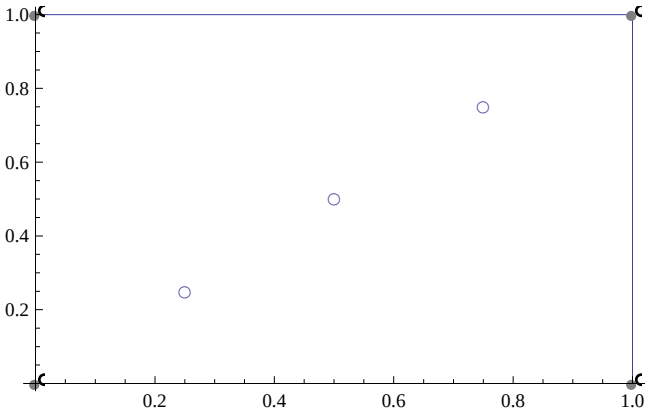
In[98]:=
```
a0 = 1.; aa = 1.; bb = 0.5;
xe = {1, 1, 0, 0}; ye = {0, 1, 1, 0};
xd = {0.25, 0.5, 0.75}; yd = {0.25, 0.5, 0.75};
nb = Length[xe]; nd = Length[xd];
tbc = {2, 1, 2, 1}; vbc = {0, 1, 0, 0};
ja = If[j == nb, 1, j + 1];
jb = If[j == 1, nb, j - 1];
dat1 = Table[{xe[[i]], ye[[i]]}, {i, 1, nb}];
dat2 = Table[{xd[[i]], yd[[i]]}, {i, 1, nd}];
p1 = ListPlot[dat1, PlotStyle → PointSize[0.02], PlotMarkers → "●", Joined → True];
p2 = ListPlot[dat2, PlotStyle → PointSize[0.02], PlotMarkers → "○"];
Show[p1, p2]
ub = Array[u, {nb}]; qb = Array[q, {2 * nb}];
jb = If[j == 1, nb, j - 1];
ja = If[j == nb, 1, j + 1];
Do[If[tbc[[j]] == 2 && tbc[[ja]] == 2, qb[[2 j - 1]] = vbc[[j]]; qb[[2 j]] = vbc[[ja]]];
 If[tbc[[j]] == 2 && tbc[[ja]] == 1, qb[[2 j - 1]] = vbc[[j]]; ub[[j]] = vbc[[ja]]];
 If[tbc[[j]] == 1 && tbc[[ja]] == 2, ub[[j]] = vbc[[j]]; qb[[2 j]] = vbc[[ja]]];
 If[tbc[[j]] == 1 && tbc[[ja]] == 1, ub[[j]] = vbc[[j]]; qb[[2 * j - 1]] = qb[[2 * j]]], {j, 1, nb}]
boundarydat = Table[{j, tbc[[j]], vbc[[j]]}, {j, 1, nb}];
TableForm[boundarydat, TableHeadings -> {None, {"Element No. ", "BC-Type", "BC-Value"}}];
r = Sqrt[(x - xi)^2 + (y - yi)^2];
us = Log[1 / r] / (2 π a0);
qs = a0 * (D[us, x] * n1 + D[us, y] * n2);
usxi = D[us, xi];
usyi = D[us, yi];
qsxi = D[qs, xi];
qsyi = D[qs, yi];
<< NumericalDifferentialEquationAnalysis`
np = 10; p = w = Table[Null, {np}];
Do[p[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 1]], {i, 1, np}]
Do[w[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 2]], {i, 1, np}]
Int[f_, z_] := Sum[(f /. z → p[[i]]) w[[i]], {i, 1, np}]
phi1 = (1 - z) / 2; phi2 = (1 + z) / 2;
Hb = Table[0, {i, 1, nb}, {j, 1, nb}];
Gb = Table[0, {i, 1, nb}, {j, 1, 2 * nb}];
xb = yb = Table[0, {i, 1, nb}];
Do[xb[[j]] = xe[[jb]] * phi1 + xe[[j]] * phi2;
  yb[[j]] = ye[[jb]] * phi1 + ye[[j]] * phi2;
  L[j] = Sqrt[(xe[[j]] - xe[[jb]])^2 + (ye[[j]] - ye[[jb]])^2]; ds[j] = L[j] / 2;
  nx[j] = (ye[[j]] - ye[[jb]]) / L[j];
  ny[j] = (xe[[jb]] - xe[[j]]) / L[j], {j, 1, nb}];
Do[Gb[[i, 2 j - 1]] =
   Int[(us * phi2 * ds[j]) /. {x → xb[[j]], y → yb[[j]], xi → xe[[i]], yi → ye[[i]]}, z];
  Gb[[i, 2 j]] = Int[(us * phi1 * ds[ja]) /. {x → xb[[ja]], y → yb[[ja]], xi → xe[[i]], yi → ye[[i]]},
    z]; Hb[[i, j]] = Int[qs * ds[j] * phi2 /.
      {x → xb[[j]], y → yb[[j]], n1 → nx[j], n2 → ny[j], xi → xe[[i]], yi → ye[[i]]}, z] +
    Int[qs * ds[ja] * phi1 /. {x → xb[[ja]], y → yb[[ja]], n1 → nx[ja], n2 → ny[ja],
       xi → xe[[i]], yi → ye[[i]]}, z], {i, 1, nb}, {j, 1, nb}];
```

$$\text{Do}\left[\text{Gb}[[j,\,2*j-1]] = \frac{\text{L}[j]\left(3+2\,\text{Log}\left[\frac{1}{\text{L}[j]}\right]\right)}{8*\pi*\text{a0}};\ \text{Gb}[[j,\,2*j]] = \frac{\text{L}[ja]\left(3+2\,\text{Log}\left[\frac{1}{\text{L}[ja]}\right]\right)}{8*\pi*\text{a0}},\ \{j,\,1,\,nb\}\right]$$

```
Do[Hb[[j, j]] = -Sum[Hb[[j, k]], {k, 1, nb}] + Hb[[j, j]], {j, 1, nb}]
sol = Solve[Hb.ub == Gb.qb];
ub = ub /. sol[[1]]; qb = qb /. sol[[1]];
BoundarySol = Table[{i, xe[[i]], ye[[i]], ub[[i]], qb[[2 i - 1]], qb[[2 i]]}, {i, 1, nb}];
TableForm[BoundarySol, TableHeadings ->
  {None, {"Element No. ", "XE", "YE", "Potential(u)", "Flux-x(q1)", "Flux-y(q2)"}}]
Hd = Hd1 = Hd2 = Table[0, {i, 1, nd}, {j, 1, nb}];
Gd = Gd1 = Gd2 = Table[0, {i, 1, nd}, {j, 1, 2 * nb}];
Do[Gd[[i, 2 * j - 1]] =
  Int[(us * phi2 * ds[j]) /. {x → xb[[j]], y → yb[[j]], xi → xd[[i]], yi → yd[[i]]}, z];
 Gd[[i, 2 * j]] = Int[(us * phi1 * ds[ja]) /.
    {x → xb[[ja]], y → yb[[ja]], xi → xd[[i]], yi → yd[[i]]}, z];
 Hd[[i, j]] = Int[(qs * phi2 * ds[j]) /. {x → xb[[j]], y → yb[[j]], n1 → nx[j],
      n2 → ny[j], xi → xd[[i]], yi → yd[[i]]}, z] + Int[(qs * phi1 * ds[ja]) /.
     {x → xb[[ja]], y → yb[[ja]], n1 → nx[ja], n2 → ny[ja], xi → xd[[i]], yi → yd[[i]]}, z];
 Gd1[[i, 2 * j - 1]] = Int[(usxi * phi2 * ds[j]) /. {x → xb[[j]],
     y → yb[[j]], xi → xd[[i]], yi → yd[[i]]}, z];
 Gd1[[i, 2 * j]] = Int[(usxi * phi1 * ds[ja]) /. {x → xb[[ja]],
     y → yb[[ja]], xi → xd[[i]], yi → yd[[i]]}, z];
 Hd1[[i, j]] = Int[(qsxi * phi2 * ds[j]) /. {x → xb[[j]], y → yb[[j]], n1 → nx[j],
      n2 → ny[j], xi → xd[[i]], yi → yd[[i]]}, z] + Int[(qsxi * phi1 * ds[ja]) /.
     {x → xb[[ja]], y → yb[[ja]], n1 → nx[ja], n2 → ny[ja], xi → xd[[i]], yi → yd[[i]]}, z];
 Gd2[[i, 2 * j - 1]] = Int[(usyi * phi2 * ds[j]) /. {x → xb[[j]],
     y → yb[[j]], xi → xd[[i]], yi → yd[[i]]}, z];
 Gd2[[i, 2 * j]] = Int[(usyi * phi1 * ds[ja]) /. {x → xb[[ja]],
     y → yb[[ja]], xi → xd[[i]], yi → yd[[i]]}, z];
 Hd2[[i, j]] = Int[(qsyi * phi2 * ds[j]) /. {x → xb[[j]], y → yb[[j]],
      n1 → nx[j], n2 → ny[j], xi → xd[[i]], yi → yd[[i]]}, z] +
   Int[(qsyi * phi1 * ds[ja]) /. {x → xb[[ja]], y → yb[[ja]], n1 → nx[ja], n2 → ny[ja],
      xi → xd[[i]], yi → yd[[i]]}, z], {i, 1, nd}, {j, 1, nb}]
ud = (-Hd.ub + Gd.qb); q1 = (-Hd1.ub + Gd1.qb); q2 = (-Hd2.ub + Gd2.qb);
DomainSol = Table[{i, xd[[i]], yd[[i]], ud[[i]], q1[[i]], q2[[i]]}, {i, 1, nd}];
TableForm[DomainSol, TableHeadings ->
  {None, {"Node No. ", "XD", "YD", "Potential(u)", "Flux-x(q1)", "Flux-y (q2)"}}]
```

Out[109]=



Out[139]//TableForm=

Element No.	XE	YE	Potential(u)	Flux-x(q1)	Flux-y(q2)
1	1	0	1	0	0.999982
2	1	1	1	0.999982	0
3	0	1	0	0	-0.999982
4	0	0	0	-0.999982	0

Out[145]//TableForm=

Node No.	XD	YD	Potential(u)	Flux-x(q1)	Flux-y (q2)
1	0.25	0.25	0.250006	0.999928	-0.000177273
2	0.5	0.5	0.5	0.999991	-1.38778×10^{-17}
3	0.75	0.75	0.750023	1.00029	0.000187494