Basic Influence Line Equations for Continuous Beams and Rigid Frames

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Abstract: The subject matter of this paper is essentially an extension of the moment distribution method of analysis and with the aim of offering a simple approach to the construction of influence line diagrams for continuous structures, the members of which can be either of constant or of variable cross section. The distributions of the fixed-end moments are similar to the Hardy-Cross approach except that they are performed algebraically rather than by successive arithmetical approximations. The results are a series of simple equations with only one unknown, which can be easily evaluated with the aid of a desk calculator or by elementary programming for the computer. These simple equations can be expanded to include the effects of sway. They also include the effects of degrees of end fixation.

INTRODUCTION

The undergraduate syllabus for structural analysis introduces the student to both the classical approach and a comparative study of all other theorems derived, including their relative merits for analyzing indeterminate structures. Moment distribution is one such theorem and, because of its simple interpretation of the classical, it is introduced at the intermediate level of study. It is, however, rather cumbersome for constructing influence line diagrams if the initial distributions of fixed-end moments are carried out by successive arithmetical approximations. On the other hand, if the fixed-end moments are distributed algebraically, it becomes a very useful teaching aid for a study of influence lines.

ALGEBRAIC DISTRIBUTIONS

The bridge frame shown in Fig. 1(a) consists of members of different sizes and lengths deliberately chosen so that the influence line equations to be developed will be general, so that they can be applied to any three-span continuous beam, or single-bay portal. All members are of constant cross sections throughout.

Irrespective of what analytical method of analysis is used, the stiffness of each member must first be determined and, in the case of the frame shown, the distribution factors are indicated: α, β, γ, δ, λ, and μ.

In an algebraic distribution each span is treated separately as shown in Fig. 2, and each end moment M is obtained by summation to infinity. An example of a summation is as follows:

$$M_{AB} = -f_A M^F_{AB} - \frac{f_A}{2}\alpha k_1 - \frac{f_A}{8}\alpha\beta\gamma k_1 - \frac{f_A}{32}\alpha\beta^2\gamma^2 k_1 - \text{etc} \quad (1)$$

$$M_{AB} = -f_A M^F_{AB} - \frac{f_A}{2}\alpha k_1\left(1 + \frac{\beta\gamma}{4} + \frac{\beta^2\gamma^2}{16} + \text{etc}\right) \quad (2)$$

The terms within the brackets form a geometrical progression, the sum to infinity being

$$S = \frac{1}{\left(1 - \frac{\beta\gamma}{4}\right)}; \quad k_1 = \left[M^F_{BA} + \left(\frac{1 - f_A}{2}\right)M^F_{AB}\right] \quad (3, 4)$$

Therefore the influence line equation for M_{AB} when the load is on span AB can be written

$$M_{AB} = -f_A M^F_{AB} - \frac{f_A\alpha S}{2}\left[M^F_{BA} + \left(\frac{1 - f_A}{2}\right)M^F_{AB}\right] \quad (5)$$

Similar distributions are performed for spans BC and CD. Table 1 lists the complete set of equations for the end moments developed in the frame shown in Fig. 1(a). In this table provision is made for any degree of end fixation, designated by the symbol f, which can have a value from 0 to 1.

When a unit load crosses the frame, the fixed-end moments can be expressed as follows:

$$M^F_{AB} = a(1 - a)^2 l_1 \quad (6)$$

and

$$M^F_{BA} = a^2(1 - a)l_1 \quad (7)$$

where a = coefficient of l_1, which can have values from 0 to 1. Other fixed-end moments are similarly transformed. If end A is hinged and D is not restricted horizontally, a common condition in practice, Table 1 reduces to Table 2. In each equation there is only one variable, a; all other elements have fixed values for a specific beam or frame.

In Table 2 there are four basic influence curves that can be used for both static and moving loads. In the case of point loads, the end moments are obtained by multiplying each load by the appropriate end-moment coefficient and the ordinate to the curve. In the case of uniformly distributed loads, the end moments are obtained by multiplying the intensity of loading by the appropriate coefficient and the area of the curve immediately above the load. Evaluation of fixed-end moments and distributions of the same are avoided when applying the equations to members of constant cross sections.

CARRY-OVER MOMENT

In practical design, it may be agreed to limit the amount of carry-over moment to foundations in which case the carry-over factor c will have a value somewhere between 0 and 0.5. The stiffness at the top of the member is therefore affected, the stiffness factor lying between 3 and 4. When the value of the carry-over moment is decided and consequently the carry-over factor c, the value of the stiffness factor, ρ, can be obtained from the following relationship:

$$\rho = \frac{6}{2 - c} \quad (8)$$

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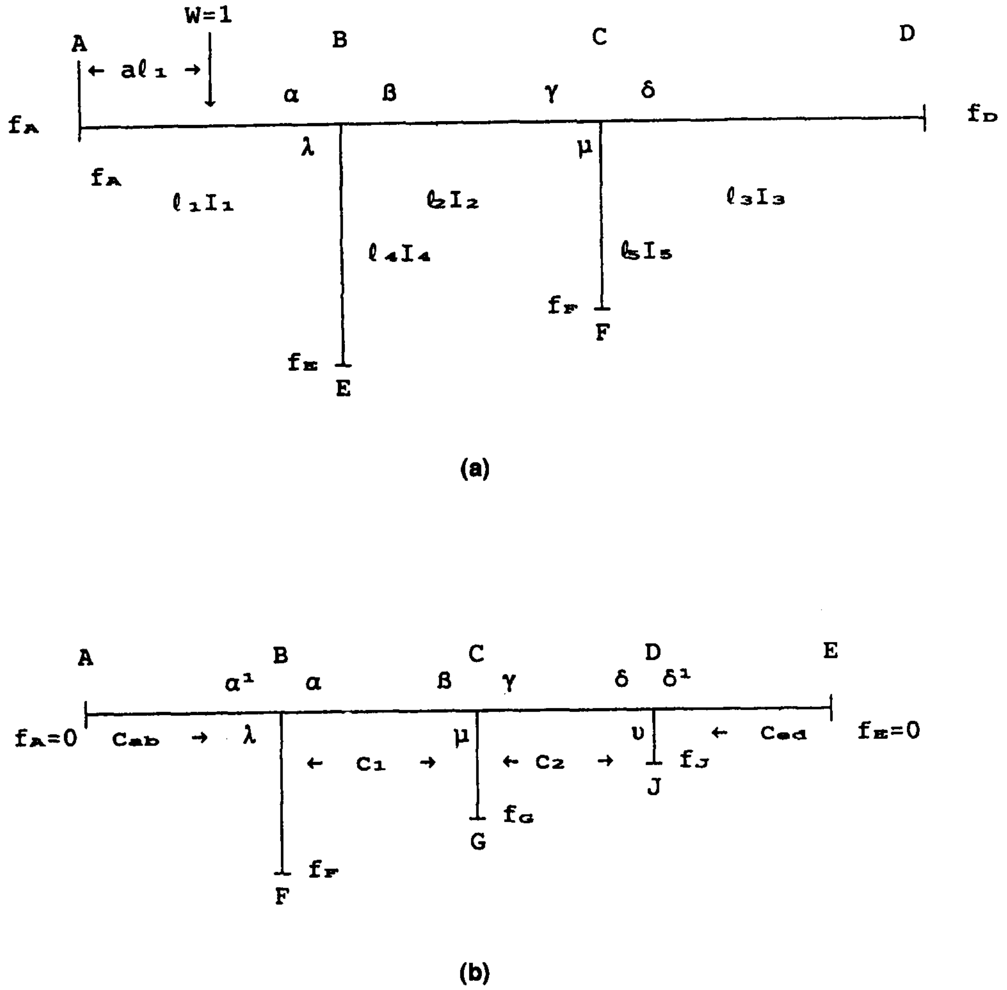

FIG. 1. Bridge Frame: (a) Members of Different Sizes and Lengths; (b) Members of Variable Cross Section

If ends A and D of the frame in Fig. 1(a) are freely supported, the equations can be modified to include the effects of sway by using the standard relationship

$$M = \pm M^{ns} \pm \frac{H_A}{F_A} M^s \tag{9}$$

where M^{ns} = end moment in the nonsway condition; M^s = corresponding sway moment due to F_A, the horizontal force at end A causing sway; H_A = horizontal reaction at A in the nonsway condition; and M = final moment. The values of M^{ns} and H_A vary depending on the value of a.

When the bridge frame shown in Fig. 1(a) consists of members of variable cross sections, the basic influence line equations for the end moments in the horizontal members are listed in Table 3; $f_A = f_D = 0$. For such structures it is necessary to evaluate the fixed-end moments and the carry-over and stiffness factors. The distributions of these fixed-end moments follow the same procedure as for members of constant cross sections, except that the symbol c is used for the carry-over factor instead of 1/2. The usual practice is to obtain the values of beam constants and fixed-end moments from published tables. The reading of such tables should present no difficulty to intermediate level students, but their derivation is usually postponed until senior level when the slope-deflection equations for members of variable cross sections are derived showing how beam constants and fixed-end moments are interrelated.

By applying the same algebraic method of distributions, basic influence line equations can be derived for two-bay portal frames. One such bridge frame is shown in Fig. 1(b), the members of which are of variable cross sections. The distribution of the fixed-end moments when span BC is loaded is shown in Fig. 3; $f_A = f_E = 0$. Table 4 lists the equations for the end moments, the fixed end moments being the result of a unit load crossing the frame from A to E.

It will be seen that for the two-bay portal frame or for a four-span continuous beam, S in the distributions of fixed-end moments equals $1/1 - (\alpha\beta c_1^2 + \gamma\delta c_2^2)$. Table 5 lists the equations for a similar frame, but consisting of members of constant cross sections. The equations are obtained by equating the carry-over factors to 0.5. In this case

$$S = \frac{1}{1 - \left(\dfrac{\alpha\beta + \gamma\delta}{4}\right)} \tag{10}$$

CONCLUSION

In each of the aforementioned tables, there is a similarity of structure between the different curves. This similarity becomes more obvious when the beam or frame under consideration is symmetrical about its centerline.

Finally, the foregoing method for constructing influence line equations can, without difficulty, be introduced at intermediate project levels, and as a teaching aid, it will enable students to approach with confidence the advanced level of their studies with the Mueller-Breslau method.

A	BA	BE	BC	CB	CF	CD	D
	α	λ	β	γ	μ	δ	
$-M^F_{AB}$	$+M^F_{BA}$						
$+(1-f_A)M^F_{AB}$ $\rightarrow$	$+\frac{(1-f_A)}{2}M^F_{AB}$						
$\frac{-f_A\alpha K_1}{2}$ $\leftarrow$	$-\alpha K_1$	$-\lambda K_1$	$-\beta K_1$ $\rightarrow$	$\frac{-\beta K_1}{2}$			
			$\frac{+\beta\gamma K_1}{4}$ $\leftarrow$	$\frac{+\beta\gamma K_1}{2}$	$\frac{+\beta\mu K_1}{2}$	$\frac{+\beta\delta K_1}{2}$ $\rightarrow$	$\frac{+f_D\beta\delta K_1}{4}$
$\frac{-f_A\alpha\beta\gamma K_1}{8}$ $\leftarrow$	$\frac{-\alpha\beta\gamma K_1}{4}$	$\frac{-\beta\gamma\lambda K_1}{4}$	$\frac{-\beta^2\gamma K_1}{4}$ $\rightarrow$	$\frac{-\beta^2\gamma K_1}{8}$			
			$\frac{+\beta^2\gamma^2 K_1}{16}$ $\leftarrow$	$\frac{+\beta^2\gamma^2 K_1}{8}$	$\frac{+\beta^2\gamma\mu K_1}{8}$	$\frac{+\beta^2\gamma\delta K_1}{8}$ $\rightarrow$	$\frac{+f_D\beta^2\gamma\delta K_1}{16}$
$\frac{-f_A\alpha\beta^2\gamma^2 K_1}{32}$ $\leftarrow$	$\frac{-\alpha\beta^2\gamma^2 K_1}{16}$	$\frac{-\beta^2\gamma^2\lambda K_1}{16}$	$\frac{-\beta^3\gamma^2 K_1}{16}$ $\rightarrow$	$\frac{-\beta^3\gamma^2 K_1}{32}$			
			$\frac{+\beta^3\gamma^3 K_1}{64}$ $\leftarrow$	$\frac{+\beta^3\gamma^3 K_1}{32}$	$\frac{+\beta^3\gamma^2\mu K_1}{32}$	$\frac{+\beta^3\gamma^2\delta K_1}{32}$ $\rightarrow$	$\frac{+f_D\beta^3\gamma^2\delta K_1}{64}$
$\frac{-f_A\alpha\beta^3\gamma^3 K_1}{128}$ $\leftarrow$	$\frac{-\alpha\beta^3\gamma^3 K_1}{64}$	$\frac{-\beta^3\gamma^3\lambda K_1}{64}$	$\frac{-\beta^4\gamma^3 K_1}{64}$ $\rightarrow$	$\frac{-\beta^4\gamma^3 K_1}{128}$			
			$+\frac{\beta^4\gamma^4 K_1}{256}$ $\leftarrow$	$\frac{+\beta^4\gamma^4 K_1}{128}$	$\frac{+\beta^4\gamma^3\mu K_1}{128}$	$\frac{+\beta^4\gamma^3\delta K_1}{128}$ $\rightarrow$	$\frac{+f_D\beta^4\gamma^3\delta K_1}{256}$
$\frac{-f_A\alpha\beta^4\gamma^4 K_1}{512}$ $\leftarrow$	$\frac{-\alpha\beta^4\gamma^4 K_1}{256}$	$\frac{-\beta^4\gamma^4\lambda K_1}{256}$	$\frac{-\beta^5\gamma^4 K_1}{256}$				

NOTE: $K_1 = [M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}]$

FIG. 2. Algebraic Distribution of Fixed-End Moments Due to Load on Span AB of Frame Shown in Fig. 1(a)

TABLE 1. Equations for Support Moments in Frame Shown in Fig. 1(a) Due to Loading on Any One Span

End moment (1)	Load on AB (2)	Load on BC (3)	Load on CD (4)
M_{AB}	$-f_A M^F_{AB} - \frac{f_A\alpha S}{2}\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$+\frac{f_A}{2}\alpha S\left[M^F_{BC} + \frac{\gamma}{2}M^F_{CB}\right]$	$-\frac{f_A}{4}\alpha\gamma S\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{BA}	$+(1-\alpha S)\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$+\alpha S\left[M^F_{BC} + \frac{\gamma}{2}M^F_{CB}\right]$	$-\frac{\alpha\gamma S}{2}\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{BE}	$-\lambda S\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$+\lambda S\left[M^F_{BC} + \frac{\gamma}{2}M^F_{CB}\right]$	$-\frac{\lambda\gamma S}{2}\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{EB}	$-\frac{f_E\lambda S}{2}\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$+\frac{f_E}{2}\lambda S\left[M^F_{BC} + \frac{\gamma}{2}M^F_{CB}\right]$	$-\frac{f_E\lambda\gamma S}{4}\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{BC}	$-\beta\left(1 - \frac{\gamma}{4}\right)S\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$-(1-\beta)S\left[M^F_{BC} + \frac{\gamma}{2}M^F_{CB}\right]$	$+\frac{\gamma(1-\beta)S}{2}\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{CB}	$-\frac{\beta}{2}(1-\gamma)S\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$+(1-\gamma)S\left[M^F_{CB} + \frac{\beta}{2}M^F_{BC}\right]$	$+\gamma\left(1 - \frac{\beta}{4}\right)S\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{CF}	$+\frac{\beta\mu S}{2}\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$-\mu S\left[M^F_{CB} + \frac{\beta}{2}M^F_{BC}\right]$	$+\mu S\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{FC}	$+\frac{f_F\beta\mu S}{4}\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$-\frac{f_F\mu S}{2}\left[M^F_{CB} + \frac{\beta}{2}M^F_{BC}\right]$	$+\frac{f_F\mu S}{2}\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{CD}	$+\frac{\beta\delta S}{2}\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$-\delta S\left[M^F_{CB} + \frac{\beta}{2}M^F_{BC}\right]$	$-(1-\delta S)\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$
M_{DC}	$+\frac{f_D\beta\delta S}{4}\left[M^F_{BA} + \frac{(1-f_A)}{2}M^F_{AB}\right]$	$-\frac{f_D}{2}\delta S\left[M^F_{CB} + \frac{\beta}{2}M^F_{BC}\right]$	$+f_D M^F_{DC} + \frac{f_D\delta S}{2}\left[M^F_{CD} + \frac{(1-f_D)}{2}M^F_{DC}\right]$

Note: + sign indicates clockwise direction around joint; and $S = 1/[1 - (\beta\gamma/4)]$.

TABLE 2. Influence Line Equations for Support Moments in Frame Shown in Fig. 1(a) Due to Unit Load Crossing Frame from A to D

End moment (1)	Load on AB (2)	Load on BC (3)	Load on CD (4)
M_{BA}	$+(1-\alpha S)\left(\frac{a-a^3}{2}\right)l_1$	$+\alpha S\left(\frac{2-2a+\gamma a}{2}\right)(a-a^2)l_2$	$-\frac{\alpha\gamma S}{2}\frac{(2a-3a^2+a^3)l_3}{2}$
M_{BC}	$-\beta\left(1-\frac{\gamma}{4}\right)S\left(\frac{a-a^3}{2}\right)l_1$	$-(1-\beta)S\left(\frac{2-2a+\gamma a}{2}\right)(a-a^2)l_2$	$+\frac{\gamma}{2}(1-\beta)S\frac{(2a-3a^2+a^3)l_3}{2}$
M_{BE}	$-\lambda S\left(\frac{a-a^3}{2}\right)l_1$	$+\lambda S\left(\frac{2-2a+\gamma a}{2}\right)(a-a^2)l_2$	$-\gamma\frac{\lambda}{2}S\frac{(2a-3a^2+a^3)l_3}{2}$
M_{CB}	$-\frac{\beta}{2}(1-\gamma)S\left(\frac{a-a^3}{2}\right)l_1$	$+(1-\gamma)S\left(\frac{2a+\beta-\beta a}{2}\right)(a-a^2)l_2$	$+\gamma\left(1-\frac{\beta}{4}\right)S\frac{(2a-3a^2+a^3)l_3}{2}$
M_{CD}	$+\frac{\beta\delta S}{2}\left(\frac{a-a^3}{2}\right)l_1$	$-\delta S\left(\frac{2a+\beta-\beta a}{2}\right)(a-a^2)l_2$	$-(1-\delta S)\frac{(2a-3a^2+a^3)l_3}{2}$
M_{CF}	$+\frac{\beta\mu S}{2}\left(\frac{a-a^3}{2}\right)l_1$	$-\mu S\left(\frac{2a+\beta-\beta a}{2}\right)(a-a^2)l_2$	$+\mu S\frac{(2a-3a^2+a^3)l_3}{2}$

Note: "a" has values of between 0 and 1; + sign indicates clockwise direction around joint; $f_A = f_D = 0$; and $S = 1/[1-(\beta\gamma/4)]$.

TABLE 3. Influence Line Equations for Support Moments in Frame Similar to Fig. 1(a), Consisting of Members of Variable Cross Section

End moment (1)	Load on AB (2)	Load on BC (3)	Load on CD (4)
M_{BA}	$+(1-\alpha S)(c_{ab}M^F_{AB}+M^F_{BA})$	$+\alpha S(M^F_{BC}+\gamma c M^F_{CB})$	$-\alpha\gamma c S(M^F_{CD}+c_{dc}M^F_{DC})$
M_{BC}	$-\beta(1-\gamma c^2)S(c_{ab}M^F_{AB}+M^F_{BA})$	$-(1-\beta)S(M^F_{BC}+\gamma c M^F_{CB})$	$+\gamma c(1-\beta)S(M^F_{CD}+c_{dc}M^F_{DC})$
M_{BE}	$-\lambda S(c_{ab}M^F_{AB}+M^F_{BA})$	$+\lambda S(M^F_{BC}+\gamma c M^F_{CB})$	$-\gamma\lambda c S(M^F_{CD}+c_{dc}M^F_{DC})$
M_{CB}	$-\beta c(1-\gamma)S(c_{ab}M^F_{AB}+M^F_{BA})$	$+(1-\gamma)S(\beta c M^F_{BC}+M^F_{CB})$	$+\gamma(1-\beta c^2)S(M^F_{CD}+c_{dc}M^F_{DC})$
M_{CD}	$+\beta\delta c S(c_{ab}M^F_{AB}+M^F_{BA})$	$-\delta S(\beta c M^F_{BC}+M^F_{CB})$	$-(1-\delta S)(M^F_{CD}+c_{dc}M^F_{DC})$
M_{CF}	$+\beta\mu c S(c_{ab}M^F_{AB}+M^F_{BA})$	$-\mu S(\beta c M^F_{BC}+M^F_{CB})$	$+\mu S(M^F_{CD}+c_{dc}M^F_{DC})$

Note: c_{ab} is the carry-over factor from A to B; c_{dc} is the carry-over factor from D to C; c is the carry-over factor from B to C and from C to B; + sign indicates clockwise direction around joint; and $S = 1/(1-\beta\gamma c^2)$.

A BA	BC		CB	CD		DC	DE E
α^1	α	c_1	β	γ	c_2	δ	δ^1
	$-M^F_{BC}$		$+M^F_{CB}$				
$+\alpha^1 M^F_{BC}$	$+\alpha M^F_{BC}$		$-\beta M^F_{CB}$	$-\gamma M^F_{CB}$		$-\gamma c_2 M^F_{CB}$	
	$-\beta c_1 M^F_{CB}$		$+\alpha c_1 M^F_{BC}$				
$+\alpha^1\beta c_1 M^F_{CB}$	$+\alpha\beta c_1 M^F_{CB}$		$-\alpha\beta c_1 M^F_{BC}$	$-\alpha\gamma c_1 M^F_{BC}$		$+\gamma\delta c_2 M^F_{CB}$	$+\gamma\delta^1 c_2 M^F_{CB}$
	$-\alpha\beta c_1^2 M^F_{BC}$		$+\alpha\beta c_1^2 M^F_{CB}$	$+\gamma\delta c_2^2 M^F_{CB}$		$-\alpha\gamma c_1 c_2 M^F_{BC}$	
			$B=(\alpha\beta c_1^2+\gamma\delta c_2^2)$				
$+\alpha^1\alpha\beta c_1^2 M^F_{BC}$	$+\alpha^2\beta c_1^2 M^F_{BC}$		$-\beta B M^F_{CB}$	$-\gamma B M^F_{CB}$		$+\alpha\gamma\delta c_1 c_2 M^F_{BC}$	$+\alpha\gamma\delta^1 c_1 c_2 M_{BC}$
	$-\beta c_1 B M^F_{CB}$		$+\alpha^2\beta c_1^3 M^F_{BC}$	$+\alpha\gamma\delta c_1 c_2^2 M^F_{BC}$		$-\gamma c_2 B M^F_{CB}$	
$+\alpha^1\beta c_1 B M^F_{CB}$	$+\alpha\beta c_1 B M^F_{CB}$		$-\alpha\beta c_1 B M^F_{BC}$	$-\alpha\gamma c_1 B M^F_{BC}$		$+\gamma\delta c_2 B M^F_{CB}$	$+\gamma\delta^1 c_2 B M^F_{CB}$
	$-\alpha\beta c_1^2 B M^F_{BC}$		$+\alpha\beta c_1^2 B M^F_{CB}$	$+\gamma\delta c_2^2 B M^F_{CB}$		$-\alpha\gamma c_1 c_2 B M^F_{BC}$	
$+\alpha^1\alpha\beta c_1^2 B M^F_{BC}$	$+\alpha^2\beta c_1^2 B M^F_{BC}$		$-\beta B^2 M^F_{CB}$	$-\gamma B^2 M^F_{CB}$		$+\alpha\gamma\delta c_1 c_2 B M^F_{BC}$	$+\alpha\gamma\delta^1 c_1 c_2 B M^F_{BC}$
	$-\beta c_1 B^2 M^F_{CB}$		$+\alpha^2\beta c_1^3 B M^F_{BC}$	$+\alpha\gamma\delta c_1 c_2^2 B M^F_{BC}$		$-\gamma c_2 B^2 M^F_{CB}$	
$+\alpha^1\beta c_1 B^2 M^F_{CB}$	$+\alpha\beta c_1 B^2 M^F_{CB}$		$-\alpha\beta c_1 B^2 M^F_{BC}$	$-\alpha\gamma c_1 B^2 M^F_{BC}$		$+\gamma\delta c_2 B^2 M^F_{CB}$	$+\gamma\delta^1 c_2 B^2 M^F_{CB}$

FIG. 3. Algebraic Distribution of Fixed-End Moments Due to Load on Span BC of Frame Shown in Fig. 1(b)

TABLE 4. Influence Line Equations for Support Moments in Frame Similar to Fig. 1(b), Consisting of Members of Variable Cross Section

End moment (1)	Load on AB (2)	Load on BC (3)	Load on CD (4)	Load on DE (5)
M_{BA}	$+[1 - \alpha^1(1 + \alpha\beta c_1^2 S)][c_{ab}M_{AB}^F + M_{BA}^F]$	$+\alpha^1[M_{BC}^F + \beta c_1 S(\alpha c_1 M_{BC}^F + M_{CB}^F)]$	$-\alpha^1\beta c_1 S[M_{CD}^F + \delta c_2 M_{DC}^F]$	$+\alpha^1\beta\delta c_1 c_2 S(M_{DE}^F + c_{ed}M_{ED}^F)$
M_{BC}	$-\alpha[1 - (1 - \alpha)\beta c_1^2 S][c_{ab}M_{AB}^F + M_{BA}^F]$	$-(1 - \alpha)[M_{BC}^F + \beta c_1 S(\alpha c_1 M_{BC}^F + M_{CB}^F)]$	$+\beta c_1 S(1 - \alpha)[M_{CD}^F + \delta c_2 M_{DC}^F]$	$-\beta\delta c_1 c_2 S(1 - \alpha)(M_{DE}^F + c_{ed}M_{ED}^F)$
M_{CB}	$-\alpha c_1[1 - \beta S(1 - \alpha c_1^2)[c_{ab}M_{AB}^F + M_{BA}^F]$	$+[1 - \beta S(1 - \alpha c_1^2)][\alpha c_1 M_{BC}^F + M_{CB}^F]$	$+\beta S(1 - \alpha c_1^2)[M_{CD}^F + \delta c_2 M_{DC}^F]$	$-\beta\delta c_2 S(1 - \alpha c_1^2)(M_{DE}^F + c_{ed}M_{ED}^F)$
M_{CD}	$+\alpha\gamma c_1 S(1 - \delta c_2^2)[c_{ab}M_{AB}^F + M_{BA}^F]$	$-\gamma S(1 - \delta c_2^2)[\alpha c_1 M_{BC}^F + M_{CB}^F]$	$-[1 - \gamma S(1 - \delta c_2^2)][M_{CD}^F + \delta c_2 M_{DC}^F]$	$+\delta c_2[1 - \gamma S(1 - \delta c_2^2)](M_{DE}^F + c_{ed}M_{ED}^F)$
M_{DC}	$+\alpha\gamma c_1 c_2 S(1 - \delta)[c_{ab}M_{AB}^F + M_{BA}^F]$	$-\gamma c_2 S(1 - \delta)[\alpha c_1 M_{BC}^F + M_{CB}^F]$	$+(1 - \delta)[M_{DC}^F + \gamma c_2 S(M_{CD}^F + \delta c_2 M_{DC}^F)]$	$+\delta[1 - (1 - \delta)\gamma c_2^2 S](M_{DE}^F + c_{ed}M_{ED}^F)$
M_{DE}	$-\alpha\gamma\delta^1 c_1 c_2 S[c_{ab}M_{AB}^F + M_{BA}^F]$	$+\gamma\delta^1 c_2 S[\alpha c_1 M_{BC}^F + M_{CB}^F]$	$-\delta^1[M_{DC}^F + \gamma c_2 S(M_{CD}^F + \delta c_2 M_{DC}^F)]$	$-[1 - \delta^1(1 + \gamma\delta c_2^2 S)](M_{DE}^F + c_{ed}M_{ED}^F)$

Note: c_{ab} is the carry-over factor from A to B; c_{ed} is the carry-over factor from E to D; c_1 is the carry-over factor from B to C and from C to B; c_2 is the carry-over factor from C to D and from D to C; + sign indicates clockwise direction around joint; and $S = 1/[1 - (\alpha\beta c_1^2 + \gamma\delta c_2^2)]$.

TABLE 5. Influence Line Equations for Support Moments in Frame Similar to Fig. 1(b), Consisting of Members of Constant Cross Section

End moment (1)	Load on AB (2)	Load on BC (3)	Load on CD (4)	Load on DE (5)
M_{BA}	$+\left[1 - \alpha_1\left(1 + \frac{\alpha\beta S}{4}\right)\right]\left(\frac{b}{2} + a\right) abl_1$	$+\alpha^1\left[b + \frac{\beta S}{2}\left(\frac{\alpha b}{2} + a\right)\right] abl_2$	$-\frac{\alpha\beta S}{2}\left(b + \frac{\delta a}{2}\right) abl_3$	$+\frac{\alpha^1\beta\delta S}{4}\left(b + \frac{a}{2}\right) abl_4$
M_{BC}	$-\alpha\left[1 - (1 - \alpha)\frac{\beta S}{4}\right]\left(\frac{b}{2} + a\right) abl_1$	$-(1 - \alpha)\left[b + \frac{\beta S}{2}\left(\frac{\alpha b}{2} + a\right)\right] abl_2$	$+\frac{\beta S}{2}(1 - \alpha)\left(b + \frac{\delta a}{2}\right) abl_3$	$-\frac{\beta\delta S}{4}(1 - \alpha)\left(b + \frac{a}{2}\right) abl_4$
M_{CB}	$-\frac{\alpha}{2}\left[1 - \beta S\left(1 - \frac{\alpha}{4}\right)\right]\left(\frac{b}{2} + a\right) abl_1$	$+\left[1 - \beta S\left(1 - \frac{\alpha}{4}\right)\right]\left(\frac{\alpha b}{2} + a\right) abl_2$	$+\beta S\left(1 - \frac{\alpha}{4}\right)\left(b + \frac{\delta a}{2}\right) abl_3$	$-\frac{\beta\delta S}{2}\left(1 - \frac{\alpha}{4}\right)\left(b + \frac{a}{2}\right) abl_4$
M_{CD}	$+\frac{\alpha\gamma S}{2}\left(1 - \frac{\delta}{4}\right)\left(\frac{b}{2} + a\right) abl_1$	$-\gamma S\left(1 - \frac{\delta}{4}\right)\left(\frac{\alpha b}{2} + a\right) abl_2$	$-\left[1 - \gamma S\left(1 - \frac{\delta}{4}\right)\right]\left(b + \frac{\delta a}{2}\right) abl_3$	$+\frac{\delta}{2}\left[1 - \gamma S\left(1 - \frac{\delta}{4}\right)\right]\left(b + \frac{a}{2}\right) abl_4$
M_{DC}	$+\frac{\alpha\gamma S}{4}(1 - \delta)\left(\frac{b}{2} + a\right) abl_1$	$-\frac{\gamma S}{2}(1 - \delta)\left(\frac{\alpha b}{2} + a\right) abl_2$	$+(1 - \delta)\left[a - \frac{\gamma S}{2}\left(b + \frac{\delta a}{2}\right)\right]$	$+\delta\left[1 - (1 - \delta)\frac{\gamma S}{4}\right]\left(b + \frac{a}{2}\right) abl_4$
M_{DE}	$-\frac{\alpha\gamma\delta^1 S}{4}\left(\frac{b}{2} + a\right) abl_1$	$+\frac{\gamma\delta^1 S}{2}\left(\frac{\alpha b}{2} + a\right) abl_2$	$-\delta^1\left[a - \frac{\gamma S}{2}\left(b + \frac{\delta a}{2}\right)\right]$	$-\left[1 - \delta^1\left(1 + \frac{\gamma\delta S}{4}\right)\right]\left(b + \frac{a}{2}\right) abl_4$

Note: "a" has values of from 0 to 1, and $b = (1 - a)$; + sign indicates clockwise direction around joint; and $S = 1/[1 - (\alpha\beta + \gamma\delta)/4]$.

APPENDIX I. NOTATION

The following symbols are used in this paper:

c = carry-over factor;
E = Young's modulus;
f = degree of end fixation;
H = horizontal reaction;
l = length of member;
M = design moment;
M^F = fixed-end moment;
M^S = nonsway moment;
S = sum to infinity of geometrical progression;
α, β, γ = distribution factors; and
ρ = stiffness factor.