

LAB 3-B : ROOTS OF POLYNOMIAL AND TRIGONOMETRIC EQUATIONS EQUATIONS

ROOTS OF POLYNOMIAL AND TRIGONOMETRIC EQUATIONS EQUATIONS

There are three simple built-in functions:

- 1- Solve[f(x)==0,x]
- 2- NSolve[f(x)==0,x]
- 3- FindRoot[f(x)==0,{x,x0}] where x0 is an initial guess.

The first built in function gives exact roots for most of polynomials. Examples:

$$f = 26 - 15x - 12x^2 + x^3$$

$$26 - 15x - 12x^2 + x^3$$

$$\text{Solve}[26 - 15x - 12x^2 + x^3 == 0, x]$$

$$\{\{x \rightarrow -2\}, \{x \rightarrow 1\}, \{x \rightarrow 13\}\}$$

$$\text{Solve}[f == 0, x]$$

$$\{\{x \rightarrow -2\}, \{x \rightarrow 1\}, \{x \rightarrow 13\}\}$$

$$\text{Solve}[-15x - 12x^2 + x^3 == -26, x]$$

$$\{\{x \rightarrow -2\}, \{x \rightarrow 1\}, \{x \rightarrow 13\}\}$$

$$\text{Solve}[-12x^2 + x^3 == -26 + 15x, x]$$

$$\{\{x \rightarrow -2\}, \{x \rightarrow 1\}, \{x \rightarrow 13\}\}$$

$$f2 = 3 + 3x - 7x^2 - x^3 + 2x^4 + 3x^7 - 3x^8 - x^9 + x^{10}$$

$$3 + 3x - 7x^2 - x^3 + 2x^4 + 3x^7 - 3x^8 - x^9 + x^{10}$$

$$\text{Solve}[f2 == 0, x]$$

$$\{\{x \rightarrow 1\}, \{x \rightarrow -\sqrt{3}\}, \{x \rightarrow \sqrt{3}\}, \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 1]\}, \\ \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 2]\}, \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 3]\}, \\ \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 4]\}, \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 5]\}, \\ \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 6]\}, \{x \rightarrow \text{Root}[1 + 2\sqrt{1 + \sqrt{1^7}}, 7]\}\}$$

$$\text{NSolve}[1 + 2x + x^7 == 0, x]$$

$$\{\{x \rightarrow -0.868688 - 0.585282i\}, \{x \rightarrow -0.868688 + 0.585282i\}, \\ \{x \rightarrow -0.496292\}, \{x \rightarrow 0.0763556 - 1.14095i\}, \{x \rightarrow 0.0763556 + 1.14095i\}, \\ \{x \rightarrow 1.04048 - 0.56735i\}, \{x \rightarrow 1.04048 + 0.56735i\}\}$$

To get a numerical value, we have to use the second built-in function:

NSolve[f2 == 0, x]

```
{ {x → -1.73205}, {x → -0.868688 - 0.585282 i},
  {x → -0.868688 + 0.585282 i}, {x → -0.496292},
  {x → 0.0763556 - 1.14095 i}, {x → 0.0763556 + 1.14095 i}, {x → 1.},
  {x → 1.04048 - 0.56735 i}, {x → 1.04048 + 0.56735 i}, {x → 1.73205} }
```

Another example:

Solve[Exp[x] - x^2 == 0, x]

InverseFunction::ifun:

Inverse functions are being used. Values may be lost for multivalued inverses.

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Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found.

```
{ {x → -2 ProductLog[-1/2]}, {x → -2 ProductLog[1/2]} }
```

NSolve[Exp[x] - x^2 == 0, x]

InverseFunction::ifun:

Inverse functions are being used. Values may be lost for multivalued inverses.

InverseFunction::ifun:

Inverse functions are being used. Values may be lost for multivalued inverses.

Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found.

```
{ {x → -0.703467}, {x → 1.58805 - 1.54022 i} }
```

Solve[Sin[x] - x^2 == 0, x]

Solve::tdep:

The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

```
Solve[-x^2 + Sin[x] == 0, x]
```

Let us try 'NSolve':

```
NSolve[Sin[x] - x2 == 0, x]
```

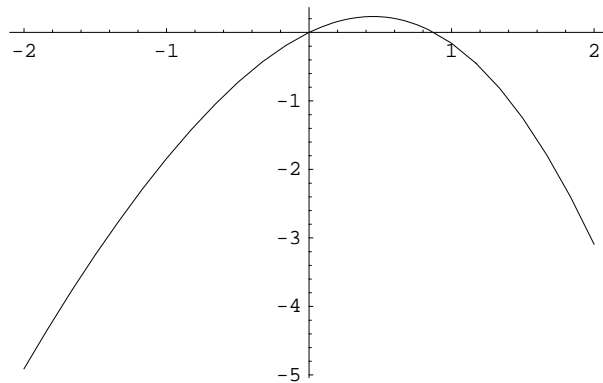
```
Solve::tdep:
```

The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

```
NSolve[-x2 + Sin[x] == 0, x]
```

We need to use another built-in function called: 'FindRoot'. But we need an initial guess. Let us plot the function

```
Plot[Sin[x] - x2, {x, -2, 2}]
```



- Graphics -

A reasonable guess is 1:

```
FindRoot[Sin[x] - x2 == 0, {x, 1}]
```

```
{x → 0.876726}
```

```
FindRoot[Sin[x] - x2 == 0, {x, 5}]
```

```
{x → 0.876727}
```

```
FindRoot[Sin[x] - x2 == 0, {x, -1}]
```

```
{x → -1.20947 × 10-11}
```

```
FindRoot[Sin[x] - x2 == 0, {x, -10}]
```

```
{x → -1.98508 × 10-12}
```

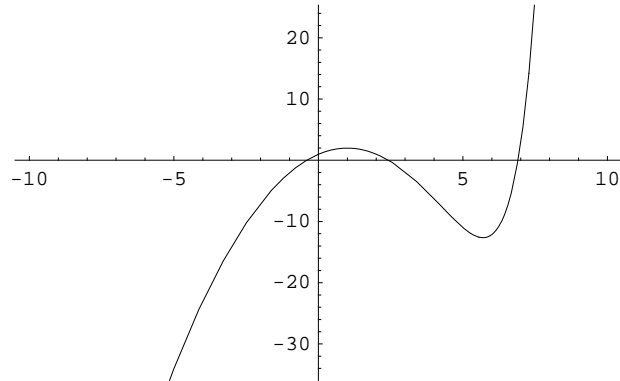
```
FindRoot[Sin[x] - x2 == 0, {x, .3}]
```

```
{x → -1.4361 × 10-8}
```

$$f = 1 + e^{-5+x} (-2 + x) + 2x - x^2$$

$$1 + e^{-5+x} (-2 + x) + 2x - x^2$$

`Plot[f, {x, -10, 10}]`



- Graphics -

`FindRoot[f == 0, {x, -5}]`

{x → -0.410399}

`FindRoot[f == 0, {x, 2}]`

{x → 2.42563}

`FindRoot[f == 0, {x, 10}]`

{x → 6.90177}

`Solve[{x^2 - y^2 == 1, x^3 + 2y^2 == 5}, {x, y}]`

$$\left\{ \left\{ y \rightarrow -\sqrt{\left(-\frac{5}{9} - \frac{4}{9} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right)^{1/3} + \frac{1}{9} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right)^{2/3} + \frac{1}{9} \left(\frac{1}{2} (173 + 3\sqrt{3297})\right)^{2/3} - \frac{2}{9} 2^{2/3} (173 + 3\sqrt{3297})^{1/3} + \frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right) (173 + 3\sqrt{3297})\right)^{1/3}\right)}, \right. \\ \left. x \rightarrow -\frac{2}{3} + \frac{1}{3} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right)^{1/3} + \frac{1}{3} \left(\frac{1}{2} (173 + 3\sqrt{3297})\right)^{1/3} \right\}, \\ \left\{ y \rightarrow \sqrt{\left(-\frac{5}{9} - \frac{4}{9} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right)^{1/3} + \frac{1}{9} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right)^{2/3} + \frac{1}{9} \left(\frac{1}{2} (173 + 3\sqrt{3297})\right)^{2/3} - \frac{2}{9} 2^{2/3} (173 + 3\sqrt{3297})^{1/3} + \frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right) (173 + 3\sqrt{3297})\right)^{1/3}\right)}, \right. \\ \left. x \rightarrow \frac{2}{3} + \frac{1}{3} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2}\right)^{1/3} + \frac{1}{3} \left(\frac{1}{2} (173 + 3\sqrt{3297})\right)^{1/3} \right\} \right\}$$

$$\frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right) (173 + 3\sqrt{3297}) \right)^{1/3},$$

$$x \rightarrow -\frac{2}{3} + \frac{1}{3} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} + \frac{1}{3} \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{1/3},$$

$$\left\{ y \rightarrow -\sqrt{\left(-\frac{5}{9} + \frac{2}{9} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} + \frac{2i \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3}}{3\sqrt{3}} - \right.} \right.$$

$$\frac{1}{18} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3} + \frac{i \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3}}{6\sqrt{3}} -$$

$$\frac{1}{18} \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{2/3} + \frac{1}{9} 2^{2/3} (173 + 3\sqrt{3297})^{1/3} -$$

$$\frac{i 2^{2/3} (173 + 3\sqrt{3297})^{1/3}}{3\sqrt{3}} - \frac{i (173 + 3\sqrt{3297})^{2/3}}{6 \cdot 2^{2/3} \sqrt{3}} +$$

$$\left. \frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right) (173 + 3\sqrt{3297}) \right)^{1/3} \right\},$$

$$x \rightarrow -\frac{2}{3} - \frac{1}{6} (1 + i\sqrt{3}) \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} -$$

$$\frac{1}{6} (1 - i\sqrt{3}) \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{1/3},$$

$$\left\{ y \rightarrow \sqrt{\left(-\frac{5}{9} + \frac{2}{9} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} + \frac{2i \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3}}{3\sqrt{3}} - \right.} \right.$$

$$\frac{1}{18} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3} + \frac{i \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3}}{6\sqrt{3}} -$$

$$\frac{1}{18} \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{2/3} + \frac{1}{9} 2^{2/3} (173 + 3\sqrt{3297})^{1/3} -$$

$$\begin{aligned}
& \frac{i 2^{2/3} (173 + 3 \sqrt{3297})^{1/3}}{3 \sqrt{3}} - \frac{i (173 + 3 \sqrt{3297})^{2/3}}{6 2^{2/3} \sqrt{3}} + \\
& \left. \frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right) (173 + 3 \sqrt{3297}) \right)^{1/3} \right), \\
\mathbf{x} \rightarrow & -\frac{2}{3} - \frac{1}{6} (1 + i \sqrt{3}) \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{1/3} - \\
& \frac{1}{6} (1 - i \sqrt{3}) \left(\frac{1}{2} (173 + 3 \sqrt{3297}) \right)^{1/3} \}, \\
\left\{ \mathbf{y} \rightarrow & -\sqrt{\left(-\frac{5}{9} + \frac{2}{9} \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{1/3} - \frac{2 i \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{1/3}}{3 \sqrt{3}} - \right.} \right. \\
& \frac{1}{18} \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{2/3} - \frac{i \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{2/3}}{6 \sqrt{3}} - \\
& \frac{1}{18} \left(\frac{1}{2} (173 + 3 \sqrt{3297}) \right)^{2/3} + \frac{1}{9} 2^{2/3} (173 + 3 \sqrt{3297})^{1/3} + \\
& \frac{i 2^{2/3} (173 + 3 \sqrt{3297})^{1/3}}{3 \sqrt{3}} + \frac{i (173 + 3 \sqrt{3297})^{2/3}}{6 2^{2/3} \sqrt{3}} + \\
& \left. \frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right) (173 + 3 \sqrt{3297}) \right)^{1/3} \right), \\
\mathbf{x} \rightarrow & -\frac{2}{3} - \frac{1}{6} (1 - i \sqrt{3}) \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{1/3} - \\
& \frac{1}{6} (1 + i \sqrt{3}) \left(\frac{1}{2} (173 + 3 \sqrt{3297}) \right)^{1/3} \}, \\
\left\{ \mathbf{y} \rightarrow & \sqrt{\left(-\frac{5}{9} + \frac{2}{9} \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{1/3} - \frac{2 i \left(\frac{173}{2} - \frac{3 \sqrt{3297}}{2} \right)^{1/3}}{3 \sqrt{3}} - \right.}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{18} \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3} - \frac{i \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3}}{6\sqrt{3}} - \\
& \frac{1}{18} \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{2/3} + \frac{1}{9} 2^{2/3} (173 + 3\sqrt{3297})^{1/3} + \\
& \frac{i 2^{2/3} (173 + 3\sqrt{3297})^{1/3}}{3\sqrt{3}} + \frac{i (173 + 3\sqrt{3297})^{2/3}}{6 \cdot 2^{2/3} \sqrt{3}} + \\
& \left. \frac{1}{9} 2^{2/3} \left(\left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right) (173 + 3\sqrt{3297}) \right)^{1/3} \right), \\
\mathbf{x} \rightarrow & -\frac{2}{3} - \frac{1}{6} (1 - i\sqrt{3}) \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} - \\
& \left. \frac{1}{6} (1 + i\sqrt{3}) \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{1/3} \right\}
\end{aligned}$$

Simplify[%]

$$\begin{aligned}
& \left\{ \left\{ \mathbf{y} \rightarrow -\frac{1}{3} \sqrt{ \left(3 + \left(\frac{1}{2} (173 - 3\sqrt{3297}) \right)^{2/3} - 4 \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} + \right. \right. \right. \\
& \left. \left. \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{2/3} - 2 \cdot 2^{2/3} (173 + 3\sqrt{3297})^{1/3} \right) \right\}, \\
& \mathbf{x} \rightarrow \frac{1}{6} \left(-4 + 2^{2/3} (173 - 3\sqrt{3297})^{1/3} + 2^{2/3} (173 + 3\sqrt{3297})^{1/3} \right) \left. \right\}, \\
& \left\{ \left\{ \mathbf{y} \rightarrow \frac{1}{3} \sqrt{ \left(3 + \left(\frac{1}{2} (173 - 3\sqrt{3297}) \right)^{2/3} - 4 \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{1/3} + \right. \right. \right. \\
& \left. \left. \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{2/3} - 2 \cdot 2^{2/3} (173 + 3\sqrt{3297})^{1/3} \right) \right\}, \\
& \mathbf{x} \rightarrow \frac{1}{6} \left(-4 + 2^{2/3} (173 - 3\sqrt{3297})^{1/3} + 2^{2/3} (173 + 3\sqrt{3297})^{1/3} \right) \left. \right\}, \\
& \left\{ \left\{ \mathbf{y} \rightarrow -\frac{1}{6} \sqrt{ \left(12 + (8 + 8i\sqrt{3}) \left(\frac{1}{2} (173 - 3\sqrt{3297}) \right)^{1/3} + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2i \left(i + \sqrt{3} \right) \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3} + 4 \cdot 2^{2/3} \left(1 - i\sqrt{3} \right) \\
& \left. \left(173 + 3\sqrt{3297} \right)^{1/3} + 2^{1/3} \left(-1 - i\sqrt{3} \right) \left(173 + 3\sqrt{3297} \right)^{2/3} \right), \\
\mathbf{x} \rightarrow & -\frac{2}{3} - \frac{1}{6} \left(1 + i\sqrt{3} \right) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{1/3} + \\
& \frac{1}{6} i \left(i + \sqrt{3} \right) \left(\frac{1}{2} \left(173 + 3\sqrt{3297} \right) \right)^{1/3} \}, \\
\{ \mathbf{y} \rightarrow & \frac{1}{6} \sqrt{\left(12 + (8 + 8i\sqrt{3}) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{1/3} + \right.} \\
& 2i \left(i + \sqrt{3} \right) \left(\frac{173}{2} - \frac{3\sqrt{3297}}{2} \right)^{2/3} + 4 \cdot 2^{2/3} \left(1 - i\sqrt{3} \right) \\
& \left. \left(173 + 3\sqrt{3297} \right)^{1/3} + 2^{1/3} \left(-1 - i\sqrt{3} \right) \left(173 + 3\sqrt{3297} \right)^{2/3} \right), \\
\mathbf{x} \rightarrow & -\frac{2}{3} - \frac{1}{6} \left(1 + i\sqrt{3} \right) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{1/3} + \\
& \frac{1}{6} i \left(i + \sqrt{3} \right) \left(\frac{1}{2} \left(173 + 3\sqrt{3297} \right) \right)^{1/3} \}, \\
\{ \mathbf{y} \rightarrow & -\frac{1}{6} \sqrt{\left(12 + (8 - 8i\sqrt{3}) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{1/3} + \right.} \\
& (-2 - 2i\sqrt{3}) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{2/3} + 4 \cdot 2^{2/3} \left(1 + i\sqrt{3} \right) \\
& \left. \left(173 + 3\sqrt{3297} \right)^{1/3} + i \cdot 2^{1/3} \left(i + \sqrt{3} \right) \left(173 + 3\sqrt{3297} \right)^{2/3} \right), \\
\mathbf{x} \rightarrow & -\frac{2}{3} + \frac{1}{6} i \left(i + \sqrt{3} \right) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{1/3} - \\
& \frac{1}{6} \left(1 + i\sqrt{3} \right) \left(\frac{1}{2} \left(173 + 3\sqrt{3297} \right) \right)^{1/3} \}, \\
\{ \mathbf{y} \rightarrow & \frac{1}{6} \sqrt{\left(12 + (8 - 8i\sqrt{3}) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{1/3} + \right.} \\
& (-2 - 2i\sqrt{3}) \left(\frac{1}{2} \left(173 - 3\sqrt{3297} \right) \right)^{2/3} + 4 \cdot 2^{2/3} \left(1 + i\sqrt{3} \right)
\end{aligned}$$

$$\left((173 + 3\sqrt{3297})^{1/3} + i 2^{1/3} (i + \sqrt{3}) (173 + 3\sqrt{3297})^{2/3} \right),$$

$$x \rightarrow -\frac{2}{3} + \frac{1}{6} i (i + \sqrt{3}) \left(\frac{1}{2} (173 - 3\sqrt{3297}) \right)^{1/3} -$$

$$\frac{1}{6} (1 + i\sqrt{3}) \left(\frac{1}{2} (173 + 3\sqrt{3297}) \right)^{1/3} \left. \right\}$$

N[%]

```
{ {y → -1.02055, x → 1.42882}, {y → 1.02055, x → 1.42882},
  {y → -1.54589 + 1.55259 i, x → -1.71441 + 1.39998 i},
  {y → 1.54589 - 1.55259 i, x → -1.71441 + 1.39998 i},
  {y → -1.54589 - 1.55259 i, x → -1.71441 - 1.39998 i},
  {y → 1.54589 + 1.55259 i, x → -1.71441 - 1.39998 i}}
```

NSolve[{x^2 - y^2 == 1, x^3 + 2 y^2 == 5}, {x, y}]

```
{ {y → 1.54589 - 1.55259 i, x → -1.71441 + 1.39998 i},
  {y → -1.54589 + 1.55259 i, x → -1.71441 + 1.39998 i},
  {y → 1.54589 + 1.55259 i, x → -1.71441 - 1.39998 i},
  {y → -1.54589 - 1.55259 i, x → -1.71441 - 1.39998 i},
  {y → -1.02055, x → 1.42882}, {y → 1.02055, x → 1.42882}}
```

NSolve[{x^2 + y^2 == 1, y == x Exp[x]}, {x, y}]

InverseFunction::ifun :

Inverse functions are being used. Values may be lost for multivalued inverses.

InverseFunction::ifun :

Inverse functions are being used. Values may be lost for multivalued inverses.

InverseFunction::ifun :

Inverse functions are being used. Values may be lost for multivalued inverses.

General::stop :

Further output of InverseFunction::ifun will be suppressed during this calculation.

Solve::tdep :

The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

NSolve[{x^2 + y^2 == 1, y == e^x x}, {x, y}]

```
FindRoot[{x^2 + y^2 == 1, y == x Exp[x]}, {x, .1}, {y, .5}]
{x -> 0.513489, y -> 0.858096}
```

■ Exercise 1 :

1 - Find all the real roots of the following :

a) $x^5 - x^3 + 4 = 0$

b) $3x + \sin(x) - e^x = 0$

c) $y = x^2 - 3$ & $2y^2 = 4$

■ Exercise 2:

The total amount of water infiltrating in a particular soil was obtain as :

$$f(t) = \frac{t + 50 \cdot (1 - \text{Exp}[-2t/25])}{60}$$

where F is in inches & t is in minutes.

- a- Plot F versus time.
- b- Solve for the time at which the total infiltration will be 1 in.