Introduction to Mathematica

I.1 Numerical Computations

Mathematica can be used as a calculator:

- **Addition:**

```
2 + 3
```

```
5
```

- **Subtraction:**

```
2 - 3
```

```
-1
```

- **Multiplication:**

Use a star between multiplied numbers

```
2 * 3
```

```
6
```

You can replace the star by a blank:

```
2 3
```

```
6
```

- **Division:**

```
2 / 3
```

$$\frac{2}{3}$$

```
2. / 3
```

```
0.666667
```

```
N[2 / 3]
```

```
0.666667
```

- **Powers**

```
2^5
```

```
32
```

```
2^-5
```

$$\frac{1}{32}$$

```
5^(1 / 3)
```

$5^{1/3}$

To find the numerical value, use the command "N"

```
N[5^(1 / 3)]
```

1.70998

```
2.5 * 10^12
```

2.5×10^{12}

```
Sqrt[2]
```

$\sqrt{2}$

```
N[Sqrt[2]]
```

1.41421

```
Sin[1 / 2]
```

$\text{Sin}\left[\frac{1}{2}\right]$

- **Sequence of Operations**

```
1 + 2 * 3
```

7

```
2. + 3 / 4
```

2.75

```
(2. + 3) / 4
```

1.25

```
(2. + 3) / 5 + 1
```

2.

```
2 * 3^2 + 1
```

19

```
32^1 / 5
```

$\frac{32}{5}$

```
32^(1 / 5)
```

2

- **Precision of Numbers**

```
Pi
```

π

To get a numerical value, you can use the built-in function N:

```
N[Pi]
```

```
3.14159
```

```
Pi // N
```

```
3.14159
```

```
N[2 / 3]
```

```
0.666667
```

```
2 / 3 // N
```

```
0.666667
```

```
N[Sqrt[2]]
```

```
1.41421
```

```
Sin[Pi / 3]
```

$$\frac{\sqrt{3}}{2}$$

```
N[Sin[Pi / 3]]
```

```
0.866025
```

You can have the precision up to n decimal points:

```
N[Pi, 30]
```

```
3.14159265358979323846264338328
```

```
N[Sqrt[2], 20]
```

```
1.4142135623730950488
```

- **Building Up Calculations**

```
2.5^2
```

```
6.25
```

You can call the last result using %:

```
% + 1
```

```
7.25
```

```
2 * %
```

```
14.5
```

You can call the result from out[n] using %n where n is the number of the output line:

```
%34
```

```
3.14159265358979323846264338328
```

```
3 / 4
```

$$\frac{3}{4}$$

```
N[%]
```

```
0.75
```

- **Defining Variables**

```
x = 5
```

```
5
```

```
y = 12
```

```
12
```

```
x
```

```
5
```

```
x + y
```

```
17
```

```
x / y
```

$$\frac{5}{12}$$

```
N[%]
```

```
0.416667
```

To clear the value assigned, use ".":

```
x =.
```

```
y =.
```

```
x
```

```
x
```

```
y
```

```
y
```

If you do not want to have some of the output to be printed, you can end the input by ' ; '

```
x = 1; y = 2;
```

```
f = (x + y)^2
```

```
9
```

```
g = f^2
```

```
81
```

```
x =.; y =.
```

I.2 Symbolic Computations

```
f = 1 + x^2
```

```
1 + x^2
```

```
g = f^2
```

$$(1 + x^2)^2$$

```
x = 3
```

```
3
```

```
f
```

```
10
```

```
g
```

```
100
```

```
x =.
```

```
f
```

$$1 + x^2$$

```
f /. x → 3
```

```
10
```

```
f
```

$$1 + x^2$$

- **Functions**

Let us clear the assignment given to f earlier:

```
f =.
```

To define a function:

```
f[x_] := 1 + x^2
```

```
f[3]
```

```
10
```

```
f[2]
```

```
5
```

```
f[a]
```

$$1 + a^2$$

```
f[a + 3]
```

$$1 + (3 + a)^2$$

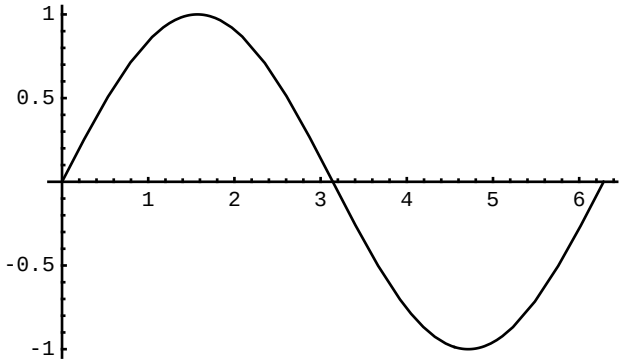
```
f[a + b + c]
```

$$1 + (a + b + c)^2$$

I.3 Graphics

To plot one function along x-axis:

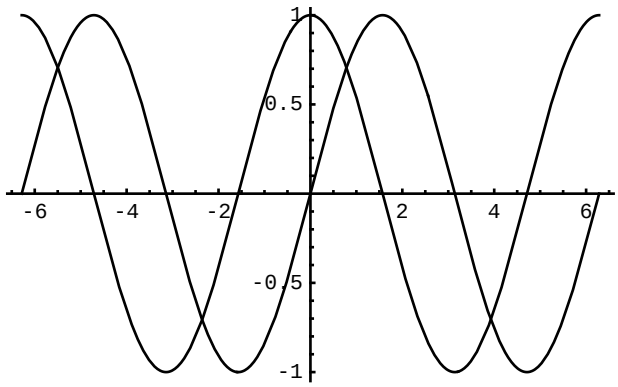
```
Plot[Sin[x], {x, 0, 2 Pi}]
```




```
- Graphics -
```

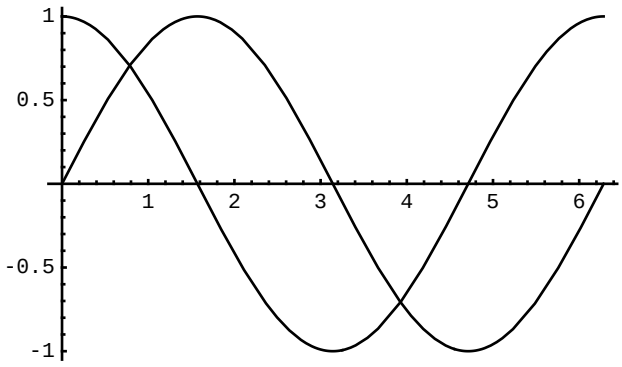
To plot more than one function:

```
Plot[{Sin[x], Sin[ x + Pi / 2]}, {x, -2 Pi, 2 Pi}]
```




```
- Graphics -
```

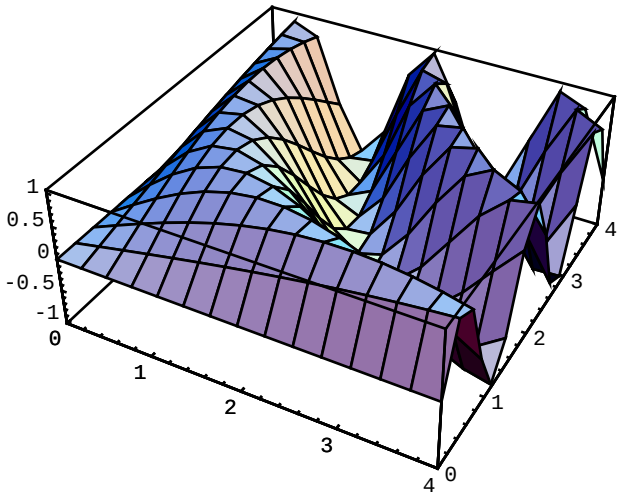
```
Plot[{Sin[x], Cos[x]}, {x, 0, 2 Pi}]
```




```
- Graphics -
```

To do 3-D plot:

```
Plot3D[Sin[x * y], {x, 0, 4}, {y, 0, 4}]
```




```
- SurfaceGraphics -
```

Excercise1

Define the function $g = x^2 - y^2$. Evaluate the function at $x = 2$ and $y = 3$ without destroying the definition. Then, Plot the function for x ranges from -1 to 1 and y from -1 to 1.

1.4 Simplification of Polynomials and Rational Functions

Let us define two polynomials p1, p2

```
p1 = 1 + 2 x + x^2 - x^3
```

$1 + 2\,x + x^2 - x^3$

```
p2 = 2 - x^4
```

$2 - x^4$

We can multiply p1 times p2:

```
p3 = p1 p2
```

$\left(1 + 2\,x + x^2 - x^3\right)\left(2 - x^4\right)$

To expand the result, we use the built-in function 'Expand':

```
Expand[p3]
```

$2 + 4\,x + 2\,x^2 - 2\,x^3 - x^4 - 2\,x^5 - x^6 + x^7$

To have it factorized, we use the built-in function 'Factor:

```
Factor[%]
```

$\left(-1 - 2\,x - x^2 + x^3\right)\left(-2 + x^4\right)$

```
Expand[(1 + x)^6]
```

$$1 + 6\,x + 15\,x^2 + 20\,x^3 + 15\,x^4 + 6\,x^5 + x^6$$

```
Factor[%]
```

$$(1 + x)^6$$

You can perform mathematical operations on polynomials:

```
p4 = p1 / p2 + p3
```

$$\frac{1 + 2\,x + x^2 - x^3}{2 - x^4} + \left(1 + 2\,x + x^2 - x^3\right)\left(2 - x^4\right)$$

You can have a common denomenator:

```
Together[p4]
```

$$\frac{-5 - 10\,x - 5\,x^2 + 5\,x^3 + 4\,x^4 + 8\,x^5 + 4\,x^6 - 4\,x^7 - x^8 - 2\,x^9 - x^{10} + x^{11}}{-2 + x^4}$$

1.5 Vectors & Matrices

```
v1 = {1, -2, 5}; v2 = {3, 0, 1};
v1 + v2
```

```
{4, -2, 6}
```

```
v1 - v2
```

```
{-2, -2, 4}
```

```
v1.v2
```

```
8
```

```
Dot[v1, v2]
```

```
8
```

```
Cross[v1, v2]
```

```
{-2, 14, 6}
```

```
m1 = {{3, 6, 4}, {5, 2, 7}, {2, 1, 5}}
```

```
{{3, 6, 4}, {5, 2, 7}, {2, 1, 5}}
```

```
MatrixForm[m1]
```

$$\begin{pmatrix} 3 & 6 & 4 \\ 5 & 2 & 7 \\ 2 & 1 & 5 \end{pmatrix}$$

```
m1[[1]]
```

```
{3, 6, 4}
```

```
m1[[2]]
```

```
{5, 2, 7}
```

```
m1[[1, 2]]
```

```
6
```

m2 = 2 m1

{{6, 12, 8}, {10, 4, 14}, {4, 2, 10}}

m3 = m1 + m2

{{9, 18, 12}, {15, 6, 21}, {6, 3, 15}}

Dot[m1, m2]

{{94, 68, 148}, {78, 82, 138}, {42, 38, 80}}

m1.m2

{{94, 68, 148}, {78, 82, 138}, {42, 38, 80}}

Transpose[m1]

{{3, 5, 2}, {6, 2, 1}, {4, 7, 5}}

Inverse[m1]

$$\left\{\left\{-\frac{3}{53}, \frac{26}{53}, -\frac{34}{53}\right\}, \left\{\frac{11}{53}, -\frac{7}{53}, \frac{1}{53}\right\}, \left\{-\frac{1}{53}, -\frac{9}{53}, \frac{24}{53}\right\}\right\}$$

MatrixForm[%]

$$\begin{pmatrix} -\frac{3}{53} & \frac{26}{53} & -\frac{34}{53} \\ \frac{11}{53} & -\frac{7}{53} & \frac{1}{53} \\ -\frac{1}{53} & -\frac{9}{53} & \frac{24}{53} \end{pmatrix}$$

%.m1

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

MatrixForm[%]

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Det[m1]

−53

m1.v1

{11, 36, 25}

Excercise2

Define two 3 x3 matrices (a and b). Determine their dot product c = a.b. Determine the inverse of c and show that the $c.c^{-1}$ = the identity matrix.

1.6 Differentiation

```
D[x^5, x]
```

$$5\,x^4$$

```
D[x^5, {x, 2}]
```

$$20\,x^3$$

```
D[x^5, {x, 6}]
```

$$0$$

```
D[x^n, {x, 3}]
```

$$(-2+n)\,(-1+n)\,n\,x^{-3+n}$$

```
D[(Tan[x] + x)/(Sin[x] + x^2) + Log[x^2 + Cos[x]])^3, x]
```

$$3\left(\frac{2x-\text{Sin}[x]}{x^2+\text{Cos}[x]}+\frac{1+\text{Sec}[x]^2}{x^2+\text{Sin}[x]}-\frac{(2x+\text{Cos}[x])\,(x+\text{Tan}[x])}{(x^2+\text{Sin}[x])^2}\right)\left(\text{Log}[x^2+\text{Cos}[x]]+\frac{x+\text{Tan}[x]}{x^2+\text{Sin}[x]}\right)^2$$

1.7 Integration

```
Integrate[x^2, x]
```

$$\frac{x^3}{3}$$

```
Integrate[x^2, {x, a, b}]
```

$$-\frac{a^3}{3}+\frac{b^3}{3}$$

```
Integrate[x^2, {x, 0, 1}]
```

$$\frac{1}{3}$$

```
Integrate[Exp[x]/x, {x, 1, 2}]
```

$$-\text{Gamma}[0,-2]+\text{Gamma}[0,-1]$$

```
NIntegrate[Exp[x]/x, {x, 1, 2}]
```

$$3.05912$$

```
∫x^2 𝕕x
```

$$\frac{x^3}{3}$$

$$\int_0^1 x\text{^}2 \, dx$$

$$\frac{1}{3}$$

$$\int_0^1 \int_0^1 x\, y \, dx \, dy$$

$$\frac{1}{4}$$

Excercise3

1 – Compute $\int_0^1 x^x \, dx$ (Answer = 0.783431)

2 – Compute $\int_0^1 \int_0^1 \left(x^2 + y^2\right) dx \, dx$ (Answer = 2 / 3)