2-100. Determine the angle θ between the two position vectors.

2-105. A force of F = 80 N is applied to the handle of the wrench. Determine the magnitudes of the components of the force acting along the axis AB of the wrench handle and perpendicular to it.

2-116. The force **F** = {25**i** – 50**j** + 10**k** N acts at the end *A* of the pipe assembly. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of *AB* and perpendicular to it.

2-100

Position vector:

$\mathbf{r}_1 = [0 - (-8)]\mathbf{i} + (15 - 5)\mathbf{j} + (10 - 8)\mathbf{k}$

$= \{8\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}\}$ ft

$r_1 = \sqrt{8^2 + 10^2 + (-8)^2} = \sqrt{228 \text{ ft}} \approx 15.1$ ft

$\mathbf{r}_2 = [0 - (-8)]\mathbf{i} + (0 - 5)\mathbf{j} + (6 - 8)\mathbf{k}$

$= \{8\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}\}$ ft

$r_2 = \sqrt{8^2 + (-5)^2 + (-2)^2} = \sqrt{93 \text{ ft}} \approx 9.64$ ft

$$\cos\theta = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}$$

$$= \frac{(8\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}) \cdot (8\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})}{15.1 * 9.64}$$

$$= \frac{8(8) + 10(-5) + (-8)(-2)}{145.6} = 0.2060$$

$\theta = 78.1^{o}$ Ans

2-105

$\mathbf{u}_F = -\cos 30^\circ \sin 45^\circ \mathbf{i} + \cos 30^\circ \cos 45^\circ \mathbf{j} + \sin 30^\circ \mathbf{k}$

$= -0.6124\mathbf{i} + 0.6124\mathbf{j} + 0.5\mathbf{k}$

$\mathbf{u}_{AB} = -\mathbf{j}$

$\mathbf{F} = F\mathbf{u}_F = 80(-0.6124\mathbf{i} + 0.6124\mathbf{j} + 0.5\mathbf{k})$

$= (-48.990\mathbf{i} + 48.99\mathbf{j} + 40\mathbf{k})$ N

$F_{AB} = \mathbf{F} \cdot \mathbf{u}_{AB} = |(-48.990\mathbf{i} + 48.99\mathbf{j} + 40\mathbf{k}) \cdot (-\mathbf{j})|$

$= 49.0$ N Ans

Negative sign indicates that F_{AB} acts in the direction opposite to that of $\mathbf{u}_{AB}$.

$$F_{per.} = \sqrt{F^2 - F_A^{e}B}$$

$$= \sqrt{80^2 - (-49.0)^2} = 63.2 \text{ N}$$ Ans

Also, from prob. 2-104 $\theta = 128^\circ$

$F_{AB} = F \cos\theta = |\,80 \cos 128^\circ\,| = 49.0$ N

$F_{per.} = F \sin\theta = |\,80 \sin 128^\circ\,| = 63.2$ N

2-116

$$\mathbf{u}_{AB} = \frac{(0-0)\mathbf{i}+(5-9)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(5-9)^2+(0-6)^2}} = \frac{-4\mathbf{j}-6\mathbf{k}}{\sqrt{52}}$$

$$F_1 = \mathbf{F}\cdot\mathbf{u}_{AB} = (25\mathbf{i}-50\mathbf{j}+10\mathbf{k})\cdot\left(\frac{-4\mathbf{j}-6\mathbf{k}}{\sqrt{52}}\right)$$

$$= \frac{25(0)+(-50)(-4)+(10)(-6)}{\sqrt{52}}$$

$$= 19.4 \text{ N}$$ Ans

$$F = \sqrt{25^2+(-50)^2+10^2} = 56.79 \text{ N}$$

$$F_2 = \sqrt{F^2-F_1^2}$$

$$= \sqrt{56.79^2-19.41^2} = 53.4 \text{ N}$$ Ans

Ch. 3 Equilibrium of a Particle

- Resolve a force into components
- Express a force as a Cartesian vector

3.1 Conditions for the equilibrium of a Particle

A particle is in equilibrium:

$\rightarrow$ at rest, if originally at rest.

$\rightarrow$ has constant velocity, if originally in motion.

<u>Static equilibrium</u> or <u>equilibrium</u> , mostly at rest, if originally at rest.

- To maintain a state of equilibrium, it is necessary to satisfy first Law of Motion.
  [if resultant force acting on a particle is zero, then particle is in equilibrium].

$$\sum \vec{F} = 0$$

vector sum of all the forces acting on the particle

necessary condition of equilibrium & sufficient $\rightarrow$ $\sum \vec{F} = m\vec{a}$

satisfied $\Rightarrow$ $m\vec{a} = 0 \Rightarrow \vec{a} = 0$

constant velocity or at rest.

3.2 Free-Body Diagram

ΣF includes known & unknown forces acting on the particle.

(1) Draw the free body diagram of the particle sketch of the particle, which represents it as being isolated or cut "force" from its surroundings.

* on sketch show all forces acting on the particle (action & reaction forces)

(2) Apply equation of equilibrium

Connections:

1. Springs: Linear elastic spring support

change in length α force acting on it.

$\Delta L = (L - L_o) \quad = \quad S \quad \alpha F$

L – deformed

L_o – undeformed

$S = \frac{1}{K} F$

K – spring constant or stiffness

$F = KS$

S – Elongation or reduction

+ve ⇒ F is pull

−ve ⇒ F is push

$K = 500$ N/m

$F_1 = 500 * .2 = 100$ N

$F_2 = 500 * (-2) = -100$ N

2. Cables & pulleys

Assume 1) cable's weight is negligible
2) cable's can't be stretched

- Cable can support only tension (pulling) force

  * acting in the direction of the cable

  * tension in a continuous cable passing over frictionless pulley has constant magnitude, to keep cable in equilibrium.

T regardless of θ

  * cable is subjected to a constant tension T throughout its length.

3. Others supports, later

Procedure for Drawing a Free-Body Diagram

1. Imagine the particle isolated, cut, free, from its surroundings.
   Draw / sketch its outline shape.

2. On the sketch, indicate all forces acting on the particle.

Forces:
- active forces: tend to set particle in motion, e.g. weight, magnetic/electro static interaction
- reactive forces: tend to prevent motion, caused by constraints or support.

  - o trace around the particle's boundary, noting each force acting on it.

3. • Label known forces with their proper magnitudes & directions

   • Represent unknown forces by letters

     - If a force has a known line of action (direction), a sense can be assumed. Then found

     - $\therefore$ mag. of $\vec{F}$ is +ve. $\Rightarrow$ $-$ve $\equiv$ opposite sense

Equilibrium

- Apply equilibrium eqn. on the free-body diagram.

$$\sum \vec{F} = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = 0$

Objectives:

1. Introduce the concept of F.B.D. for a particle.
2. Solve particle problems using the equation of equilibrium

3.3 Coplanar Force Systems

$$\sum \vec{F} = 0$$

$$\sum F_x \hat{i} + \sum_y \hat{j} = 0$$

$$\Rightarrow \left.\begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{array}\right\} \text{scalar equilibrium eqns.}$$

⇒ algebraic sum of x & y components of all the forces acting on the particle = 0.

2 equations & 2 unknowns (mag. & direction)

<u>Scalar Notation:</u>

sense ⇒ algebraic sign ≡ arrowhead

magnitude & sense can be <u>assmed</u>

∴ $\vec{F}$ is always +ve, ⇒ if (−ve) result
≡ sense is opposite to that which was assumed.

F
→•→ 10 N

$\Sigma F_x = 0 \Rightarrow F + 10 = 0 \Rightarrow F = -10$ N

(−: opposite sense)

Procedure for Analysis: (planar forces & particle equilibrium)

1. Free body diagram:

- identify the body / particle
- draw F.B.D. of the particle
    - isolation
    - known & unknown forces (mag. & angles) to be labeled)
    - assume sense for unknown forces

2. Equations of Equilibrium

- x-y axes in any direction
- $\xrightarrow{+}$ $\Sigma F_x = 0$
  $+\uparrow \Sigma F_y = 0$
- Spring $F = Ks$