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An overlapping domain decomposition approach for coupling the finite and boundary element methods

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Abstract

An overlapping iterative domain decomposition approach for the coupling of the finite element method (FEM) and the boundary element method (BEM) is presented in this paper. In this proposed method, the domain of the original problem is subdivided into a FEM sub-domain and a BEM sub-domain, such that the two sub-domains partially overlap over a common region. The common region is modeled by both methods. A brief discussion on the existing iterative coupling methods and their limitations are given in the first part of this paper. In the second part, the proposed overlapping method is described and the convergence conditions are presented. Two numerical examples are given to demonstrate the capability of the proposed method for handling cases where the Neumann boundary conditions are specified on the entire external boundary of the FEM or BEM sub-domains.

Keywords: Boundary element method; Finite element method; Domain decomposition; Iterative methods; Coupling

1. Introduction

The finite element method (FEM) and the boundary element method (BEM) are well known as powerful numerical techniques for solving a wide range of problems in applied science and engineering. Each method has its own advantages and disadvantages, so that it is desirable to develop a combined FEM/BEM technique, which makes use of their advantages and reduces or completely eliminates their disadvantages, and to use the combined technique in situations where it is appropriate. The first proposed coupling formulation was presented by Zeinkiewicz and co-authors [1]. Subsequent contributions came from Atluri and Grannel [2] and Brebbia and Georgion [3]. Unfortunately, the FEM and BEM lead to very different kinds of systems that cannot be linked as they stand. The conventional coupling methods employ an entire unified equation for the whole domain by altering the formulation of one of the methods to make it compatible with the other. There are many variations of the conventional coupling methods. See, e.g. Refs. [1–18] not to mention many others.

More recently, coupling the BEM and FEM has been achieved through the iterative domain decomposition methods. In these coupling methods there is no need to combine the coefficient matrices of the FEM and the BEM sub-domains, as required in most of the conventional coupling methods. Separate computing for each sub-domain and successive renewal of the variables on the interface of both sub-domains are performed to reach the final convergence. A second advantage is that different formulations for the FEM and BEM can be adopted as base programs for coupling the computer codes only.

Gerstle et al. [19] presented a solution method, which was iterative in nature. In each iteration the sub-domains were analyzed independently by applying trial displacements to degrees of freedom on the interface. The conjugate gradient domain decomposition solver was used to predict a new set of trial interfacial displacements for the next iteration. Their method, however, was only applicable to the symmetric BEM formulation. Perera et al. [20] presented a parallel method based on the interface equilibrium of Steklov Poincare. Kamiya et al. [21] employed the renewal methods known as Schwarz Neumann–Neumann and Schwarz Dirichlet–Neumann methods. It should be noted, however, that the above methods presented in Refs. [20,21] are not applicable for problems where Neumann boundary conditions are specified on the entire external boundary of the FEM sub-domain as will be explained later. Kamiya and Iwase [22] introduced an iterative analysis using conjugate gradient and condensation, which again had the same limitation of being applicable only to the symmetric BEM formulation. Computations for all the above iterative methods

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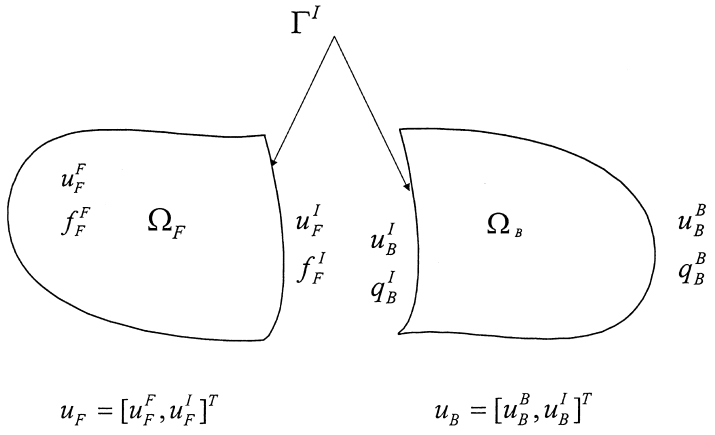

Fig. 1. Domain decomposition.

were performed in parallel. Lin et al. [23] and Feng and Owen [24] proposed a method similar to the Schwarz Dirichlet–Neumann method. The method was based on assigning an arbitrary displacement vector to the interface of the BEM sub-domain. Then, the energy equivalent nodal forces of the obtained interface tractions were treated as boundary conditions for the FEM sub-domain to solve for the interfacial displacements. The procedure was iterated until convergence is achieved.

In this paper, we review and discuss the limitations of some of the available iterative coupling methods. We also propose an overlapping iterative coupling method that overcomes these limitations.

2. Iterative coupling methods

For simplicity, let us Consider the 2-D region of Fig. 1, which is governed by the Laplace equation, i.e. $K_i \nabla^2 u = 0$ in Ω_i, where K_i is the material property in the sub-domain Ω_i and u is the potential. The boundary conditions are such that the potential u, the flux $q = K\nabla u$ or their combination is prescribed at each point on the boundary. The decomposed portions are modeled using the BEM and FEM. Now, let us define the following vectors (Fig. 1):

$u_{\mathrm{B}}^{\mathrm{I}}$: potential on the FEM/BEM interface, approached from the BEM sub-domain
$u_{\mathrm{B}}^{\mathrm{B}}$: potential in the BEM sub-domain except $u_{\mathrm{B}}^{\mathrm{I}}$

$$u_{\mathrm{B}} = [u_{\mathrm{B}}^{\mathrm{B}}, u_{\mathrm{B}}^{\mathrm{I}}]^{\mathrm{T}}$$

$u_{\mathrm{F}}^{\mathrm{I}}$: potential on the FEM/BEM interface, approached from the FEM sub-domain
$u_{\mathrm{F}}^{\mathrm{F}}$: potential in the FEM sub-domain except $u_{\mathrm{F}}^{\mathrm{I}}$

$$u_{\mathrm{F}} = [u_{\mathrm{F}}^{\mathrm{F}}, u_{\mathrm{F}}^{\mathrm{I}}]^{\mathrm{T}}$$

Similarly, one can define the flux and integrated flux vectors for the BEM and the FEM sub-domains, respectively. The corresponding boundary integral equation for the BEM sub-domain can be written in sub-structured form as

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u_{\mathrm{B}}^{\mathrm{B}} \\ u_{\mathrm{B}}^{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} q_{\mathrm{B}}^{\mathrm{B}} \\ q_{\mathrm{B}}^{\mathrm{I}} \end{bmatrix} \tag{1}$$

where H and G are influence coefficient matrices. After solving for the unknowns in Eq. (1), a potential vector inside the domain can be obtained as

$$u = \begin{bmatrix} H_{11}^* & H_{12}^* \\ H_{12}^* & H_{22}^* \end{bmatrix} \begin{bmatrix} u_{\mathrm{B}}^{\mathrm{B}} \\ u_{\mathrm{B}}^{\mathrm{I}} \end{bmatrix} + \begin{bmatrix} G_{11}^* & G_{12}^* \\ G_{21}^* & G_{22}^* \end{bmatrix} \begin{bmatrix} q_{\mathrm{B}}^{\mathrm{B}} \\ q_{\mathrm{B}}^{\mathrm{I}} \end{bmatrix} \tag{2}$$

For the FEM sub-domain, the assembled element equations are given by

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u_{\mathrm{F}}^{\mathrm{F}} \\ u_{\mathrm{F}}^{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} f_{\mathrm{F}}^{\mathrm{F}} \\ f_{\mathrm{F}}^{\mathrm{I}} \end{bmatrix} \tag{3}$$

where K is the stiffness matrix for the system. At the interface, the compatibility and equilibrium conditions are

$$u_{\mathrm{B}}^{\mathrm{I}} = u_{\mathrm{F}}^{\mathrm{I}} \in \Gamma^{\mathrm{I}}, \tag{4}$$

$$f_{\mathrm{F}}^{\mathrm{I}} + [M]\{q_{\mathrm{B}}^{\mathrm{I}}\} = 0 \in \Gamma^{\mathrm{I}} \tag{5}$$

where M is the converting matrix due to the weighing of the boundary fluxes by the interpolation function on the interface. It should be noted that similar coupling equations can be obtained for the elasticity problem by substituting the displacement and force vectors for the potential and integrated flux vectors, respectively. The procedures for solving the above equations using the available non-overlapping iterative coupling methods are given below.

2.1. Parallel Schwarz Neumann–Neumann method

In this method, the flux values (Neumann data) are assumed simultaneously on the interface of each sub-domain [21]. Then, the computations for FEM and BEM are performed in parallel as described below:

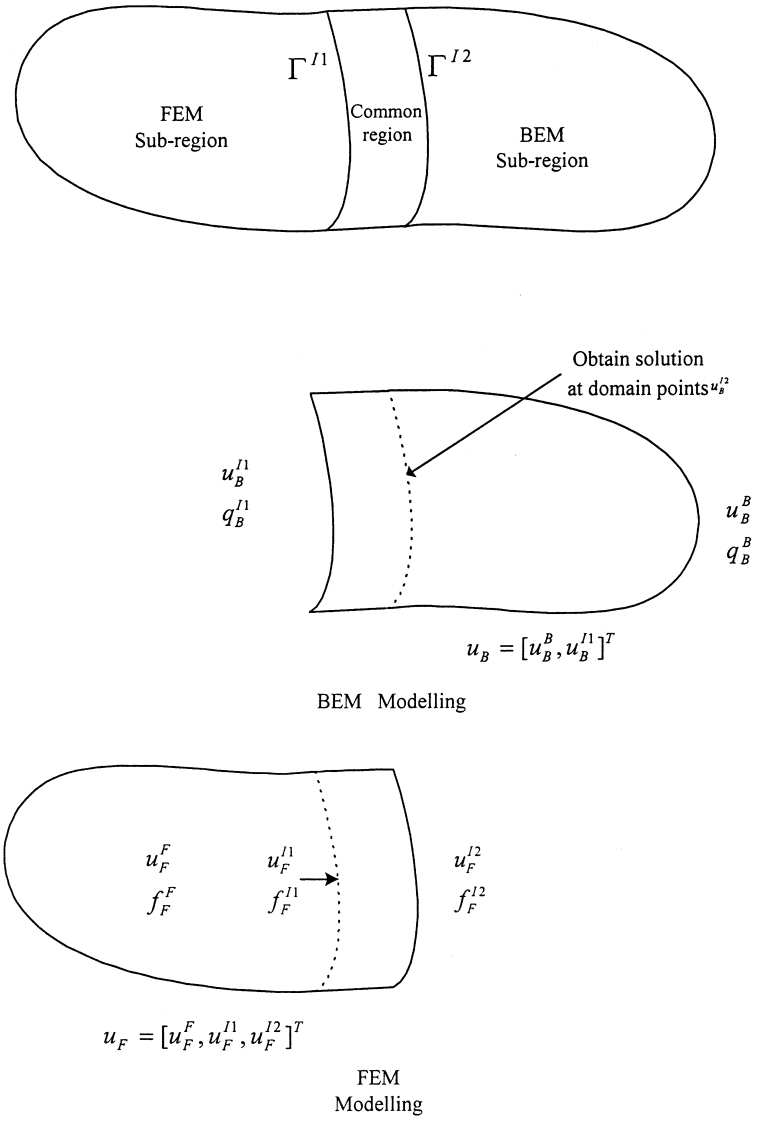

Fig. 2. Overlapping domain decomposition.

1. Set initial values $q^{\mathrm{I}}_{\mathrm{B},0} = \bar{q}$ and $f^{\mathrm{I}}_{\mathrm{F},0} = -Mq^{\mathrm{I}}_{\mathrm{B},0}$
2. For $n = 0, 1, 2, \ldots$, do

Solve
$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{B}}_{\mathrm{B}} \\ u^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} q^{\mathrm{B}}_{\mathrm{B}} \\ q^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{F}}_{\mathrm{F}} \\ u^{\mathrm{I}}_{\mathrm{F},n} \end{bmatrix} = \begin{bmatrix} f^{\mathrm{F}}_{\mathrm{F}} \\ f^{\mathrm{I}}_{\mathrm{F},n} \end{bmatrix}$$

Get $u^{\mathrm{I}}_{\mathrm{B},n}$ and $u^{\mathrm{I}}_{\mathrm{F},n}$

Apply $q^{\mathrm{I}}_{\mathrm{B},n+1} = q^{\mathrm{I}}_{\mathrm{B},n} + \beta(u^{\mathrm{I}}_{\mathrm{F},n} - u^{\mathrm{I}}_{\mathrm{B},n})$

$f^{\mathrm{I}}_{\mathrm{F},n+1} = -Mq^{\mathrm{I}}_{\mathrm{B},n}$

where β is a relaxation parameter.

Until $$\frac{\|u^{\mathrm{I}}_{\mathrm{B},n+1} - u^{\mathrm{I}}_{\mathrm{B},n}\|}{\|u^{\mathrm{I}}_{\mathrm{B},n+1}\|} < \epsilon \text{ (given tolerance)}$$

A drawback of this method is that it requires a parameter β, the selection of which requires some trial and error and deep experience. Another major drawback is that the method produces non-unique solutions for certain problems where Neumann boundary conditions are specified on the entire external boundary of the FEM or BEM sub-domain.

2.2. Parallel Schwarz Dirichlet–Neumann method

In this method [22], the potential (Dirichlet data) is assumed on the BEM interface while the flux (Neumann data) is assumed on the FEM interface. The computations for FEM and BEM are performed in parallel. The iterative method can be described as follows:

1. Set initial values $u^{\mathrm{I}}_{\mathrm{B},0} = \bar{u}$ and $f^{\mathrm{I}}_{\mathrm{F},0} = \bar{f}$.
2. For $n = 0, 1, 2, \ldots$, do

Solve $$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{B}}_{\mathrm{B}} \\ u^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} q^{\mathrm{B}}_{\mathrm{B}} \\ q^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{F}}_{\mathrm{F}} \\ u^{\mathrm{I}}_{\mathrm{F},n} \end{bmatrix} = \begin{bmatrix} f^{\mathrm{F}}_{\mathrm{F}} \\ f^{\mathrm{I}}_{\mathrm{F},n} \end{bmatrix}$$

Get $q^{\mathrm{I}}_{\mathrm{B},n}$ and $u^{\mathrm{I}}_{\mathrm{F},n}$

Apply $$u^{\mathrm{I}}_{\mathrm{B},n+1} = (1-\alpha)u^{\mathrm{I}}_{\mathrm{B},n} + \alpha u^{\mathrm{I}}_{\mathrm{F},n}$$

$$f^{\mathrm{I}}_{\mathrm{F},n+1} = -Mq^{\mathrm{I}}_{\mathrm{B},n}$$

where α is a relaxation parameter.

Until $$\frac{\|u^{\mathrm{I}}_{\mathrm{B},n+1} - u^{\mathrm{I}}_{\mathrm{B},n}\|}{\|u^{\mathrm{I}}_{\mathrm{B},n+1}\|} < \epsilon \text{ (given tolerance)}$$

The method may not be suited for problems where Neumann boundary conditions are specified on the entire external boundary of the FEM sub-domain due to the same reason given earlier.

2.3. Sequential Schwarz Dirichlet–Neumann method

The iterative method can be described as follows [23,24]:

1. Set initial values $u^{\mathrm{I}}_{\mathrm{B},0} = \bar{u}$.
2. For $n = 0, 1, 2, \ldots$, do

Solve $$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{B}}_{\mathrm{B}} \\ u^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} q^{\mathrm{B}}_{\mathrm{B}} \\ q^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix}$$

Get $q^{\mathrm{I}}_{\mathrm{B},n}$

Solve $$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{F}}_{\mathrm{F}} \\ u^{\mathrm{I}}_{\mathrm{F},n} \end{bmatrix} = \begin{bmatrix} f^{\mathrm{F}}_{\mathrm{F}} \\ -Mq^{\mathrm{I}}_{\mathrm{B},n} \end{bmatrix}$$

Get $u^{\mathrm{I}}_{\mathrm{F},n}$

Apply $$u^{\mathrm{I}}_{\mathrm{B},n+1} = (1-\alpha)u^{\mathrm{I}}_{\mathrm{B},n} + \alpha u^{\mathrm{I}}_{\mathrm{F},n}$$

where α is a relaxation parameter.

Until $$\frac{\|u^{\mathrm{I}}_{\mathrm{B},n+1} - u^{\mathrm{I}}_{\mathrm{B},n}\|}{\|u^{\mathrm{I}}_{\mathrm{B},n+1}\|} < \epsilon \text{ (given tolerance)}$$

The method has the same limitation as that of the parallel Shwarz Dirichlet–Neumann method.

3. An overlapping iterative domain decomposition method

It is clear from the previous section that the parallel Neumann–Neumann method may not be suited for problems where the Neumann boundary conditions are specified on either the entire external boundary of the FEM or BEM sub-domains. Also, the other two methods are not suited for problems where the Neumann boundary conditions are specified on the entire external boundary of the FEM sub-domain.

In this section we propose an overlapping domain decomposition method that can handle such situations. The domain of the original problem is subdivided into FEM and BEM sub-domains, such that the two sub-domains partially overlap over a common region that is modeled by both methods (Fig. 2). In general, the size of the common region is chosen arbitrarily. However, its minimum width should not be too small in order to avoid the computation of singular boundary integrals, but not too large to avoid the reduction in the efficiency of the method, due to the increase in the number of the coupling equations. A reasonable decision is to choose a common region with a minimum width being equal to the length of the boundary element on the BEM interface. To describe the proposed method, let us define the following vectors:

$u^{\mathrm{I1}}_{\mathrm{B}}$: displacement on Γ^{I1} approached from the BEM sub-domain
$u^{\mathrm{B}}_{\mathrm{B}}$: displacement in the BEM sub-domain except $u^{\mathrm{I1}}_{\mathrm{B}}$
$u^{\mathrm{I2}}_{\mathrm{B}}$: displacement on Γ^{I2} calculated as internal points for the BEM sub-domain

$$u_{\mathrm{B}} = [u^{\mathrm{B}}_{\mathrm{B}}, u^{\mathrm{I1}}_{\mathrm{B}}]^{\mathrm{T}}$$

$u^{\mathrm{I1}}_{\mathrm{F}}$: displacement on Γ^{I1} approached from the FEM sub-domain
$u^{\mathrm{I2}}_{\mathrm{F}}$: displacement on Γ^{I2} approached from the FEM sub-domain
$u^{\mathrm{F}}_{\mathrm{F}}$: displacement in the FEM sub-domain except $u^{\mathrm{I1}}_{\mathrm{F}}$ and

$$u_{\mathrm{F}} = [u^{\mathrm{F}}_{\mathrm{F}}, u^{\mathrm{I1}}_{\mathrm{F}}, u^{\mathrm{I2}}_{\mathrm{F}}]^{\mathrm{T}}$$

Similarly, one can denote the BEM flux vector by $q^{\mathrm{B}}_{\mathrm{B}}$ and $q^{\mathrm{I1}}_{\mathrm{B}}$ and FEM integrated flux vectors by $f^{\mathrm{F}}_{\mathrm{F}}$, $f^{\mathrm{I1}}_{\mathrm{F}}$ and $f^{\mathrm{I2}}_{\mathrm{F}}$.

The proposed iterative method can be described as follows:

1. Set initial values $u^{\mathrm{I1}}_{\mathrm{B},0} = \bar{u}$.
2. For $n = 0, 1, 2, \ldots$, do

Solve $$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u^{\mathrm{B}}_{\mathrm{B}} \\ u^{\mathrm{I1}}_{\mathrm{B},n} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} q^{\mathrm{B}}_{\mathrm{B}} \\ q^{\mathrm{I1}}_{\mathrm{B},n} \end{bmatrix}$$

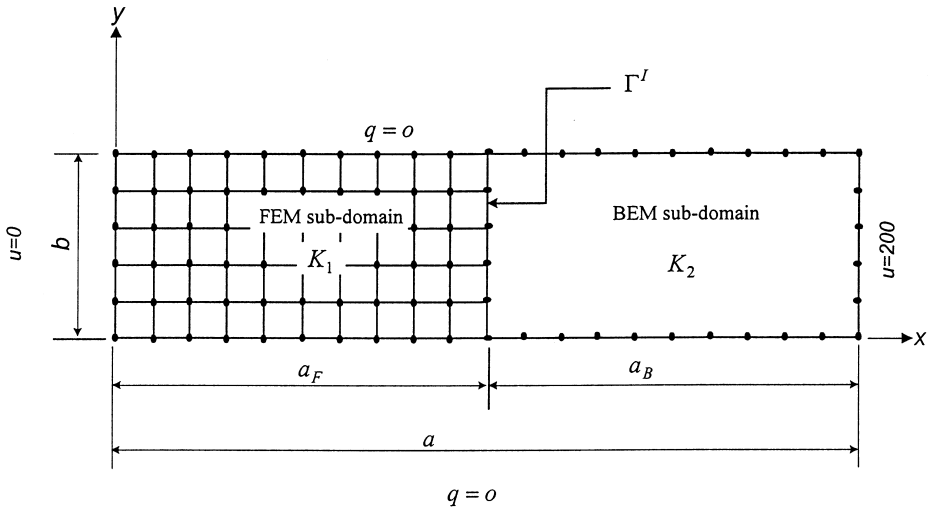

Fig. 3. Example 1 using the sequential Dirichlet–Neumann method.

Get $q^{I1}_{B,n}$ and the boundary unknowns

Solve

$$u^{I2}_{B,n} = \begin{bmatrix} H^*_{11} & H^*_{12} \\ H^*_{12} & H^*_{22} \end{bmatrix} \begin{bmatrix} u^B_B \\ u^{I1}_{B,n} \end{bmatrix} + \begin{bmatrix} G^*_{11} & G^*_{12} \\ G^*_{21} & G^*_{22} \end{bmatrix} \begin{bmatrix} q^B_B \\ q^{I1}_{B,n} \end{bmatrix}$$

Solve
$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} u^F_F \\ u^{I1}_{F,n} \\ u^{I2}_{B,n} \end{bmatrix} = \begin{bmatrix} f^F_F \\ f^{I1}_F \\ f^{I2}_F \end{bmatrix}$$

Get $u^{I1}_{F,n}$

Apply $u^{I1}_{B,n+1} = (1-\alpha)u^{I1}_{B,n} + \alpha u^{I1}_{F,n}$

where α is a relaxation parameter

Until $$\frac{\|u^{I1}_{B,n+1} - u^{I1}_{B,n}\|}{\|u^{I1}_{B,n+1}\|} < \epsilon \text{ (given tolerance)}$$

The current proposed method avoids the prescription of Neumann boundary conditions on the interface Γ^{I1} or Γ^{I2} and therefore it overcomes the problem encountered in the previous iterative methods.

Now we wish to conduct a convergence analysis of the proposed overlapping iterative method. After applying boundary conditions, rearranging and conducting a series of matrix operations, the vector of unknowns $u^{I2}_{B,n}$ is obtained as

$$[u^{I2}_{B,n}] = [A_{11}\ A_{12}] \begin{bmatrix} C_B \\ u^{I1}_{B,n} \end{bmatrix} \quad (6)$$

Similarly for the FEM one can apply boundary conditions, rearrange and perform a series of matrix operations to obtain

$$\begin{bmatrix} u^F_F \\ u^{I1}_{F,n} \\ u^{I2}_{F,n} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C^F_F \\ C^{I1}_F \\ u^{I2}_{B,n} \end{bmatrix} \quad (7)$$

Note that C_B, C^F_F and C^{I1}_F are vectors of known values. Using the second row of Eq. (7) and substituting in Eq. (6), gives

$$u^I_{F,n} = C u^{I1}_{B,n} + c \quad (8)$$

where

$C = F_{23}A_{12}$

and

$c = F_{21}C^F_F + F_{22}C^{I1}_F + F_{23}A_{11}C_B$

Substituting for $u^I_{F,n}$ in the iterations $u^{I1}_{B,n+1} = (1-\alpha)u^{I1}_{B,n} +$

Table 1
Applicable range and optimum α

K_2/K_1		Relaxation parameter	
		α (sequential Dirichlet–Neumann method)	α (overlapping iterative method)
0.1	Range	0.02–1.8	0.02–3.62
	Optimum	0.9	1.81
0.5	Range	0.02–1.32	0.02–8.0
	Optimum	0.66	4.0
1.0	Range	0.02–0.98	0.02–11
	Optimum	0.5	5.5
2.0	Range	0.02–0.64	0.02–14
	Optimum	0.34	7.0
8.0	Range	0.02–0.20	0.02–18
	Optimum	0.12	9.0

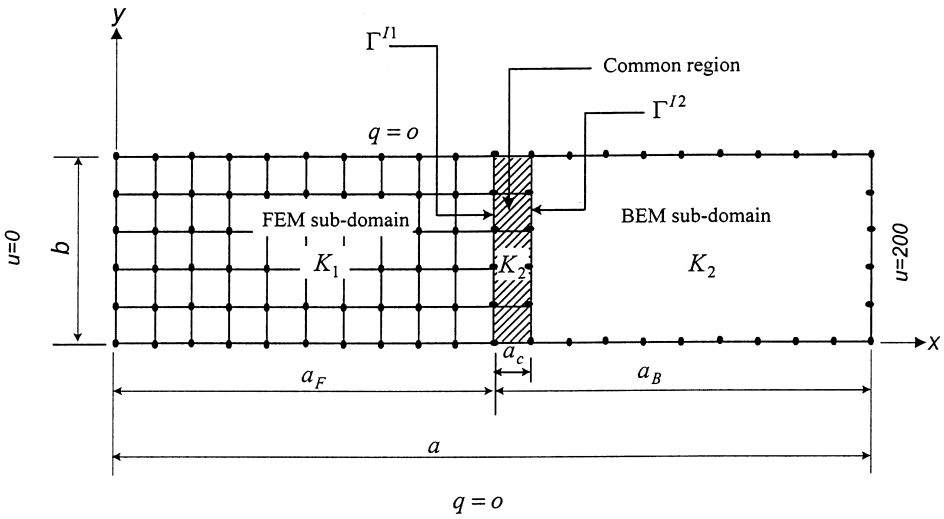

Fig. 4. Example 1 using the overlapping iterative method.

$\alpha u^{I1}_{F,n}$, gives

$$u^{I1}_{B,n+1} = [(1-\alpha)I + \alpha C]u^{I1}_{B,n} + \alpha c \tag{9}$$

which has the same form as obtained in a previous investigation [25]. We conclude that if $\lambda_1 = x_1 + iy_1, \ldots, \lambda_N = x_N + iy_N$ are the eigenvalues of C, then,

$$\alpha < \min_{1\le i\le N}\left\{\frac{2(1-x_i)}{(1-x_i)^2+y_i^2}\right\} \qquad x_i < 1, \quad i = 1,2,\ldots,N \tag{10}$$

are the necessary conditions for the convergence of the overlapping iterative coupling method. The optimum α is obtained as

$$\bar{\alpha} = -\frac{\mathrm{Re}(\underline{1}^{\mathrm{T}}(\underline{\lambda}-\underline{1}))}{\|\underline{\lambda}-\underline{1}\|^2} \tag{11}$$

4. Numerical examples

In this section we give two numerical examples that show the applicability of the overlapping iterative coupling method. It should be noted that the proposed method is versatile and capable of handling more complex problems than the examples presented in this section.

4.1. Example 1

Consider the simple case of a potential flow in a rectangular domain (Fig. 3). For the Sequential Dirichlet–Neumann method, the rectangular domain is decomposed to the FEM and BEM sub-domains. The problem is modeled using 30 linear boundary elements and 50 linear quadrilateral finite elements. The geometry of the problem is such that $0 \le x \le a$ and $0 \le y \le b$ while the boundary conditions are $u(0,y) = 0$, $u(a,y) = 200$ and zero flux elsewhere. The values of a_1, a_1 and K_1 are fixed to unity. The same

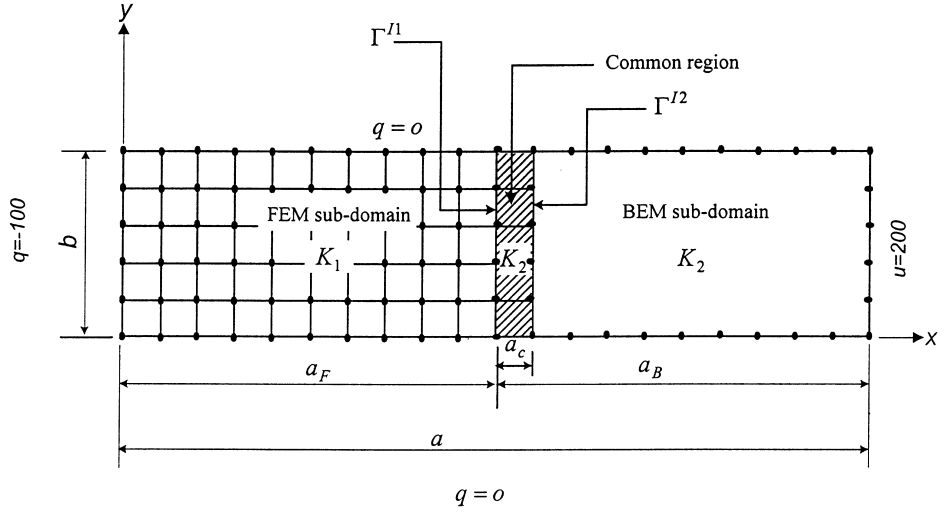

Fig. 5. Example 1 using the overlapping iterative method (Neumann boundary conditions are specified for the entire FEM sub-domain).

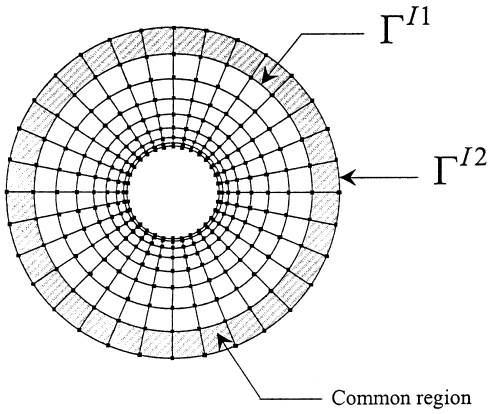

Fig. 6. Discretization for the tunnel problem.

problem is reinvestigated with the overlapping iterative method with an overlapping distance $a_c = 0.1$ (Fig. 4). The problem is modeled using 30 linear boundary elements and 55 linear quadrilateral finite elements. Due to the simplicity of the problem, both methods agree very well with the exact solution and therefore, the results are not given here. Since the parameter α is an important parameter for all iterative coupling methods, especially for problems with very dissimilar materials [25], a comparative analysis is performed with different values of K_2/K_1 using the two methods. The optimum value and the applicable range of the parameter α for both methods are given in Table 1. The results in Table 1 indicate that the overlapping iterative method provides a wider applicable range of α, which is more advantageous. As an example, the applicable range is 0.02–0.2 for $K_2/K_1 = 8$ using the sequential Dirichlet–Neumann method as compared to 0.02–18 using the overlapping iterative method.

In order to show the applicability of the new method to problems involving Neumann boundary conditions on the entire external boundary of the FEM sub-domain, let us investigate the same problem with the following boundary conditions: $q(0, y) = -100$, $u(a, y) = 200$ and zero flux elsewhere (Fig. 5). The geometry and the material properties are such that, a_1, a_2, K_1 and K_2 are fixed to unity. The problem considered here cannot be solved using any of the iterative methods presented in Section 2. The overlapping iterative method gives a solution for such a case that is in good agreement with the exact solution. The range for the parameter α is obtained as 0.02–20 with an optimum value of 10.

4.2. Example 2

Consider an elasticity problem where the excavation of a circular tunnel opening in a geological medium is modeled. The tunnel is deeply inserted in an intact rock. The plane strain condition is assumed to prevail. The radius of the tunnel R is taken as 100 units. The material properties employed are as follows: Young's modulus $E = 2.1 \times 10^4$ units, Poisson's ratio $\nu = 0.18$. The problem was modeled using 32 linear boundary elements and 256 linear quadrilateral finite elements (Fig. 6). The stress condition in the geological medium is assumed to be hydrostatic and the stress is taken to be 10 units. At the boundary of the tunnel, the forces corresponding to the in situ state of stress condition are computed at the nodal points and applied in the opposite direction to simulate the excavation of the opening.

Fig. 7 shows an excellent agreement between the results of the overlapping iterative method and the exact solution for the radial displacements. It should be noted that this problem cannot be solved using the iterative methods presented in Section 2 due to the reasons explained earlier.

5. Conclusions

An overlapping iterative domain decomposition method for coupling the FEM and BEM is presented. Unlike the

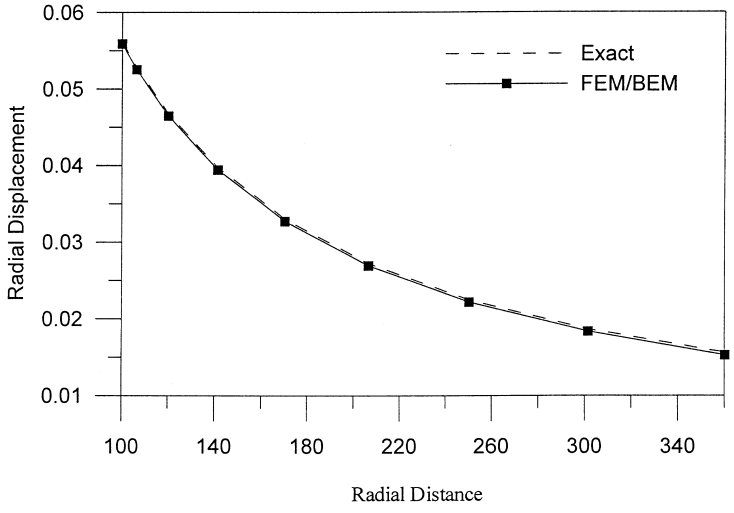

Fig. 7. Radial displacement for the tunnel problem.

previous iterative coupling methods, the proposed one has the capability of handling problems where Neumann boundary conditions are specified on the entire external boundary of the FEM or BEM sub-domains. In addition, the proposed method has the advantage of providing a wider range for selecting the relaxation parameter, which is an important factor for the convergence of the iterative methods. The proposed method is tested through two simple numerical examples. The present study will be extended to consider problems having more complicated geometry and material properties.

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