* **Comments on:**

o <u>Effective stress law for rocks</u>:

1) $\sigma_{total} = \sigma' + u$

$u = z_w \gamma_w$

This is OK for soil, but for rock is not. Back to the original derivation:

$\sigma_{total} = \sigma' + u\,(1 - a_s)$

a/A

in soil as is negligible.

But in rock as may not be as negligible as in soil, especially in cemented rock.

But how to find as !!!

2) For low porosity rocks, pore water pressure may not be continuous.

<u>Anisotropy</u>: angle of bedding plane, foliation

Fig. 3.23 for continuous rock.

1) ϕ = 30 corresponds to failure angle ≈ (45 - ϕ/2)

2) <u>Cracked rock</u>

- Fig. 3.21 size effect $\underset{\text{large}}{q_u} = \underset{\text{small}}{\dfrac{q_u}{4}}$

some reasons as k, V

∴ encountered more as cracks …….

$\Downarrow$

∴ use high factor of safety i.e.

8.0 In-situ stresses:

What are σ_1 and σ_3 ?
Can we predict them?
Why we want to measure them?

- in soils we generally assume $\sigma_x = \sigma_y \neq \sigma_z$
o in rock it is less likely that $\sigma_x = \sigma_y$.
because $\rightarrow$ tectonic forces
$\rightarrow$ layer are not horizontal

1) <u>Excavation</u>

plan view : excavate along

$\therefore$ Excavate // to larger stress

2) Tunneling

if you have a choice to build (1) or (2)
plan view
aim : safety

Rock burst is likely to happened more in tunnel (1)

$\therefore$ choose (2) $\therefore$ (1) is unsafe

if safety is of concern.

$\therefore$ make tunnel $\perp$ to larger stress.

- Flat topography:

  σ_v & σ_H generally correspond to the principal stresses.

- Hilly topography:

  principal stresses at the surface will follow the topography.

* <u>Vertical stresses:</u>

For flat unfolded earth

$$\sigma_v = \underbrace{\gamma}_{\text{unit wt. of rock}} \cdot \underbrace{Z}_{\text{depth}}$$

* <u>Horizontal stress</u>: generally much more difficult to predict than σ_r.

let $\quad K = \dfrac{\sigma_H}{\sigma_v}$

look at τ - vs σ.

if σ_v is given $\rightarrow$ min. σ_H before failure
$\rightarrow$ max. σ_H before failure

$$K_{min} = \frac{\sigma_{H,\,min}}{\sigma_v} = K_{active}$$

$$K_{max} = \frac{\sigma_{H,\,max}}{\sigma_v} = K_{passive}$$

These two values are associated with different tectonic movements.

1)

$$\sigma_v = \sigma_1$$

$$\sigma_H = \sigma_3 = K_a\,\sigma_v$$

Normal fault

2)

$$\sigma_v = \sigma_3$$

$$\sigma_H = \sigma_1 = K_p\sigma_v$$

Reverse fault or thrust fault

<u>For K_a:</u> $$\sigma_1 = q_u + \sigma_3 \tan^2 (45+\phi/2)$$

$$1 = \frac{q_u}{\sigma_1} + \frac{\sigma_3}{\sigma_1} \tan^2 (45+\phi/2)$$

$$1 = \frac{q_u}{\sigma_1} + K_a \tan^2 (45+\phi/2)$$

$$K_a = \left(1 - \frac{q_u}{\sigma_1}\right) / \tan^2 (45+\phi/2)$$

or $$K_a = \tan^2 (45-\phi/2) - \frac{q_u}{\sigma_1} \tan^2 (45-\phi/2)$$ where $\sigma_1 = \sigma_v$

<u>For K_p:</u> $$\sigma_1 = q_u + \sigma_3 \tan^2 (45+\phi/2)$$

$$\frac{\sigma_1}{\sigma_3} = \frac{q_u}{\sigma_3} + \tan^2 (45 + \phi/2)$$

$$K_p = \frac{q_u}{\sigma_3} + \tan^2 (45 + \phi/2)$$ where $\sigma_3 = \sigma_v$

σ_v is easier to find, but not always, see text, Fig. 4.3.

stiffer layer

loads transferred through stiffer layer to this point

σ_v maybe $3 * \gamma Z$

- Fig. 10.6 California…….
- Fig. 4.7

near surface

$\gamma_H > \sigma_v \Rightarrow K > 1.0$

$\therefore \sigma_v = 0$ near surface and σ_H is more

* Change in K due to erosion: σ_v decrease $\Rightarrow$ K increase.

Assume elasticity & isotropy

Horizontal $$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{xx} + \nu(\varepsilon_{yy} + \varepsilon_{zz})\right]$$

Vertical $$\sigma_{zz} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{zz} + \nu(\varepsilon_{xx} + \varepsilon_{yy})\right]$$

$$\therefore \quad K = \frac{\sigma_{xx}}{\sigma_{zz}} = \frac{(1-\nu)\varepsilon_{xx} + \nu(\varepsilon_{yy} + \varepsilon_{zz})}{(1-\nu)\varepsilon_{zz} + \nu(\varepsilon_{xx} + \varepsilon_{yy})}$$

assume $\varepsilon_{xx} = \varepsilon_{yy} = 0$ (no change in horizontal strain)

$$\therefore \quad K = \frac{\nu\,\varepsilon_{zz}}{(1-\nu)\,\varepsilon_{zz}} = \frac{\nu}{(1-\nu)}$$

Before erosion After erosion

$$K_o = \frac{\sigma_{x_o}}{\sigma_{z_o}} \qquad K = \frac{\sigma_x}{\sigma_z}$$

$$K = \frac{\sigma_x}{\sigma_z} = \frac{\sigma_{x_o} - \Delta\sigma_x}{\sigma_{z_o} - \Delta\sigma_z} = \frac{K_o Z_o \gamma - K\gamma\Delta Z}{\gamma Z_o - \gamma\Delta Z}$$

assuming γ doesn't change

$$K = \frac{K_o Z_o - \frac{\nu}{1-\nu} \Delta Z}{Z_o - \Delta Z}$$

since $Z_o = Z + \Delta Z$

$$\therefore \quad K = \frac{K_o (Z + \Delta Z) - \frac{\nu}{1-\nu} \Delta Z}{Z + \Delta Z - \Delta Z} = \frac{K_o Z + K_o \Delta Z - \frac{\nu}{1-\nu} \Delta Z}{Z}$$

$$K = K_o + \left[K_o - \frac{\nu}{1-\nu} \right] \frac{\Delta Z}{Z} \qquad \text{Eq. 4.3}$$

i.e $K > K_o$

the higher the Z the less the change in K-value. $\Rightarrow$

This explain $K_a \approx K_p$ in Fig. 4.7.

HW # 4. Ch. 4, # 1, 2, 6, 9.

We have talked about in-situ stresses, and we'll talk today about tech. to measure them.

See course pack pp. 30

Tech. → Overcoring
→ Doorstopper methoc
→ Flat Jack method

8.2 Measurement of In-Situ Stresses

8.2.1 Overcoring (strain relief method)

1. drill a hole into rock mass (with diameter d)

2. insert torpedo which measure strain in 3-directions

3. drill a larger hole with ($D > 2$ d)

- three sets of deformation buttons @ 0^o, 60^o, 120^o
- test data yields deformation, Δd @ 3 diameters 60^o apart

* From theory of elasticity and isotropic behavior assumed:

$$\Delta d(\theta) = \sigma_x f_1 + \sigma_y f_2 + \sigma_z f_3 + \tau_{xz} f_4$$

where $f_1 = d\,(1 + 2\cos 2\theta)\,\frac{1-\nu^2}{E} + \frac{d\nu^2}{E}$

$$f_2 = \frac{d\nu}{E}$$

$$f_3 = d\,(1 - 2\cos 2\theta)\,\frac{1-\nu^2}{E} + \frac{d\nu^2}{E}$$

$$f_4 = d\,(4\sin 2\theta)\,\frac{1-\nu^2}{E}$$

$\Rightarrow$ we get equations $\Rightarrow$ $\Delta d\,(\theta = 0^o) = \ldots..$
$\Delta d\,(\theta = 60^o) = \ldots..$
$\Delta d\,(\theta = 120^o) = \ldots..$

Therefore, we have 3 equations, but for unknowns, σ_x, σ_y, σ_z, τ_{xz}

Solution

a) for vertical hole

i) assume one of the stresses
i.e. $\sigma_v = \gamma Z = \sigma_x$

for horizontal
ii) assume $\sigma_y = 0$

b) for deep

Drill three boreholes for three tests and
hope it is in the same rock.
They should (at least 10′ apart) so that one will not affect other.

$$\begin{Bmatrix} \Delta d(\theta_1) - f_2\sigma_y \\ \Delta d(\theta_1 + 60°) - f_2\sigma_y \\ \Delta d(\theta_1 + 120°) - f_2\sigma_y \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{13} & f_{14} \\ f_{21} & f_{23} & f_{24} \\ f_{31} & f_{33} & f_{34} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix}$$

From overcoring and assumed σ_y — E, υ, geometry — need to solve for

need τ_{yz} , τ_{xy} ……. perform test in different directions

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \ldots\ldots\ldots\ldots\ldots\ldots \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix}$$

8.2.2 Doorstopper Technique: strain rosette is installed on a flat rock surfaces and overcored.

Strain from three oriented at 60^o angle, are used to determine ε_x, ε_y, γ_{xy}, which can e used in the theory of elasticity to find change in stress and the in-situ stresses.

SEE Ch. 4 & Appendix 2 of text for equations.

8.2.3 Flat-Jack Method simplest one

1. mount extensometer to measure change in length.

2. cut a square slot into the rock.

3. install flat jack into slot, and apply pressure to reopen, slot to d_o.

8.2.4 <u>Hydraulic Fracturing:</u>

Fig. 4.10 pp.31 course pack.

For the element $\rightarrow$

$$\sigma_\theta = 3\sigma_{h_{min}} - \sigma_{h_{max}} \qquad (1)$$

one pressure is applied internally of magnitude p,
Lane's solution tells us that: σ_θ borehole wall must decrease by p. i.e.

$$\sigma_\theta = 3\sigma_{h_{min}} - \sigma_{h_{max}} - P \qquad (2)$$

σ_θ goes from compression to tension.

when tensile stress is reached $\sigma_\theta = -T_o$ (3)

$\Rightarrow$ $P = P_{c1}$ in text (4)

$\therefore$ subst. 3 into (2)

$$-T_0 = 3\sigma_{h_{min}} - \sigma_{h_{max}} - p_{c1} \quad (5)$$

Once the crack forms, it will continue cracking until the water pressure is reduced to $P_{shut\ in}$. At this point we have stress equilibrium and

$$P_{shut\ in} = \sigma_{h_{min}} \quad (6)$$

Now wee drop the pressure, allowing the crack to close; and once again raise the pressure to above $P_{shut\ in}$. The new max pressure that can be reach is P_{c2} at which point, the tensile strength in the crack is zero. i.e. reopen closed crack (not propagating the crack).

$$\therefore \quad 0 = 3\sigma_{h_{min}} - \sigma_{h_{max}} - P_{c2} \quad (7)$$

Combine Eq. (5) & (7)

$$T_0 = P_{c1} - P_{c2}$$

Then go back into Eq. (5) to solve for $\sigma_{h_{max}}$.

<u>Limitation</u>

1. not be used in shallow foundation ………… vertical stress in the min. hori. crack.

- This test is good in oil …..
- Fig. 4.7

highest principle & stress ⊥ falt….. falt is sliding

direction of $\sigma_{h_{max}}$ & $\sigma_{h_{min}}$?? $\rightarrow$ lower a TV camera

* overcoring

make sure that annular area
is large enough $\Rightarrow$ not to break
is small enough $\Rightarrow$ not to have sufficient self-wt.

* for deep

- Find stresses in local coordinate
- Transform them to global coordinate

$\Rightarrow$ $\Delta d(\theta) = f\ (\sigma_x , \sigma_y , \sigma_z , \tau_{xz}, \tau_{zy})$ $\Rightarrow$ get 3 eqns. from one test
$\Rightarrow$ $\Delta d(\theta) = f$ $\Rightarrow$ get 3 eqns
$\Rightarrow$ $\Delta d(\theta) = f$ $\Rightarrow$ get 3 eqns.

(9 eqns. 6 are independent)

Therefore the stresses can be determined uniquely.

9.0 Aspects of Structural Geology

The study of earth beneath & geometry.

9.1 Definition

Dip: the angle at which a plane is downward inclined to a horizontal surface. It is the largest vertical angle, and is therefore measured in a plane perpendicular to the strike.

Strike: the direction (bearing) of a horizontal line on an inclined plane.
If ground surface is flat, the outcrop of the plane corresponds to the strike.

δ : Dip angle
α : apparent dip (will always e smaller than δ)
β : angle between strike and the apparent dip

$\tan\alpha = \tan\delta \sin\beta$ $\rightarrow$ see Nomogram pp.74 course pack.

or $\tan\delta = \dfrac{\tan\alpha}{\sin\beta}$

Nomenclature: in USA N45°E 25°S

First letter always N or S

0 to 90°

Second letter always E or W

0 to 90

N or S exception if strike is then dip will E or W.

Strike : from North 45° towards east.

Dip : 25° downward toward S.

In <u>Europe</u>: use azimuth 0^o to 360^o such that dip is always ton the right.

Ex. 295^o , 30^o

dip: downward to right of strike

Strike from North rightwards

Same as N65^oW, 30^oN

<u>Thickness</u>: perpendicular distance between two parallel planes bounding a layer. (t)

a) <u>outcrop measurement ⊥ strike</u>.

$$\sin \delta = \frac{t}{w}$$

$$t = w \sin \delta$$

b) <u>if outcrop measurement is not ⊥ strike:</u>

$$w = d \sin \beta$$

$$t = d \sin \beta \sin \delta$$

c) if outcrop measurement is not $\perp$ strike and is on sloping ground.

$t = s \mid \sin \delta \ \sigma \sin \beta \pm \sin \sigma \cos \delta \mid$

use + if σ and δ are in opposite directions

use – if σ and δ are in the same directions

9.2 Dip, strike and outcrop patterns from drill hole data:

<u>Ex. 1</u>

Assume that we hit same rock at 300′, 300′ and 100′ in A, B & C.

(elevation not depth)

$\therefore$ Strike is the line connecting A & B because they hit same rock at same elevation.

- get $\perp$ to AB from C ……………… D $\Rightarrow$ in horizontal plane
- make cE // AB where cE = 300-100

  (vertical in plane)
- connect ED $\Rightarrow$ δ

  (in dipping plane)

Ex. 2

Outcrop Pattern of a Dipping Bed

The outcrop pattern of a horizon* can be predicted if a contour map showing the topography is available, if the dip and strike of the horizon are known, and if the location of one exposure of the horizon is given. This is possible, however, only if the horizon is truly a plane surface – that is, if its dip and strike are constant.

The figure below illustrates the procedure that may be followed. The horizon outcrops at X. The ground surface is represented by 100-foot contours. Inasmuch as the horizon is known to strike N.75^{o}W. and the dip 20^{o}S., it is possible to predict its position at any place in the area.

Starting about an inch from the left border of the map extend a line SS′ through the outcrop X parallel to the strike of the horizon (N.75^{o}W.). Inasmuch as the outcrop is at an altitude of 800 ft, at every place on this line the horizon has an altitude of 800 feet. Now make a vertical section at right angles to the strike by drawing AB perpendicular to the strike of the bed at any convenient distance to the left of the map. The intersection of AB and SS′ may be designated by C. At C lay off the angle BCE equal to the dip of the horizon, in this instance 20 degrees. CE is the trace of the horizon on the vertical section. Along SS′ from point C lay off 100-foot units (equal to the topographic contour interval), using the same scale as that of the map.

Through each 100-foot point above or below C draw a line parallel to AB to an intersection with line CE. The intersections are points on the bedding plane; they are 100 feet apart vertically. From each of these intersections draw lines parallel to SS′. These lines are 100-foot structure contours on the horizon. A each point where a structure contour intersects a topographic contour of the same altitude, the horizon will outcrop. Mark the locations of these intersections with small circles. When connected, these circles, show the predicted outcrop pattern.

* “Horizon” refers to a surface having no thickness.

Outcrop pattern intersect ground plane.

H.W. 5 (# 5 pp. 36 course pack).

9.3 Structural Features and Outcrop Patterns:

Def:

9.3.1 Folds: a distortion of a volume of material that manifests itself as a bend or nest of bends in linear or planer elements within the material.

1) Consider early geology deposition system

2) By tectonic stresses

3) By erosion
- glacical
- water
- wind
- …..

∨ <u>syncline</u> : youngest at the middle

∧ anticline : oldest at the middle

Map pattern

Syncline: Folds that are concave upward

- o youngest rock in center

Anticline: Folds are concave downward

- o older rocks are in the center

in addition to fold, hinge line might dip from the horizontal by an angle called plunge.

plunge: dip of the hinge line.

By erosion

Map patters: →

Plunging anticline:

X-section

plunging cencling

if tectonic stresses are too high ∴

9.3.2 Fault: a fracture surface in a rock body along which one side has been offset relative to the other.

Fault zone: a zone of sheared, crushed or foliated rock in which numerous small dislocations have taken.

Place adding up to an appreciable total offset of the deformed walls.

A faults “attitude” is given by its dip & strike.

Relative motion:

1. Normal fault, a gravity fault

   Ka : caused by gravity force

2. Reverse fault, thrust fault

   Kp : caused by high
   horiz. pressure

* Displacement:

AB = net slip
BC = dip slip
BD = strike slip
AE = throw
ED = heave
< CAB = rake

- For a strike-slip fault

  $\overline{BC} \cong 0$ and rake $\cong 0°$

- For a dip-slip fault

  $\overline{BD} = 0°$ and rake $\cong 90°$

SE course pack pp. 38

Erosion old at top in footwall Fig. 111.

Fig. 112 not much in pattern

Fig. 113 as if but beds are originally inclined.

Fig. 114

Fig. 115

pp. 39

Fig. 116 dip & slip movements // bedding planes

Fig. 117 re-occuring pattern

Fig. 118

Fig. 119

Fig. 120 bedding planes are not // ….

Figs. 111-120: we know B, we have to interpret A.

Graphical representation of

Stereo net: is projection of 3-D plane into 2-D.

pp. 40 course pack.

9.4 Stereographic Projection (Example)

Pr. # 1: Draw a plane with N 0^oE, 30^oW

Sol.
1. Mark North & South
2. Draw the strike
3. From West count 10, 20, 30 (dip.)
4. Draw line

Pr. #2: Draw a plane with N 30^oE, 70^oN

1. Strike …………….
2. Rotate yellow until strike coincide NS
3. 70^o

Pr. #3: Project a <u>line</u> plunging 40^o towards N 30^oE

Pr. #4: Line plunging 20^o toward N 20^oW

Pr. #5: Determine the strike & dip of plane containing both lines in # 3 & # 4.
Rotate until $\rightarrow$ $\rightarrow$ they meet one great cicle.

Sol. dip = 41^o
strike = N 44^oW

Pr. #6: Find the direction and plunge of a line defined by the planes from # 1 & # 2.

strike = S 37^oW
plunge = 20^o

a plane can be represented by a single point on the stereograph which is the tip of the normal.

Pr. #7: Find normals, $\hat{n}$ to the planes from #1 & #2.

If dip = 0 $\Rightarrow$ normal

If dip = 90^o $\Rightarrow$ normal

$\therefore$ angle between plane & its normal = 90^o

$\therefore$ measure 90^o from plane.

- $\hat{n}_1$ $\hat{n}_2$

- plane containing both normals.

Pr. #8: Rotate until they are same

Pr. #9: Normal of this plane is point
$\Rightarrow$ Normal of intersection of two planes.

Planes
Lines
Intersections

Pr. #10: Find the locus of lines making 20^o with the first line of Pr. #3.

measure 20^o 20^o

Find mid point, draw a circle.

All lines meet circle is as 20^o.

* Another way

1. Find angles between two lines

H.W. #6

Read Appendix #5

Do pr. #2, #3, #6.

<u>Application</u> of stereo.net.

1. Seismic

Plot it in stereo.nets. pp. 43

$\Rightarrow$ Find the mechanism pp. 44

2. Geophysics pp.46

3. Str. geology

4. Joint survey for large projects (tunnel, dam, …..)

- Find strikes & dips
- Plot normals of planes into a stereograph $\approx$ pp.48
- Superimpose over the net pp. 42
- Find how many points at each triangle.
- Remove net
- Make contours on
  map of ore-dominant joint pattern.

10.0 Discontinuous Rock

- see next table
- see Fig. 6.1 pp. 47 course pack.
- plotting joint on stereographic pp. 48, 49

not only quantity, but also quality
joints of fault.
pp. 48

10.1 Joint testing:

a) Sampling: if you see visible joint
core it with long core.

b) Molding & casting:

Reconstruct joint geometry in the lab from plaster.

10.2 Laboratory testing:

a) direct
shear
test

Stress-strain dia: see fig. 6.12 / pp. 50.

CLASSIFICATION OF ROCK DISCONTINUITIES

Name	Typical Spacing	Method for Identification	Effects on Engineering Properties, Design
Micr-fissures	1 mm – 1 cm	magnifying glass or optical microscope Can't be seen by usual naked eye.	Reflected in E, ν, q_u, etc. of laboratory size samples. May result in strength anisotropy.
fissures	1 cm – 10 cm	visible in hand samples	Reflected in E, ν, q_u, etc. of laboratory size samples. Control strength anisotropy. May influence development of failure planes.
joints	10 cm – 10 m	Clearly visible, usually planer discontinuities. Typically exist in two or more "sets". Little to no previous movement observed along joints. May be weathered.	Controls kinematics of bock motion. E, ν, q_u, etc. of intact rock becomes almost irrelevant in stability analysis. Joint testing may be performed to determine ϕ_{joint} and s_{joint}. Critical in slope stability, tunneling, etc.
shears	1 m to 100 m	Discontinuities along which previous movement has occurred due to minor faulting or interlayer slip. Easily identified by the offset or a zone of crushed rock.	Could result in movements of large constructed facilities.
faults	10 m to 100's km	Large, sometimes continental size discontinuities along which significant movement has occurred in the past resulting in changes to the structural geology. Often shown on maps of structural geology.	Generally, a constructed facility will not induce movements, but is in potential danger due to movements of the fault.

Stability

- Constant normal force

  direct shear test with constant
  normal force.

- Dilation partially restrained

Deformable material with known E $\Rightarrow$ modeling the block.
more stiff than rock.

This is a hard test. But we can use simple shear test.

See Fig. 5.17 pp. 51 To simulate actual ……

- from Fig. 5.17b we need $\Delta V = 0$.

  if we can allow initial deformation only.

a : normal force

b : normal & shear

c : normal & shear

b) Triaxial test

$\nu + \beta = 90^o$

$$\sigma_d = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos^2\beta$$

$$\tau_d = \frac{1}{2}(\sigma_1 - \sigma_3)\sin^2\beta$$

Or $\sigma_d = \sigma_3 + (\sigma_1 - \sigma_3) \sin^2 4$

$\tau_d = (\sigma_1 - \sigma_3) \cos \Psi \sin 4$

$\therefore$ sample is difficult to obtain
can't get similar cases.

c) <u>Multi stage testing</u>

rapidly increase σ_3 as soon as joint
begins slipping and repeat test.

Result

10.3 <u>Strength Model</u> for jointed rock

1) Patton 1966: (Bilinear)

ϕ_u : friction angle of a smooth joint

i : asperity angle

@ $\sigma = \sigma_T \Rightarrow I_p = \begin{cases} \sigma_T \tan(\phi_u + i) \\ S_T + \sigma_T \tan \phi_J \end{cases}$

$$\text{equal} \Rightarrow \sigma_T = \frac{S_J}{(\tan(\phi_u + i) - \tan \phi_J)}$$

residual frictional angle

for intact rock

B_i : …… due to riding asperity

$\tau_p = \sigma \tan(\phi_u + i)$ if $\sigma < \sigma_T$

$\tau_p = S_J + \sigma \tan \phi_{joint}$ if $\sigma > \sigma_T$

It can be shown that $\sigma_T = \dfrac{S_J}{[\tan(\phi_u + i) - \tan \phi_u]}$

but asperities are not regular.

2. Ladonyi and Archambault (1970)

modification

$$\tau_{peak} = \frac{\sigma(1-a_s)(\dot{v}+\tan\phi_u) + a_s \tau_{max}}{1-(1-a_s)\dot{v}\tan\phi_u}$$

where a_s = % of area with asperities sheared
τ_{max} = shear strength of the intact rock
σ = normal stress
$\dot{v}$ = dilancy rate

$$\dot{v} = \tan i$$

o if $\sigma \to 0$, $a_s = 0$ and $\tau_p = \dfrac{\sigma(\dot{v}+\tan\phi_u)}{1-\dot{v}\tan\phi_u}$

$$\tau_p = \frac{\sigma(\tan i + \tan\phi_u)}{1-\tan i \tan\phi_u} \Rightarrow \tau_p = \sigma \tan(\phi_u + i)$$

- if $\sigma \rightarrow \infty$ then $a_s \rightarrow 1.0$

  $\tau_{peak} = \tau_{max}$

See fig. 6.14 pp. 52