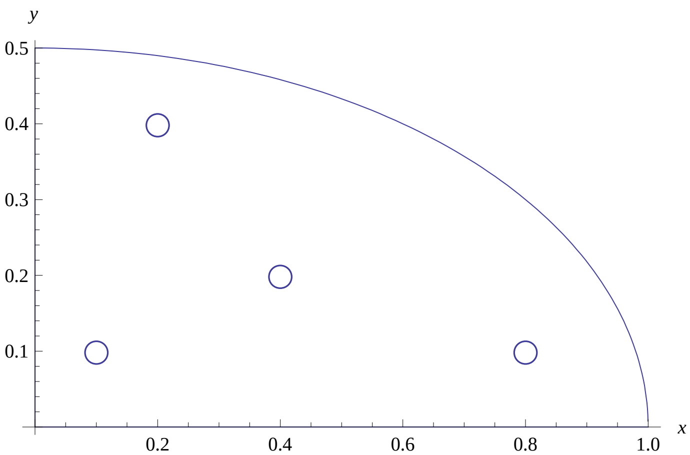
Example 1: Torsion of a solid eliptical section using "LACONBE" (Try re-running the code after changing ye[[nb1+1]] to 0.44

GDE : $\nabla^2 u = 0$

BC : $u(0, y) = u(x, 0) = 0$

$$q\left(x, \frac{1}{2}\sqrt{1-x^2}\right) = y * nx - x * ny$$

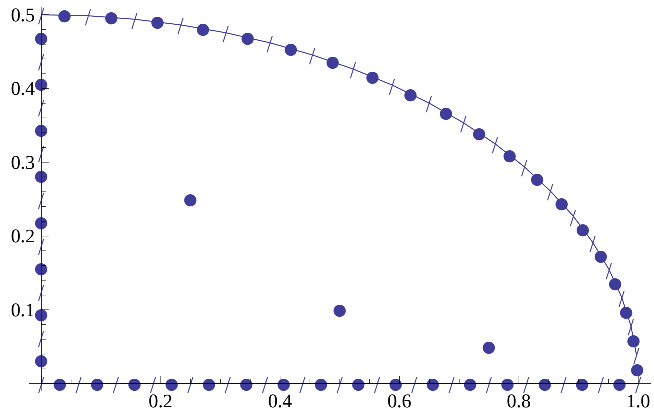


In[350]:=

```
a0 = 1.; aa = 1.; bb = 0.5;
nb1 = 20; nb2 = 8; nb3 = 16; nb = nb1 + nb2 + nb3; nd = 3;
xe = ye = Table[0, {i, 1, nb}];
Do[xe[[i]] = aa * Cos[i * Pi / nb1 / 2], {i, 1, nb1}];
Do[ye[[i]] = bb * Sin[i * Pi / nb1 / 2], {i, 1, nb1}];
Do[xe[[i + nb1]] = 0, {i, 1, nb2}];
Do[ye[[i + nb1]] = bb - bb / nb2 * i, {i, 1, nb2}];
Do[xe[[i + nb1 + nb2]] = aa / nb3 * i, {i, 1, nb3}];
Do[ye[[i + nb1 + nb2]] = 0, {i, 1, nb3}];
xd = {0.25, 0.5, 0.75}; yd = {0.25, 0.1, 0.05};
xm = ym = Table[0, {i, 1, nb}];
jb = If[j == 1, nb, j - 1];
Do[xm[[j]] = (xe[[j]] + xe[[jb]]) / 2; ym[[j]] = (ye[[j]] + ye[[jb]]) / 2, {j, 1, nb}];
tbc = vbc = Table[0, {i, 1, nb}];
Do[tbc[[i]] = 2, {i, 1, nb1}];
Do[tbc[[i + nb1]] = 1, {i, 1, nb2}];
Do[tbc[[i + nb1 + nb2]] = 1, {i, 1, nb3}];
Do[L = Sqrt[(xe[[j]] - xe[[jb]])^2 + (ye[[j]] - ye[[jb]])^2];
  nx = (ye[[j]] - ye[[jb]]) / L;
  ny = (xe[[jb]] - xe[[j]]) / L;
  vbc[[j]] = ym[[j]] * nx - xm[[j]] * ny, {j, 1, nb1}];
aa = 1; bb = 0.5; Do[vbc[[i]] = ((bb^2 - aa^2) * xm[[i]] * ym[[i]]) / Sqrt[bb^4 xm[[i]]^2 + aa^4 ym[[i]]^2], {i, 1, nb1}];
Do[vbc[[i]] = 0., {i, nb1 + 1, nb}];
dat1 = Table[{xe[[i]], ye[[i]]}, {i, 1, nb}];
```

```
dat2 = Table[{xm[[i]], ym[[i]]}, {i, 1, nb}];
dat3 = Table[{xd[[i]], yd[[i]]}, {i, 1, nd}];
p1 = ListPlot[dat1, PlotStyle → PointSize[0.02], PlotMarkers → "/", Joined → True];
p2 = ListPlot[dat2, PlotStyle → PointSize[0.02], PlotMarkers → "●"];
p3 = ListPlot[dat3, PlotStyle → PointSize[0.02], PlotMarkers → "●"];
Show[p1, p2, p3]
um = Array[u, {nb}]; qm = Array[q, {nb}];
Do[If[tbc[[i]] == 1, um[[i]] = vbc[[i]], qm[[i]] = vbc[[i]]], {i, 1, nb}]
boundarydat = Table[{i, xe[[i]], ye[[i]], xm[[i]], ym[[i]], tbc[[i]], vbc[[i]]}, {i, 1, nb}];
TableForm[boundarydat,
 TableHeadings -> {None, {"Element No. ", "XE", "YE", "XM", "YM", "BC-Type", "BC-Value"}}]
Hb = Gb = Table[0, {i, 1, nb}, {j, 1, nb}];
Hd = Gd = Hd1 = Hd2 = Gd1 = Gd2 = Table[0, {i, 1, nd}, {j, 1, nb}];
r = Sqrt[(x - xi)^2 + (y - yi)^2];
us = Log[1 / r] / (2 π a0);
qs = a0 * (D[us, x] * n1 + D[us, y] * n2);
usxi = D[us, xi];
usyi = D[us, yi];
qsxi = D[qs, xi];
qsyi = D[qs, yi];
<< NumericalDifferentialEquationAnalysis`
np = 10; p = w = Table[Null, {np}];
Do[p[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 1]], {i, 1, np}]
Do[w[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 2]], {i, 1, np}]
Int[f_, z_] := Sum[(f /. z → p[[i]]) w[[i]], {i, 1, np}]
Do[xb = 1 / 2 * (xe[[jb]] * (1 - z) + xe[[j]] * (1 + z));
 yb = 1 / 2 * (ye[[jb]] * (1 - z) + ye[[j]] * (1 + z));
 L = Sqrt[(xe[[j]] - xe[[jb]])^2 + (ye[[j]] - ye[[jb]])^2];
 ds = L / 2;
 nx = (ye[[j]] - ye[[jb]]) / L;
 ny = (xe[[jb]] - xe[[j]]) / L;
 Do[Gb[[i, j]] = Int[us * ds /. {x → xb, y → yb, xi → xm[[i]], yi → ym[[i]]}, z]; Hb[[i, j]] =
   Int[qs * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xm[[i]], yi → ym[[i]]}, z], {i, 1, nb}];
 Do[Gd[[i, j]] = Int[us * ds /. {x → xb, y → yb, xi → xd[[i]], yi → yd[[i]]}, z];
  Hd[[i, j]] = Int[qs * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xd[[i]], yi → yd[[i]]}, z];
  Gd1[[i, j]] = Int[usxi * ds /. {x → xb, y → yb, xi → xd[[i]], yi → yd[[i]]}, z];
  Hd1[[i, j]] = Int[qsxi * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xd[[i]], yi → yd[[i]]}, z];
  Gd2[[i, j]] = Int[usyi * ds /. {x → xb, y → yb, xi → xd[[i]], yi → yd[[i]]}, z];
  Hd2[[i, j]] = Int[qsyi * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xd[[i]], yi → yd[[i]]}, z];,
  {i, 1, nd}], {j, 1, nb}]; Do[Gb[[i, i]] = L / (2 * π * a0) * (Log[2 / L] + 1.), {i, 1, nb}]
Do[Hb[[i, i]] = -Sum[Hb[[i, k]], {k, 1, nb}] + Hb[[i, i]], {i, 1, nb}]
sol = Solve[Hb.um == Gb.qm]; um = um /. sol[[1]]; qm = qm /. sol[[1]];
BoundarySol = Table[{i, xm[[i]], ym[[i]], um[[i]] /. sol[[1]], qm[[i]] /. sol[[1]]}, {i, 1, nb}];
TableForm[BoundarySol,
 TableHeadings -> {None, {"Node No. ", "XM", "YM", "Potential (u)", "Flux (qn)"}}]
ud = (-Hd.um + Gd.qm); q1 = (-Hd1.um + Gd1.qm); q2 = (-Hd2.um + Gd2.qm);
DomainSol = Table[{i, xd[[i]], yd[[i]], ud[[i]], q1[[i]], q2[[i]]}, {i, 1, nd}];
TableForm[DomainSol, TableHeadings ->
  {None, {"Node No. ", "XD", "YD", "Potential(u)", "Flux-x(q1)", "Flux-y (q2)"}}]
```

Out[376]=



Out[380]//TableForm=

Element No.	XE	YE	XM	YM	BC−Type	BC−Value
1	0.996917	0.0392295	0.998459	0.0196148	2	−0.0586635
2	0.987688	0.0782172	0.992303	0.0587234	2	−0.171433
3	0.97237	0.116723	0.980029	0.09747	2	−0.271699
4	0.951057	0.154508	0.961713	0.135616	2	−0.354361
5	0.92388	0.191342	0.937468	0.172925	2	−0.417444
6	0.891007	0.226995	0.907443	0.209168	2	−0.461338
7	0.85264	0.261249	0.871823	0.244122	2	−0.487755
8	0.809017	0.293893	0.830829	0.277571	2	−0.498903
9	0.760406	0.324724	0.784711	0.309308	2	−0.496999
10	0.707107	0.353553	0.733756	0.339139	2	−0.484046
11	0.649448	0.380203	0.678277	0.366878	2	−0.461771
12	0.587785	0.404508	0.618617	0.392356	2	−0.431641
13	0.522499	0.42632	0.555142	0.415414	2	−0.394901
14	0.45399	0.445503	0.488245	0.435912	2	−0.35262
15	0.382683	0.46194	0.418337	0.453722	2	−0.305736
16	0.309017	0.475528	0.34585	0.468734	2	−0.255084
17	0.233445	0.486185	0.271231	0.480857	2	−0.201431
18	0.156434	0.493844	0.19494	0.490015	2	−0.145487
19	0.0784591	0.498459	0.117447	0.496151	2	−0.0879312
20	0	0.5	0.0392295	0.499229	2	−0.0294165
21	0	0.4375	0	0.46875	1	0.
22	0	0.375	0	0.40625	1	0.
23	0	0.3125	0	0.34375	1	0.
24	0	0.25	0	0.28125	1	0.
25	0	0.1875	0	0.21875	1	0.
26	0	0.125	0	0.15625	1	0.
27	0	0.0625	0	0.09375	1	0.
28	0	0.	0	0.03125	1	0.
29	0.0625	0	0.03125	0.	1	0.
30	0.125	0	0.09375	0	1	0.
31	0.1875	0	0.15625	0	1	0.
32	0.25	0	0.21875	0	1	0.
33	0.3125	0	0.28125	0	1	0.
34	0.375	0	0.34375	0	1	0.
35	0.4375	0	0.40625	0	1	0.
36	0.5	0	0.46875	0	1	0.
37	0.5625	0	0.53125	0	1	0.
38	0.625	0	0.59375	0	1	0.
39	0.6875	0	0.65625	0	1	0.
40	0.75	0	0.71875	0	1	0.
41	0.8125	0	0.78125	0	1	0.
42	0.875	0	0.84375	0	1	0.
43	0.9375	0	0.90625	0	1	0.
44	1.	0	0.96875	0	1	0.

Out[399]//TableForm=

Node No.	XM	YM	Potential (u)	Flux (qn)
1	0.998459	0.0196148	−0.0122479	−0.0586635
2	0.992303	0.0587234	−0.0382934	−0.171433
3	0.980029	0.09747	−0.0628423	−0.271699
4	0.961713	0.135616	−0.085017	−0.354361
5	0.937468	0.172925	−0.104283	−0.417444
6	0.907443	0.209168	−0.120324	−0.461338
7	0.871823	0.244122	−0.132962	−0.487755
8	0.830829	0.277571	−0.1421	−0.498903
9	0.784711	0.309308	−0.147695	−0.496999
10	0.733756	0.339139	−0.149746	−0.484046
11	0.678277	0.366878	−0.148298	−0.461771
12	0.618617	0.392356	−0.143443	−0.431641
13	0.555142	0.415414	−0.135328	−0.394901
14	0.488245	0.435912	−0.124156	−0.35262
15	0.418337	0.453722	−0.11019	−0.305736
16	0.34585	0.468734	−0.0937444	−0.255084
17	0.271231	0.480857	−0.0751852	−0.201431
18	0.19494	0.490015	−0.0549161	−0.145487
19	0.117447	0.496151	−0.0333573	−0.0879312
20	0.0392295	0.499229	−0.0108942	−0.0294165
21	0	0.46875	0.	0.314662
22	0	0.40625	0.	0.233691
23	0	0.34375	0.	0.202064
24	0	0.28125	0.	0.165705
25	0	0.21875	0.	0.129184
26	0	0.15625	0.	0.0924662
27	0	0.09375	0.	0.0557975
28	0	0.03125	0.	0.016393
29	0.03125	0.	0.	0.0162819
30	0.09375	0	0.	0.0556653
31	0.15625	0	0.	0.0924319
32	0.21875	0	0.	0.129428
33	0.28125	0	0.	0.166523
34	0.34375	0	0.	0.203766
35	0.40625	0	0.	0.241197
36	0.46875	0	0.	0.27887
37	0.53125	0	0.	0.316849
38	0.59375	0	0.	0.355218
39	0.65625	0	0.	0.394075
40	0.71875	0	0.	0.433533
41	0.78125	0	0.	0.473686
42	0.84375	0	0.	0.514557
43	0.90625	0	0.	0.553725
44	0.96875	0	0.	0.601827

Out[402]//TableForm=

Node No.	XD	YD	Potential(u)	Flux−x(q1)	Flux−y (q2)
1	0.25	0.25	−0.0368816	−0.148183	−0.146884
2	0.5	0.1	−0.0297785	−0.0608267	−0.297481
3	0.75	0.05	−0.0227092	−0.0323348	−0.453998