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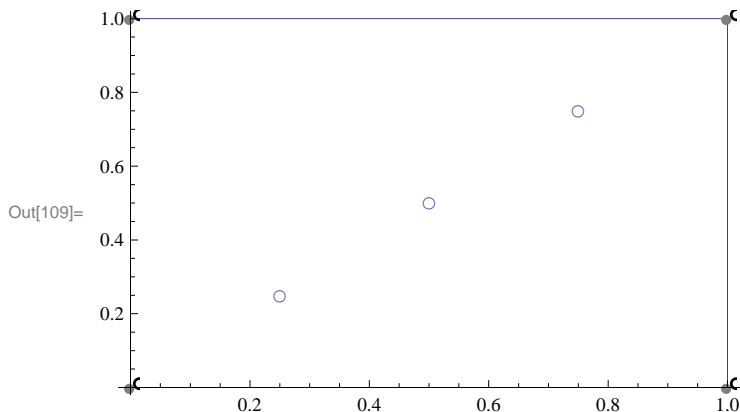
In[98]:= a0 = 1.; aa = 1.; bb = 0.5;
xe = {1, 1, 0, 0}; ye = {0, 1, 1, 0};
xd = {0.25, 0.5, 0.75}; yd = {0.25, 0.5, 0.75};
nb = Length[xe]; nd = Length[xd];
tbc = {2, 1, 2, 1}; vbc = {0, 1, 0, 0};
ja = If[j == nb, 1, j + 1];
jb = If[j == 1, nb, j - 1];
dat1 = Table[{xe[[i]], ye[[i]]}, {i, 1, nb}];
dat2 = Table[{xd[[i]], yd[[i]]}, {i, 1, nd}];
p1 = ListPlot[dat1, PlotStyle -> PointSize[0.02], PlotMarkers -> "o", Joined -> True];
p2 = ListPlot[dat2, PlotStyle -> PointSize[0.02], PlotMarkers -> "o"];
Show[p1, p2]
ub = Array[u, {nb}]; qb = Array[q, {2 * nb}];
jb = If[j == 1, nb, j - 1];
ja = If[j == nb, 1, j + 1];
Do[If[tbc[[j]] == 2 && tbc[[ja]] == 2, qb[[2 j - 1]] = vbc[[j]]; qb[[2 j]] = vbc[[ja]];
  If[tbc[[j]] == 2 && tbc[[ja]] == 1, qb[[2 j - 1]] = vbc[[j]]; ub[[j]] = vbc[[ja]];
  If[tbc[[j]] == 1 && tbc[[ja]] == 2, ub[[j]] = vbc[[j]]; qb[[2 j]] = vbc[[ja]];
  If[tbc[[j]] == 1 && tbc[[ja]] == 1, ub[[j]] = vbc[[j]]; qb[[2 * j - 1]] = qb[[2 * j]], {j, 1, nb}];
boundarydat = Table[{j, tbc[[j]], vbc[[j]]}, {j, 1, nb}];
TableForm[boundarydat, TableHeadings -> {None, {"Element No. ", "BC-Type", "BC-Value"}}];
r = Sqrt[(x - xi)^2 + (y - yi)^2];
us =  $\frac{\text{Log}[1/r]}{2 \pi a0}$ ;
qs = a0 * (D[us, x] * n1 + D[us, y] * n2);
usxi = D[us, xi];
usyi = D[us, yi];
qsxi = D[qs, xi];
qsyi = D[qs, yi];
<< NumericalDifferentialEquationAnalysis >>
np = 10; p = w = Table[Null, {np}];
Do[p[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 1]], {i, 1, np}];
Do[w[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 2]], {i, 1, np}];
Int[f_, z_] := Sum[(f /. z -> p[[i]]) w[[i]], {i, 1, np}];
phi1 = (1 - z) / 2; phi2 = (1 + z) / 2;
Hb = Table[0, {i, 1, nb}, {j, 1, nb}];
Gb = Table[0, {i, 1, nb}, {j, 1, 2 * nb}];
xb = yb = Table[0, {i, 1, nb}];
Do[xb[[j]] = xe[[jb]] * phi1 + xe[[j]] * phi2;
  yb[[j]] = ye[[jb]] * phi1 + ye[[j]] * phi2;
  L[j] =  $\sqrt{(xe[[j]] - xe[[jb]])^2 + (ye[[j]] - ye[[jb]])^2}$ ; ds[j] = L[j] / 2;
  nx[j] = (ye[[j]] - ye[[jb]]) / L[j];
  ny[j] = (xe[[jb]] - xe[[j]]) / L[j], {j, 1, nb}];
Do[Gb[[i, 2 j - 1]] =
  Int[(us * phi2 * ds[j]) /. {x -> xb[[j]], y -> yb[[j]], xi -> xe[[i]], yi -> ye[[i]]}, z];
Gb[[i, 2 j]] = Int[(us * phi1 * ds[ja]) /. {x -> xb[[ja]], y -> yb[[ja]], xi -> xe[[i]], yi -> ye[[i]]},
  z]; Hb[[i, j]] = Int[qs * ds[j] * phi2 /.
  {x -> xb[[j]], y -> yb[[j]], n1 -> nx[j], n2 -> ny[j], xi -> xe[[i]], yi -> ye[[i]]}, z] +
  Int[qs * ds[ja] * phi1 /. {x -> xb[[ja]], y -> yb[[ja]], n1 -> nx[ja], n2 -> ny[ja],
  xi -> xe[[i]], yi -> ye[[i]]}, z], {i, 1, nb}, {j, 1, nb}];

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Do[Gb[[j, 2*j - 1]] =  $\frac{L[j] \left(3 + 2 \operatorname{Log}\left[\frac{1}{L[j]}\right]\right)}{8 \pi a_0}$ ; Gb[[j, 2*j]] =  $\frac{L[ja] \left(3 + 2 \operatorname{Log}\left[\frac{1}{L[ja]}\right]\right)}{8 \pi a_0}$ , {j, 1, nb}]
Do[Hb[[j, j]] = -Sum[Hb[[j, k]], {k, 1, nb}] + Hb[[j, j]], {j, 1, nb}]
sol = Solve[Hb.ub == Gb.qb];
ub = ub /. sol[[1]]; qb = qb /. sol[[1]];
BoundarySol = Table[{i, xe[[i]], ye[[i]], ub[[i]], qb[[2 i - 1]], qb[[2 i]]}, {i, 1, nb}];
TableForm[BoundarySol, TableHeadings ->
  {None, {"Element No. ", "XE", "YE", "Potential(u)", "Flux-x(q1)", "Flux-y(q2)"}}]
Hd = Hd1 = Hd2 = Table[0, {i, 1, nd}, {j, 1, nb}];
Gd = Gd1 = Gd2 = Table[0, {i, 1, nd}, {j, 1, 2*nb}];
Do[Gd[[i, 2*j - 1]] =
  Int[(us*phi2*ds[j]) /. {x -> xb[[j]], y -> yb[[j]], xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd[[i, 2*j]] = Int[(us*phi1*ds[ja]) /.
  {x -> xb[[ja]], y -> yb[[ja]], xi -> xd[[i]], yi -> yd[[i]]}, z];
Hd[[i, j]] = Int[(qs*phi2*ds[j]) /. {x -> xb[[j]], y -> yb[[j]], n1 -> nx[j],
  n2 -> ny[j], xi -> xd[[i]], yi -> yd[[i]]}, z] + Int[(qs*phi1*ds[ja]) /.
  {x -> xb[[ja]], y -> yb[[ja]], n1 -> nx[ja], n2 -> ny[ja], xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd1[[i, 2*j - 1]] = Int[(usxi*phi2*ds[j]) /. {x -> xb[[j]],
  y -> yb[[j]], xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd1[[i, 2*j]] = Int[(usxi*phi1*ds[ja]) /. {x -> xb[[ja]],
  y -> yb[[ja]], xi -> xd[[i]], yi -> yd[[i]]}, z];
Hd1[[i, j]] = Int[(qsxi*phi2*ds[j]) /. {x -> xb[[j]], y -> yb[[j]], n1 -> nx[j],
  n2 -> ny[j], xi -> xd[[i]], yi -> yd[[i]]}, z] + Int[(qsxi*phi1*ds[ja]) /.
  {x -> xb[[ja]], y -> yb[[ja]], n1 -> nx[ja], n2 -> ny[ja], xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd2[[i, 2*j - 1]] = Int[(usyi*phi2*ds[j]) /. {x -> xb[[j]],
  y -> yb[[j]], xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd2[[i, 2*j]] = Int[(usyi*phi1*ds[ja]) /. {x -> xb[[ja]],
  y -> yb[[ja]], xi -> xd[[i]], yi -> yd[[i]]}, z];
Hd2[[i, j]] = Int[(qsyi*phi2*ds[j]) /. {x -> xb[[j]], y -> yb[[j]],
  n1 -> nx[j], n2 -> ny[j], xi -> xd[[i]], yi -> yd[[i]]}, z] +
  Int[(qsyi*phi1*ds[ja]) /. {x -> xb[[ja]], y -> yb[[ja]], n1 -> nx[ja], n2 -> ny[ja],
  xi -> xd[[i]], yi -> yd[[i]]}, z], {i, 1, nd}, {j, 1, nb}]
ud = (-Hd.ub + Gd.qb); q1 = (-Hd1.ub + Gd1.qb); q2 = (-Hd2.ub + Gd2.qb);
DomainSol = Table[{i, xd[[i]], yd[[i]], ud[[i]], q1[[i]], q2[[i]]}, {i, 1, nd}];
TableForm[DomainSol, TableHeadings ->
  {None, {"Node No. ", "XD", "YD", "Potential(u)", "Flux-x(q1)", "Flux-y (q2)"}}]

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Out[139]/TableForm=

Element No.	XE	YE	Potential(u)	Flux-x(q1)	Flux-y(q2)
1	1	0	1	0	0.999982
2	1	1	1	0.999982	0
3	0	1	0	0	-0.999982
4	0	0	0	-0.999982	0

Out[145]/TableForm=

Node No.	XD	YD	Potential(u)	Flux-x(q1)	Flux-y (q2)
1	0.25	0.25	0.250006	0.999928	-0.000177273
2	0.5	0.5	0.5	0.999991	-1.38778×10^{-17}
3	0.75	0.75	0.750023	1.00029	0.000187494