2-31. Express $\mathbf{F}_1$, and $\mathbf{F}_3$ as Cartesian vectors.

2-32. Determine the magnitude of the resultant force and its orientation measured counterclockwise from the positive *x* axis.

2-49 The crate is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive *y* axis. Set $\theta = 45^{\circ}$.

2-31

$\mathbf{F}_1$ = {15 xoa50^o**i** + 15 sin50^o**j**} kN = {9.64**i** + 11.5**j**} kN   Ans

$\mathbf{F}_2$ = $\{-\frac{12}{13}(26)\mathbf{i}+\frac{5}{13}(26)\mathbf{j}\}$ kN = $[-24\mathbf{i} = 10\mathbf{j}\}$ kN   Ans

$\mathbf{F}_3$ = {36 cos30^o**i** – 36 sin30^o**j**} kN = {31.2**i** – 18**j**} kN   Ans

2-32

$\xrightarrow{+} F_{Rx} = \sum F_x$; $F_{Rx} = 15 \cos 50° - \frac{12}{13}(26) + 36 \cos 30°$

$= 16.82 \text{ kN} \rightarrow$

$+\uparrow F_{Ry} = \sum F_y$; $F_{Ry} = 15 \sin 50° + \frac{5}{13}(26) - 36 \sin 30°$

$= 3.491 \text{ kN} \uparrow$

Magnitude

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{16.82^2 + 3.491^2} = 17.2 \text{ kN}$$ Ans

Direction

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{3.491}{16.82} = 11.7°$$ Ans

Vector Analysis:

$\mathbf{F}_1 = \{15 \cos 50^o \mathbf{i} + 15 \sin 50^o \mathbf{j}\}$ kN = $\{9.64\mathbf{i} + 11.5\mathbf{j}\}$ kN

$\mathbf{F}_2 = \{-\frac{12}{13}(26)\mathbf{i} + \frac{5}{13}(26)\mathbf{j}\}$ kN = $[-24\mathbf{i} + 10\mathbf{j}\}$ kN

$\mathbf{F}_3 = \{36 \cos 30^o \mathbf{i} - 36 \sin 30^o \mathbf{j}\}$ kN = $\{31.2\mathbf{i} - 18\mathbf{j}\}$ kN

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= \{9.64\mathbf{i} + 11.5\mathbf{j}\} + \{-24\mathbf{i} + 10\mathbf{j}\} + (31.2\mathbf{i} - 18\mathbf{j}\}$

$= \{16.8\mathbf{i} + 3.49\mathbf{j}\}$ kN

$$F_R = \sum F_x \hat{i} + \sum F_y \hat{j}$$

$$= F_{Rx} \hat{i} + F_{Ry} \hat{j}$$

2-49

$$\xrightarrow{+} F_{Rx} = \sum F_x \;; \qquad 0 = F_A \sin 30^o - F_B \sin 45^o \qquad [1]$$

$$+\uparrow \; \mathbf{F}_{Ry} = \sum F_y \;; \qquad 600 = F_A \cos 30^o + F_B \cos 45^o \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$F_B = 311$ N $\qquad F_A = 439$ N $\qquad$ Ans

$$F_A \cdot \frac{1}{2} = F_B \cdot \frac{1}{\sqrt{2}} \qquad F_A = F_B \cdot \frac{2}{\sqrt{2}} = \sqrt{2}\, F_B \qquad (3)$$

$$600 = F_A \cdot \frac{\sqrt{3}}{2} + F_B \cdot \frac{1}{\sqrt{2}}$$

$$600 = \sqrt{2} \cdot F_B \cdot \frac{\sqrt{3}}{\sqrt{2}} + F_B \cdot \frac{1}{\sqrt{2}} = F_B \left(\sqrt{\tfrac{3}{2}} + \tfrac{1}{\sqrt{2}}\right) = F_B \, \tfrac{1}{\sqrt{2}} \left(\sqrt{3} + 1\right)$$

$$F_B = 310.58 \equiv 311 \text{ N}$$

$$F_A = \sqrt{2} \cdot 310.58 = 439.23$$

$$= 439 \text{ N}$$

2.5 <u>Cartesian Vectors</u>:

- Representing 3-D vectors in Cartesian vector form
- Right-hand coordinate system

- Rectangular components of a vector

  $\vec{A}$ is directed within an octant of *x*, *y*, *z* frame

  \- Two successive application of the parallelogram law

$$\vec{A} = \vec{A}' + \vec{A}_z$$

$$\vec{A}' = \vec{A}_x + \vec{A}_y$$

$$\therefore \quad \vec{A} = \vec{A}_X = \vec{A}_y + \vec{A}_z$$

- Unit vector

  A vector having a mag. of 1

$$\vec{A}\ , \quad |\vec{A}| \neq 0$$

$$\therefore \quad \vec{u}_A = \frac{\vec{A}}{|A|}$$

$$\vec{A} = A \cdot \vec{u}_A$$

A: positive scalar

$\vec{u}_A$: dimensionless vector

Component & direction of each vector is separated.

Cartesian Unit vectors

$\hat{i}, \hat{j}, \hat{k}$ in *x*, *y*, *z* direction

Sense : ± positive / negative *x*, *y*, *z* axes

Cartesian Vector Representation

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

- Magnitude of Cartesian vector

$$A = \sqrt{A'^2 + A_z^2} \quad , \quad A'^2 = A_x^2 + A_y^2$$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ positive square root of the sum of the squares of its components.

- Direction of Cartesian vector

Orientation of $\vec{A}$ is defined by the <u>coordinate direction angles</u> (α - alpha; β - beta; γ - gamma) measured between tail of $\vec{A}$

and the positive (*x*, *y*, *z*) axes located at the tail of $\vec{A}$.

- values of α, β, γ is between 0 & 180°.

$$\left.\begin{aligned} \text{Cos}\ \alpha &= \frac{A_x}{A} \\ \text{Cos}\ \beta &= \frac{A_y}{A} \\ \text{Cos}\ \gamma &= \frac{A_z}{A} \end{aligned}\right\} \text{ direction of cosines of } \vec{A}$$

$$\alpha = \cos^{-1}\frac{A_x}{A}\ ,\ \beta = \cos^{-1}\frac{A_y}{A}\ ,\ \gamma = \cos^{-1}\frac{A_z}{A}$$

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

$$\vec{u}_A = \frac{\vec{A}}{A} = \frac{A_x}{A}\hat{i} + \frac{A_y}{A}\hat{j} + \frac{A_z}{A}\hat{k}$$ represents the direction of A

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\left|\vec{u}_A\right| = 1 = \sqrt{\text{Cos}^2\alpha + \text{Cos}^2\beta + \text{Cos}^2\gamma}$$

or $\text{Cos}^2\alpha + \text{Cos}^2\beta + \text{Cos}^2\gamma = 1$

$\vec{A} = A\ \vec{u}_A$, $\vec{u}_A = \text{Cos}\ \alpha\ \hat{i} + \text{Cos}\ \beta\ \hat{j} + \text{Cos}\ \gamma\ \hat{k}$

$= A\ \text{Cos}\ \alpha\ \hat{i} = A\,\text{Cos}\ \beta\ \hat{j} + A\ \text{Cos}\ \gamma\ \hat{k}$

$= A_x\,\hat{i} + A_y\,\hat{j} = A_z\,\hat{k}$

2.6 Addition & subtraction of Cartesian vectors

$\vec{A}$, $\vec{B}$ vectors within the positive octant of *x*, *y*, *z* frame

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

$$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j} + (A_z + B_z)\,\hat{k}$$

$$\vec{R}' = \vec{A} - \vec{B} = (A_x - B_x)\,\hat{i} + (A_y - B_y)\,\hat{j} + (A_z - B_z)\,\hat{k}$$

- Concurrent force systems

$\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, …..

$$\vec{F}_R = \sum F = \sum F_x\,\hat{i} + \sum F_y\,\hat{j} + \sum F_z\,\hat{k}$$

$\sum F_x$, $\sum F_y$, $\sum F_z$: Algebraic sum of *x*, *y*, *z* components