = 2 \text{ (RMR)} - 100$  for RMR  $\geq 55$

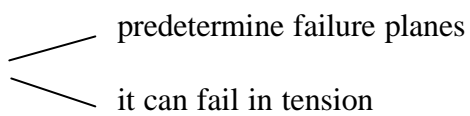
b. using Modulus Reduction Factor =  $\frac{E_{\text{rock mass}}}{E_{\text{rock}}} = f \text{ (RQD)}$

see Fig. 7.2 from US army corp of engg.

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H.W. # 8  
Ch. #4 & 9

## 12. Slope Stability in Rocks

More complicated than soil 

- predetermine failure planes
- it can fail in tension

### 12.1 Types of Failure

- plane sliding  
(one joint of orientation is significant)
- wedge-type failure  
(Two joints sets (or more) are significant)

iii) Toppling failure

### 12.2 Plane Sliding:

#### a) Kinematics

if  $\delta > a$       no failure  
if  $\delta < a$       failure



For failure to be kinematically feasible  $\delta < \alpha$

(the discontinuity must (daylight))

o stereographic plot presentation for (a)

represents dip vector  $\delta_1$

if  $\delta = \delta_1$  failure is kinematically possible

if  $\delta = \delta_2$  failure is not possible

it is enough for  $\delta$  to be  $> \alpha$  failure to failure, but  $\delta$  have to be  $> \delta_{\min}$ .

b) strength : ( $\phi$ ):

since  $\phi > \alpha$  : block will not slide.

$\therefore \phi < \alpha$  block will slide.

o stereonet presentation for (b)

If dip vector plots in this area, failure can't occur.

$\therefore$  joint is not steep enough

stereonet for (a) & (b).

Combining the 2 criteria:

failure can only occur if dip vector plots in shaded area.

joint must both daylight and its angle must exceed  $\phi_j$ .

Note: This only due to self weight of block.

- still wt. of adjacent blocks.
- still wt. of adjacent water pressure.

Example of using stereonet for vector addition:

Ex.1 given two forces:      200 = acting in the direction N80°W plunging 50°  
   600 = acting in the direction S40°W plunging 20°

Find the resultant force and its direction.

Solution:      (Procedure)

1. Find the common plane of the two forces.

rotate until they plot on same circle. → angle = 55°

2. Determine the resultant force by parallelogram theorem. (draw to scale). Find resultant. Find angle = 13°.

3. Plot direction of the resultant on stereonet.

S50°W, 290°

### Forces acting on rock blocks:-

1. Block self weight.
2. Forces transmitted from adjacent blocks.
3. Forces due to water pressure.
4. Dynamic loads.
5. Reinforcement (rock bolts).

if  $\phi < \phi_J$       stable

### How to plot $\phi$ -circle?

To plot the friction circle on stereonet:

1. Locate the normal to the failure plane,  $\hat{n}$
2. Plot two points on the diameter of the circle each being  $\phi_J^\circ$  from the normal (measured along the diameter).
3. Construct  $\phi$ -circle having the line between the two points (in step # 2) as the diameter.

If resultant vector,  $\hat{r}$  is inside the circle, failure doesn't occur.

factor of safety against sliding

$$\text{F.S.} = \frac{\tan \phi_{\text{available}}}{\tan \phi_{\text{required}}}$$

Ex. 1. A joint strikes S30°E and dips 60° NE

$$\phi_J = 25^\circ$$

resultant of all forces  $\hat{r}$  acts in the direction S50°W and plunges 20°,

Find the factor of safety.

- Procedure
1. Plot the friction circle for the joint.
  2. Plot resultant force  $\hat{r}$ .
  3. Find angle between  $\hat{r}$  and  $\hat{n}$ . (put them in same greater circle)

$$\Rightarrow \phi_{\text{reqd.}} = 15^\circ$$

4.  $\therefore \phi_{\text{available}} = \phi_J = 25^\circ$

$$\therefore \text{F.S.} = \frac{\tan 25}{\tan 15} = 1.74$$

Ex. 2 Assume that the only force acting on the block is its self-weight. Determine the minimum bolt force and direction required to raise the F.S. to 1.0 for a block weighing 20 tons.

i.e. dipping  $90^\circ \Rightarrow$  plot @ the center of stereonet.

$\hat{w}$  = weight vector

$\rightarrow \hat{w}$  not inside friction circle  $\Rightarrow$  F.S.  $\ll$  1.0.

- Procedure:
- 1)  $\hat{w}$  will plot @ center of stereonet  $\Rightarrow$  F.S.  $\ll$  1.0.
  - 2) Find the point on the  $\phi$ -circle at which  $\hat{r}$  will make the smallest angle with  $\hat{w}$ .

answer.  $35^\circ$

if bolt driven S $60^\circ$ W

- 3) Construct force diagram

own wt.	}	bolt force & resultant
angle of resultant		

4. Determine a line of minimum length from tip of  $\hat{w}$  to the  $\hat{r}$  line.

$|B| = 11$  Tons  $35^\circ$  up from horizontal.

Ex. 3. What bolt force would be required to achieve F.S. = 2.5.

Sol. 
$$\text{F.S.} = \frac{\tan \phi_{\text{available}}}{\tan \phi_{\text{reqd.}}} = \frac{\tan 25^\circ}{\tan \phi_{\text{reqd.}}} = 2.5$$

$$\tan \phi_{\text{reqd.}} = \frac{\tan 25^\circ}{2.5} = \quad \Rightarrow \phi_{\text{reqd.}} = 10.5^\circ$$

min. angle between  $\hat{w}$  and  $\phi_{\text{circle}} = 49.5^\circ$

$|B| = 15$  Tons  $49.5^\circ$  up from horizontal

Ex. 4. What both force would be required if the bolt are to be driven due west.

$$|B| = 16.5 \text{ T}$$



## Analysis of Plane Slides by traditional Block Sliding Analysis

Ref. Book (Hock & Bray 1977)

#1.

Crack intercepts crest of slope

$$W = \frac{1}{2} \gamma H^2 \left[ \left(1 - \left(\frac{Z}{H}\right)^2\right) \cot \delta - \cot \alpha \right]$$

#2.

Crack intercepts face of the slope.

$$W = \frac{1}{2} \gamma H^2 \left[ \left(1 - \frac{Z}{H}\right)^2 \cot \delta (\cot \delta \tan \alpha - 1) \right]$$

$$\text{Area of sliding surface: } A = \frac{H - Z}{\sin \delta} * 1$$

Resultant of water pressure along the vertical crack.

$$V = \int_0^{Z_w} Z dZ = \frac{1}{2} \gamma_w Z_w^2$$

Resultant of water pressure acting on the sliding

$$\text{surface} = U = \gamma_w \frac{Z_w}{2} \cdot A = \frac{1}{2} \gamma_w Z_w A.$$

### Shear Forces Along Sliding Surface:

#### Forces resisting shear:

$$(S_J + \sigma_n' \tan \phi_J) A$$

$$\sigma_n' = \sigma - u = \frac{W \cos \delta - V \sin \delta - U}{A}$$

Sliding occurs when the driving shear forces = forces resisting shear.

$$\text{F.S. } (W \sin \delta + V \cos \delta) = S_J A + (W \cos \delta - V \sin \delta - U) \tan \phi_J$$

#### For Case #1:

$$\begin{aligned} \text{F.S. } & \left[ \frac{1}{2} \gamma H^2 \left[ \left(1 - \left(\frac{Z}{H}\right)^2\right) \cot \delta - \cot \alpha \right] \sin \delta + \frac{1}{2} \gamma_w Z_w^2 \cos \delta \right] \\ & = S_J \frac{H-Z}{\sin \delta} + \left[ \frac{1}{2} \delta H^2 \left[ \left(1 - \left(\frac{Z}{H}\right)^2\right) \cot \delta - \cot \alpha \right] \cos \delta \right. \\ & \quad \left. - \frac{1}{2} \gamma_w Z_w^2 \sin \delta - \frac{1}{2} \gamma_w Z_w A \right] \tan \phi_J \end{aligned}$$

Since we have some control over  $\alpha_1$ , we can solve for it.

$$\text{Cot } \alpha = \frac{[a (\text{F.S. } \sin \delta - \cos \delta \tan \phi) + U \tan \phi + V (\sin \delta \tan \phi + \text{F.S.} \cos \delta)] - S_j A}{b (\text{F.S. } \sin \delta - \cos \delta \tan \phi)}$$

where  $a = \frac{1}{2} \gamma H^2 \left( 1 - \left( \frac{Z}{H} \right)^2 \right) \text{Cot } \delta$

$$b = \frac{1}{2} \gamma H^2$$

$\Delta S_j$  is more important for steep slopes ( $\alpha$  is large)

$\Delta \phi_j$  is more important for high slopes ( $H$  is large)

Drainage of water can be very effective in stabilizing rock slopes.

\* Wedge Failure Analysis

a) Kinematics: for sliding of a wedge, use  $\hat{I}$  (the interaction of 2 planes in place of the dip vector  $\hat{D}$ ).

$\therefore$  if the plunge of  $\hat{I}$  is less than  $\alpha$ , the wedge sliding is kinematically possible.

- Find normals to two planes  $\hat{n}_1, \hat{n}_2$
- Rotate until  $\hat{n}_1$  &  $\hat{n}_2$  on one greater circle.

Failure can occur in one of three ways.

1. if  $\hat{D}_1$  daylights
2. if  $\hat{D}_2$  daylights      plane sliding
3. if  $\hat{I}_{12}$  daylights  $\rightarrow$  sliding

If we have 3 sets of joints; Failure can occur on:

1.  $\hat{D}_1$  daylight
2.  $\hat{D}_2$  daylight
3.  $\hat{D}_3$  daylight
4.  $\hat{I}_{12}$
5.  $\hat{I}_{13}$
6.  $\hat{I}_{23}$

b) Strength: If  $\phi_J >$  plunge of  $\hat{I}$  failure cannot occur. Furthermore, for very acute (steep) wedges, considerable strength is obtained from roughness along the discontinuities, so  $\hat{I}$  can often be steeper than  $\phi_J$  without failure.

## Conventional Wedge Analysis

Ref. Hock & Bray "Rock Slope Engineering"  
Institution of Mining & Metallurgy 1981

### Geometry:

$\psi_{fi}$  = inclination of slope, slope measured in  
the view at right angles to the line of  
intersection.

$\phi_J$  = joint friction angle.

actual slope is  $> \psi_{f2}$

$\psi_I$  = plunge of line of intersection.

Sliding will occur if  $\psi_{fi} > \psi_I > \phi$

### Forces:

Assume sliding is resisted by friction only.

$$F.S. = \frac{(R_A + R_B) \tan \phi_J}{W \sin \psi_i}$$

$$\Sigma F_H \Rightarrow R_A \sin \left( \beta - \frac{1}{2} \xi \right) = R_B \sin \left( \beta + \frac{1}{2} \xi \right)$$

$$\Sigma F_r \text{ along line of intersection} \Rightarrow R_A \cos \left( \beta - \frac{1}{2} \xi \right) - R_B \cos \left( \beta + \frac{1}{2} \xi \right) = W \cos \psi_i$$

Solving for  $R_A$  &  $R_B$  and adding

$$R_A + R_B = \frac{W \cos \psi_i \sin \beta}{\sin \frac{1}{2} \xi}$$

$$\therefore \text{F.S.} = \frac{\sin \beta}{\sin \frac{1}{2} \xi} \cdot \frac{\tan \phi_J}{\tan \psi_i}$$

f.s. for plane sliding

$$(\text{F.S.})_{\text{wedge}} = K * (\text{F.S.})_{\text{plane}}$$

Fig. 96:

$\xi$  : angle between joints

$\beta$  : angle of tilt of line of intersection

Note: This is limited to joints having their normals coplanar (same plane)  $180^\circ$  in phase.

$$\text{F.S.} = A \tan \phi_A + B \tan \phi_B$$

A & B see pp.211 - end.

<u>Ex.</u>	Dip	Dip. direction	$\phi_J$
Plane A	$40^\circ$	165	$35^\circ$
Plane B	$70^\circ$	285	20
	-----	-----	
Difference	$30^\circ$	120	

$$A = 1.5 , \quad B = .7$$

$$F.S. = 1.5 \tan 35 + 0.7 \tan 20 = 1.3$$

This method is good better :  $\xi$  &  $\beta$  is difficult to find in field.

If  $F.S. \geq 2$   $\therefore$  Wedge failure is almost impossible.

o Wedge Analysis on stereographic projection.

Text pp.---- Fig. 8.16

1. Find  $\hat{n}_1$  and,  $\hat{n}_2$  then  $\hat{I}_{12}$
2. Draw greats circle through  $\hat{n}_1, I_{12}$  and  $\hat{n}_2, I_{12}$
3. Construct  $\phi$  circles for  $n_1$  and  $n_2$ .  
This gives the four points of intersection: p, q, s & t.
4. Construct great circles connecting p.s. and q.t.

if resultant of all forces.

## Toppling Failure

a) flexural toppling

b) block toppling

c) combination of flexural and toppling

- Static Analysis of Block Toppling

a. Single block:

if W is to left of  $\Rightarrow$  Failure

condition if impending failure

$$\text{Cot } \alpha = \frac{\Delta y}{\Delta x}$$

$$\text{Cot } \alpha < \frac{\Delta y}{\Delta x} \Rightarrow \text{failure will occur.}$$

Long narrow blocks on steep slopes are more susceptible to failure.

b. Multiple blocks:



Resisting Moments:

$$P_{n-1} (L_n) + \mu P_n \Delta x + W \cos \alpha \frac{\Delta x}{2}$$

Overturing Moments

$$P_n (M_n) + \sin \alpha \frac{\Delta y}{2}$$

Setting R.M. = O.M. , toppling will occur if

$$P_{n-1} < \frac{P_n (M_n - \mu \Delta x) + \frac{W}{2} (\Delta y \sin \alpha - \Delta x \cos \alpha)}{L_n}$$

Many equations

Ref. Zanbak, Caner “Design Charts for Rock Slopes Susceptible to Toppling” J. of Geot. Engg., ASCE, Vol. 109, No. 8.

## Stereographic Analysis for Flexural Toppling:

major principle direction  
∴ no stress normal to slope

Toppling will occur only if there is relative motion between dipping layer.

slippage must occur

Criterion for toppling failure: resultant should be outside  $\phi$ -circle.

If we lay off an angle  $\phi_J$  from the normal to the dipping beds, and it falls outside of the slope, then failure will occur because the direction of applied compression is outside of the  $\phi$ -circle.

Toppling will occur if  $(90 - \delta) + \phi_J < \alpha$

Or  $\delta > 90 + \phi_J - \alpha$

$\phi_J$

$\phi_J$

Toppling

No Toppling

If  $\hat{n}$  in this region, flexural toppling will occur if strike is

if  $\quad > 30^\circ \quad \therefore \quad$  toppling

is not towards excavation.

H.W. #8

Ch. 8

Prob. # 1 : Use: Set 1 Strike N40°E

Prob. # 2: Use  $\phi_J = 30^\circ$

Prob. # 3: Use: dip 55°NE  
P = 600 Tons  
 $\phi = 35^\circ$

### 13.0 Foundation on Rock

- Excessive settlement → compressibility of joints
- Bearing capacity

Fig. 9.1

Karstic: Ca O<sub>2</sub>, dissolvable sinkable in Florida

#### a) Shallow Foundations:

spread footings

#### b) Deep Foundations

H. piles  
Precast conc. — Piles  
Pipe piles

Pier  
Socketed  
into rock

### 13.1 Shallow Foundations:

For intact rock in its elastic range:

Settlement may be predicted by:

$$u = \frac{C.P (1 - \nu^2)a}{E} \quad : \text{ plate bearing}$$

where

$$C = \begin{array}{l} - \pi/2 \text{ if rigid} \\ - 1.7 \text{ if flexible} \end{array}$$

P : applied stress

E : Young's Modulus

$\nu$  : Poisson's ratio

a : radius of footing (or equivalent)

- o If rock is not homogeneous ..... se same as soil

Stress distribution beneath footings.

- a) for homogeneous isotropic rock, elasticity solution are generally available.  $\rightarrow$  point load ..... integrate to get it for any shape.
- b) for heterogeneous, anisotropic rock, finite element methods may be required.

c) Fig. 9.7 Line load

Vertical

Horizontal

if there is a joint then tension  
will in opening crack  $\Rightarrow$  stress will not be  
transmitted if total stress is tension.

Inclined

Fig. 9.8 not intact rock

resultant can't go  
outside  $\phi_J$

$\Rightarrow$  slippage  $\Rightarrow$  realignment of stresses

## Close Form Solution

1. Resolve Q & P

// &  $\perp$  to pidding planes

1977 Bray

$$\sigma_r = \frac{h}{\pi r} \left[ \frac{X \cos \beta + yg \sin \beta}{(\cot \beta - g \sin^2 \beta)^2 + h^2 \sin^2 \beta \cos^2 \beta} \right]$$

$$\text{where } g = \left( 1 + \frac{E}{(1-\nu^2)k_n S} \right)^{1/2}$$

$$h = \left\{ \left( \frac{E}{1-\nu^2} \right) \left[ \frac{2(1+\nu)}{E} + \frac{1}{k_s \cdot S} \right] + 2 \left( g - \frac{\nu}{1-\nu} \right) \right\}^{1/2}$$

E,  $\nu$  : are intact rock properties

$k_n$ ,  $k_s$  : are joint stiffnesses

S = Spacing

Fig. 9.9

Fig. 9.10

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\* Strains and settlements beneath loaded rock masses.

Procedure

1. determine the stress distribution.
2. determine the equivalent elastic properties

$$G_{ns} = G_{nt} = \frac{G_{\text{Rock}} K_s S}{G_{\text{Rock}} + K_s S}$$

Similarly

$$E_n = \frac{E_{\text{Rock}} K_n S}{E_{\text{Rock}} + K_n S}$$

$$E_s = E_t = E_{\text{rock}}$$

$$v_{sn} = v_{tn} = \frac{\epsilon_n}{\epsilon_s} = v_{\text{rock}}$$

by symmetry,

$$\begin{aligned} \frac{v_{sn}}{E_s} &= \frac{v_{ns}}{E_n} & \therefore v_{ns} &= v_{nt} = \frac{E_n}{E_s} v_{sn} \\ & & &= \frac{E_{\text{rock}} K_n S}{E_{\text{rock}} + K_n S} \cdot \frac{v_{\text{rock}}}{E_s = E_{\text{rock}}} \\ & & &= \frac{K_n S \cdot v_{\text{rock}}}{E_{\text{rock}} + K_n S} \end{aligned}$$



3. Employ the constitutive relationship for transversely isotropic media:

$$\begin{Bmatrix} \epsilon_n \\ \epsilon_s \\ \epsilon_t \\ \gamma_{ns} \\ \gamma_{nt} \\ \epsilon_{st} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_n} & \frac{-\nu_{sn}}{E_s} & \frac{-\nu_{sn}}{E_s} & 0 & 0 & 0 \\ \frac{-\nu_{sn}}{E_s} & \frac{1}{E_s} & \frac{-\nu_{st}}{E_s} & 0 & 0 & 0 \\ \frac{-\nu_{sn}}{E_s} & \frac{-\nu_{st}}{E_s} & \frac{1}{E_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_n \\ \sigma_s \\ \sigma_t \\ \tau_{ns} \\ \tau_{nt} \\ \tau_{st} \end{Bmatrix}$$

## Bearing Capacity of Shallow Foundations on Rock.

Fig. 9.5

∴ The largest horizontal stress that can be developed to support the rock beneath the footing is the unconfined compressive strength,  $q_u$ .

$$\text{since } \sigma_{1f} = \sigma_{3f} \tan^2 (45 + \phi/2) + q_u$$

$$\therefore q_f = q_u \tan^2 (45 + \phi/2) + q_u$$

$$q_f = q_u (N_\phi + 1)$$

bearing capacity factor  $\Rightarrow N_\phi = \tan^2 (45 + \frac{\phi}{2})$

$$q_{\text{all}} = \frac{q_f}{\text{F.S.}} = \frac{q_f}{3} \quad \text{especially F.S.} = 3.0$$

o Table 9.2: most rock of the region is very conservative

New York Detroit → largest values → highest buildings.

Drop Foundations on Rock:

- a) bearing capacity increases with depth due to increase in confinement.

$$\sigma_v \neq 0 \Rightarrow$$

- b) bearing capacity doesn't increase with depth.

Settlement of Deep Foundations: - rock  
- pier: pile or corrosion

- a) due to rock deflection for

elastic material : 
$$w_{\text{base}} = \frac{\pi P_{\text{end}} (1 - \nu^2) a}{E \cdot n}$$

$n = f$  (embedment depth, radius and  $\nu$ )

$\nu \backslash l/a$	0	2	4	6	8	14
0	1.0	1.4	2.1	2.2	2.3	2.4
.3	1.0	1.6	1.8	1.8	1.9	2.0
.5	1.0	1.4	1.6	1.6	1.7	1.8

b) due to concrete deformation:

$$\omega_{\text{concrete}} = \frac{P \ell_c + \ell}{E_c}$$

But P is not constant along the pile.

c) Correction for load transfer due to side friction:-  
Neglect side friction through the soil.

derived in Appendix : same as one for consolidation

$$\sigma_z = P_{\text{max}} \cdot e^{-\left[ \frac{2 v_c \tan \phi_{rc}}{1 - v_c + (1 + v_r) \frac{E_c}{E_r}} \cdot \frac{Z}{a} \right]}$$

$v_c, v_r$  : Poisson's ratio of concrete & rock

$\phi_{cv}$  : friction angle for rock/concrete & interface

$E_c, E_r$  : Young's moduli for concrete & rock.

$$\therefore \Delta w = \frac{1}{E_c} \int_0^\ell (P_{\text{max}} - \sigma_z) dZ$$

d)  $\therefore$  Total Deformation (settlement) =  $w_{\text{base}} + w_{\text{concrete}} - \Delta w$

Determining depth of “socketing” into rock to insure against bearing capacity failure.

- (1) Ref: Osterberg & Gill  
F.E.M. study (1971)

found that even small embedment (socketing) into rock greatly reduces  $P_{end}$  by taking load in side friction.

Therefore shaft diameters can be reduced.

Fig. 9.18

- (2) Ladanyi (1977) proposed a procedure for determining socketing depth.

o Assumptions:

- a) no load transfer along concrete/soil interface.
- b) bond between concrete and rock broken  
⇒ use residual strength.

$$\sigma_{max} = \frac{P}{\pi a^2}$$

$$\sigma_z = \sigma_{max} e$$

For soft rock:

$$\tau_{residual} = \alpha S_u$$

$S_u$  : undrained shear strength

$\alpha$  : reduction factor,  $0.3 < \alpha < 0.9$  , typically  $\alpha = 0.5$

For hard rock:  $\tau_{res.} \approx \frac{q_u}{20}$

where  $q_u$  = unconfined comp. strength.

Procedure:

1. Determine shaft diameter "a" based on concrete strength.
2. Assume that  $Q_p = 0$ , then  $l = l_1 = \frac{P}{2 \pi a \tau_{all}}$
3. Choose  $l = l_2$  such that  $l_2 < l_1$ .
4. Compute  $\sigma_Z$  @  $Z = l_2$ .
5. If  $\sigma_Z$  @  $Z = l_2 > q_{all}$ , then assume a new  $l_2$ .
6. If  $\sigma_Z$  @  $Z = l_2 \leq q_{all}$ , determine  $\tau$  along perimeter of pile

$$\tau = \frac{1}{2 \pi a l_2} \left[ \left( 1 - \frac{\sigma_Z @ l_2}{\sigma_{max}} \right) P \right]$$

7. Repeat the procedure to find the shortest "l" for which  $\sigma @ l \leq q_{all}$  and  $\tau \leq \tau_{all}$

$q_{all} ??? \therefore$  for shallow foundation. No vertical stress

$$q_{all} = \frac{q_u (N_\phi + 1)}{F.S.}$$

### For Deep Foundation

Since  $q_{all}$  as determined for shallow foundations would be very conservative,  $\therefore$  use a smaller factor of safety, say 1.5 to 2.

Final : open book & notes.

(1) Geological Rock =  
texture

(2) Permeability

(3) Testing - Brazilian  
- 2 point load

(4) Triaxial

$$\sigma_3 =$$

$$\sigma_1 = \frac{\Delta V}{V} = \epsilon_1 = \epsilon_2 + \epsilon_3$$

$$\frac{\Delta L_1}{L_0}$$

$$\phi \quad V \quad \& \quad \epsilon_2 = \epsilon_3 = \underline{\quad}$$

$$E \quad \epsilon_1 = f(\sigma_1 \sigma_2 \sigma_3 E, Y)$$

$$\epsilon_2 =$$

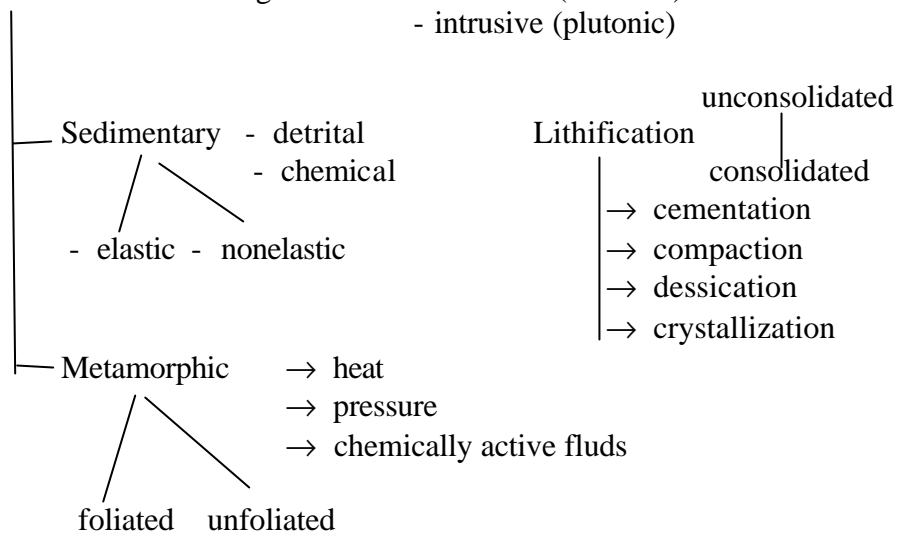
$$E, V \rightarrow G$$

$$G = \frac{E}{2(1+\nu)}$$

(5) Denatoric - stress  
- strain



- Disc. Faults → shears → joints → fissures → microfissures
- Geo. class → Geometric → Igneous → - extrusive (volcanic)  
- intrusive (plutonic)



shale  
↓  
slate  
↓  
phyllite  
↓  
schist  
↓  
gneiss

- $n = \frac{V_p}{V_t}$
- $G_{s,r} = \sum_{i=1}^n G_{s,i} V_i$
- $\gamma_{dry} = G_{s,r} \gamma_w (1 - n)$

- $k = K.F. = K \frac{\gamma_w}{\mu} \quad q_i = k_i \frac{dh}{dx_i} A_L$

$$k = \frac{q \cdot \ln(R_2/R_1)}{dx_i}$$

$$k = \frac{\gamma_w}{6\mu} \left( \frac{e^2}{S} \right)$$

- Point load test  
d = 54 mm  
L = 1.5 D  
 $I_s = P/D^2$   
 $Q_u = C I_s$

- $V_p^2 = E/\delta$

$$\frac{1}{V_\ell^*} = \sum_{i=1}^n \frac{C_i}{V \ell_i} \Rightarrow I_Q \% = \frac{V_\ell}{V_\ell^*} * 100\% \quad \text{no fissure } I_Q = 100 - 1.6 n \%$$

- $RQD = \frac{\sum LX''}{\sum L}$
- Failure → Tension  
→ Shear  
→ Compaction
- Unconfined comp. test  $\frac{L}{D} = 2 - 3 \Rightarrow q_u = \frac{P_{max}}{A}$
- Triaxial Testing: strength = f (confining p.)
- Brazilian split cylinder test  $\sigma_t = \frac{2P}{\pi dt}$
- Flex  $\sigma_t = \frac{16}{3} \frac{PL}{\pi d^3}$   $I = \frac{\pi d^4}{64}$
- Ring shear test  $\tau_{max} = \frac{2P}{\pi d^2}$

\*  $\sigma_{ij} = \frac{1}{3} \sigma_{kk} \delta_{ij} + \tau_{ij}$

$$\left. \begin{aligned} D_1 &= \frac{\Delta \bar{\sigma}}{\Delta \bar{\epsilon}} = 3K \Rightarrow \\ D_2 &= \frac{\Delta \sigma_{1, dev}}{\Delta \epsilon_{1, dev}} = 2G \Rightarrow \end{aligned} \right\} E = \frac{3D_1 D_2}{D_2 + 2D_1}, \quad \nu = \frac{D_1 - D_2}{2D_1 + D_2}$$

$$G_{yx} = \frac{E_x}{2(1 + \nu_{yx})} \Rightarrow G = \frac{E}{2(1 + \nu)} \Rightarrow 2G = \frac{E}{1 + \nu}$$

$$K = \frac{E}{3(1 - 2\nu)} \Rightarrow 3K = \frac{E}{(1 - 2\nu)}$$

$$\lambda = \frac{E\nu}{(1 + \nu)(1 - 2\nu)}$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \lambda+2G & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2G & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

(1) Failure

(2)

(3)

(4)

1) Road

2) Foundation

### Multistage

I	$\sigma_3$	$\sigma_1$
II	$\sigma_3$	$\sigma_1$
III	$\sigma_3$	$\sigma_1$

1) Draw joint failure

2) He gave us  
 $\tau = S_i + \sigma \tan \phi$   
for intact rock.

For failure occurs on the joint  $\beta_1 < \beta < \beta_2$

For failure occurs on the intact  $\beta < \beta_1$   
 $\beta > \beta_2$

Disadvantage of overcoring:

1. The linear dependence of the stresses upon, the one elastic const.  $t$ .
2. Large drill cores to make the rock not crack.