Example: Infinite flow around a circular cylinder using "LACONBE"

GDE : $\nabla^2 u = 0$

BC : $q = 0$ on Γ

$u = -x$ on Γ_∞

Exact solution :

$$u = -x/R \left(R + a^2/r\right)$$

Velocity, $vx = -\frac{\partial u}{\partial x} = 1 - \frac{a^2 \left(x^2 - y^2\right)}{R^4}$ for $a = 5$

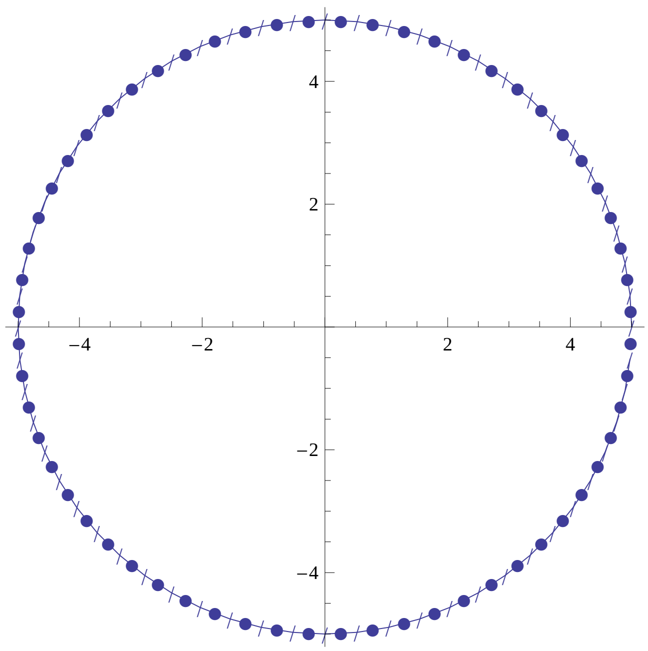
(vx) max $= 2$ at $(0, \mp a)$

$X0 = 1$

```
R = Sqrt[x^2 + y^2]; ue = -x / R * (R + rad^2/R); uex = FullSimplify[D[ue, x]]
```

$$-1 + \frac{rad^2 \left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}$$

```
a0 = 1.; rad = 5.;
nb = 60; nd = 1;
xe = ye = Table[0, {i, 1, nb}];
Do[xe[[i]] = rad * Cos[-i * 2 * Pi / nb], {i, 1, nb}];
Do[ye[[i]] = rad * Sin[-i * 2 * Pi / nb], {i, 1, nb}];
xd = {x}; yd = {y};
xm = ym = Table[0, {i, 1, nb}];
jb = If[j == 1, nb, j - 1];
Do[xm[[j]] = (xe[[j]] + xe[[jb]]) / 2; ym[[j]] = (ye[[j]] + ye[[jb]]) / 2, {j, 1, nb}];
tbc = vbc = Table[0, {i, 1, nb}];
Do[tbc[[i]] = 2, {i, 1, nb}];
Do[vbc[[i]] = -Cos[-i * 2. * Pi / nb], {i, 1, nb}];
dat1 = Table[{xe[[i]], ye[[i]]}, {i, 1, nb}];
dat2 = Table[{xm[[i]], ym[[i]]}, {i, 1, nb}];
dat3 = Table[{xd[[i]], yd[[i]]}, {i, 1, nd}];
p1 = ListPlot[dat1, PlotStyle → PointSize[0.02], PlotMarkers → "/", Joined → True];
p2 = ListPlot[dat2, PlotStyle → PointSize[0.02], PlotMarkers → "●"];
Show[p1, p2, AspectRatio → 1]
```



```
um = Array[u, {nb}]; qm = Array[q, {nb}];
Do[If[tbc[[i]] == 1, um[[i]] = vbc[[i]], qm[[i]] = vbc[[i]]], {i, 1, nb}]
boundarydat = Table[{i, xe[[i]], ye[[i]], xm[[i]], ym[[i]], tbc[[i]], vbc[[i]]}, {i, 1, nb}];
TableForm[boundarydat,
 TableHeadings -> {None, {"Element No. ", "XE", "YE", "XM", "YM", "BC-Type", "BC-Value"}}]
Hb = Gb = Table[0, {i, 1, nb}, {j, 1, nb}];
Hd = Gd = Hd1 = Hd2 = Gd1 = Gd2 = Table[0, {i, 1, nd}, {j, 1, nb}];
r = Sqrt[(x - xi)^2 + (y - yi)^2];
us = Log[1/r] / (2 π a0);
qs = a0 * (D[us, x] * n1 + D[us, y] * n2);
usxi = D[us, xi];
usyi = D[us, yi];
qsxi = D[qs, xi];
qsyi = D[qs, yi];
<< NumericalDifferentialEquationAnalysis`;
np = 20; p = w = Table[Null, {np}];
Do[p[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 1]], {i, 1, np}]
Do[w[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 2]], {i, 1, np}]
Int[f_, z_] := Sum[(f /. z → p[[i]]) w[[i]], {i, 1, np}]
Do[xb = 1/2 * (xe[[jb]] * (1 - z) + xe[[j]] * (1 + z));
 yb = 1/2 * (ye[[jb]] * (1 - z) + ye[[j]] * (1 + z));
 L = Sqrt[(xe[[j]] - xe[[jb]])^2 + (ye[[j]] - ye[[jb]])^2];
 ds = L/2;
 nx = (ye[[j]] - ye[[jb]]) / L;
 ny = (xe[[jb]] - xe[[j]]) / L;
 Do[Gb[[i, j]] = Int[us * ds /. {x → xb, y → yb, xi → xm[[i]], yi → ym[[i]]}, z]; Hb[[i, j]] =
   Int[qs * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xm[[i]], yi → ym[[i]]}, z], {i, 1, nb}];
 Do[Gd[[i, j]] = Int[us * ds /. {x → xb, y → yb, xi → xd[[i]], yi → yd[[i]]}, z];
  Hd[[i, j]] = Int[qs * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xd[[i]], yi → yd[[i]]}, z];
  Gd1[[i, j]] = Int[usxi * ds /. {x → xb, y → yb, xi → xd[[i]], yi → yd[[i]]}, z];
  Hd1[[i, j]] = Int[qsxi * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xd[[i]], yi → yd[[i]]}, z];
  Gd2[[i, j]] = Int[usyi * ds /. {x → xb, y → yb, xi → xd[[i]], yi → yd[[i]]}, z];
  Hd2[[i, j]] = Int[qsyi * ds /. {x → xb, y → yb, n1 → nx, n2 → ny, xi → xd[[i]], yi → yd[[i]]}, z];,
  {i, 1, nd}], {j, 1, nb}]; Do[Gb[[i, i]] = L / (2 * π * a0) * (Log[2 / L] + 1.), {i, 1, nb}]
Do[Hb[[i, i]] = 0.5, {i, 1, nb}]
sol = Solve[Hb.um == Gb.qm]; um = um /. sol[[1]]; qm = qm /. sol[[1]];
BoundarySol = Table[{i, xm[[i]], ym[[i]], um[[i]], qm[[i]]}, {i, 1, nb}];
TableForm[BoundarySol,
 TableHeadings -> {None, {"Node No. ", "XM", "YM", "Potential (u)", "Flux (qn)"}}]
ud = (-Hd.um + Gd.qm);
```

Element No.	XE	YE	XM	YM	BC-Type	BC-Value
1	4.97261	-0.522642	4.9863	-0.261321	2	-0.994522
2	4.89074	-1.03956	4.93167	-0.7811	2	-0.978148
3	4.75528	-1.54508	4.82301	-1.29232	2	-0.951057
4	4.56773	-2.03368	4.6615	-1.78938	2	-0.913545
5	4.33013	-2.5	4.44893	-2.26684	2	-0.866025
6	4.04508	-2.93893	4.18761	-2.71946	2	-0.809017

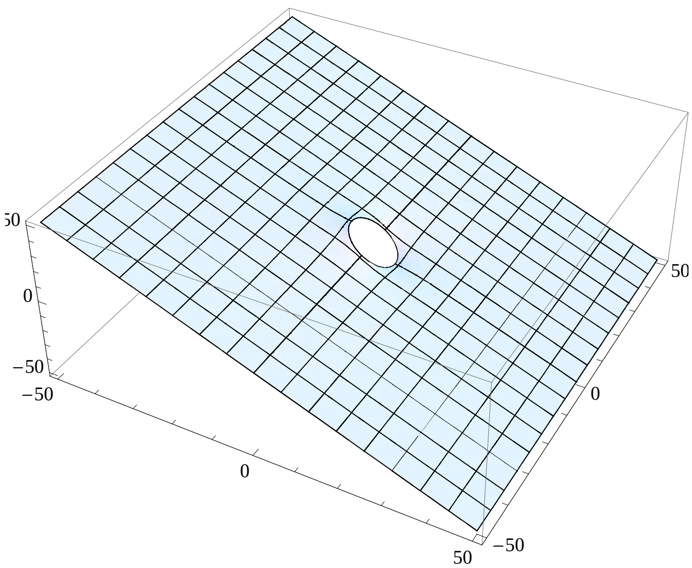
7	3.71572	−3.34565	3.8804	−3.14229	2	−0.743145
8	3.34565	−3.71572	3.53069	−3.53069	2	−0.669131
9	2.93893	−4.04508	3.14229	−3.8804	2	−0.587785
10	2.5	−4.33013	2.71946	−4.18761	2	−0.5
11	2.03368	−4.56773	2.26684	−4.44893	2	−0.406737
12	1.54508	−4.75528	1.78938	−4.6615	2	−0.309017
13	1.03956	−4.89074	1.29232	−4.82301	2	−0.207912
14	0.522642	−4.97261	0.7811	−4.93167	2	−0.104528
15	0	−5.	0.261321	−4.9863	2	-6.12323×10^{-17}
16	−0.522642	−4.97261	−0.261321	−4.9863	2	0.104528
17	−1.03956	−4.89074	−0.7811	−4.93167	2	0.207912
18	−1.54508	−4.75528	−1.29232	−4.82301	2	0.309017
19	−2.03368	−4.56773	−1.78938	−4.6615	2	0.406737
20	−2.5	−4.33013	−2.26684	−4.44893	2	0.5
21	−2.93893	−4.04508	−2.71946	−4.18761	2	0.587785
22	−3.34565	−3.71572	−3.14229	−3.8804	2	0.669131
23	−3.71572	−3.34565	−3.53069	−3.53069	2	0.743145
24	−4.04508	−2.93893	−3.8804	−3.14229	2	0.809017
25	−4.33013	−2.5	−4.18761	−2.71946	2	0.866025
26	−4.56773	−2.03368	−4.44893	−2.26684	2	0.913545
27	−4.75528	−1.54508	−4.6615	−1.78938	2	0.951057
28	−4.89074	−1.03956	−4.82301	−1.29232	2	0.978148
29	−4.97261	−0.522642	−4.93167	−0.7811	2	0.994522
30	−5.	0	−4.9863	−0.261321	2	1.
31	−4.97261	0.522642	−4.9863	0.261321	2	0.994522
32	−4.89074	1.03956	−4.93167	0.7811	2	0.978148
33	−4.75528	1.54508	−4.82301	1.29232	2	0.951057
34	−4.56773	2.03368	−4.6615	1.78938	2	0.913545
35	−4.33013	2.5	−4.44893	2.26684	2	0.866025
36	−4.04508	2.93893	−4.18761	2.71946	2	0.809017
37	−3.71572	3.34565	−3.8804	3.14229	2	0.743145
38	−3.34565	3.71572	−3.53069	3.53069	2	0.669131
39	−2.93893	4.04508	−3.14229	3.8804	2	0.587785
40	−2.5	4.33013	−2.71946	4.18761	2	0.5
41	−2.03368	4.56773	−2.26684	4.44893	2	0.406737
42	−1.54508	4.75528	−1.78938	4.6615	2	0.309017
43	−1.03956	4.89074	−1.29232	4.82301	2	0.207912
44	−0.522642	4.97261	−0.7811	4.93167	2	0.104528
45	0	5.	−0.261321	4.9863	2	1.83697×10^{-16}
46	0.522642	4.97261	0.261321	4.9863	2	−0.104528
47	1.03956	4.89074	0.7811	4.93167	2	−0.207912
48	1.54508	4.75528	1.29232	4.82301	2	−0.309017
49	2.03368	4.56773	1.78938	4.6615	2	−0.406737
50	2.5	4.33013	2.26684	4.44893	2	−0.5
51	2.93893	4.04508	2.71946	4.18761	2	−0.587785
52	3.34565	3.71572	3.14229	3.8804	2	−0.669131
53	3.71572	3.34565	3.53069	3.53069	2	−0.743145
54	4.04508	2.93893	3.8804	3.14229	2	−0.809017
55	4.33013	2.5	4.18761	2.71946	2	−0.866025
56	4.56773	2.03368	4.44893	2.26684	2	−0.913545
57	4.75528	1.54508	4.6615	1.78938	2	−0.951057

58	4.89074	1.03956	4.82301	1.29232	2	−0.978148
59	4.97261	0.522642	4.93167	0.7811	2	−0.994522
60	5.	0	4.9863	0.261321	2	−1.

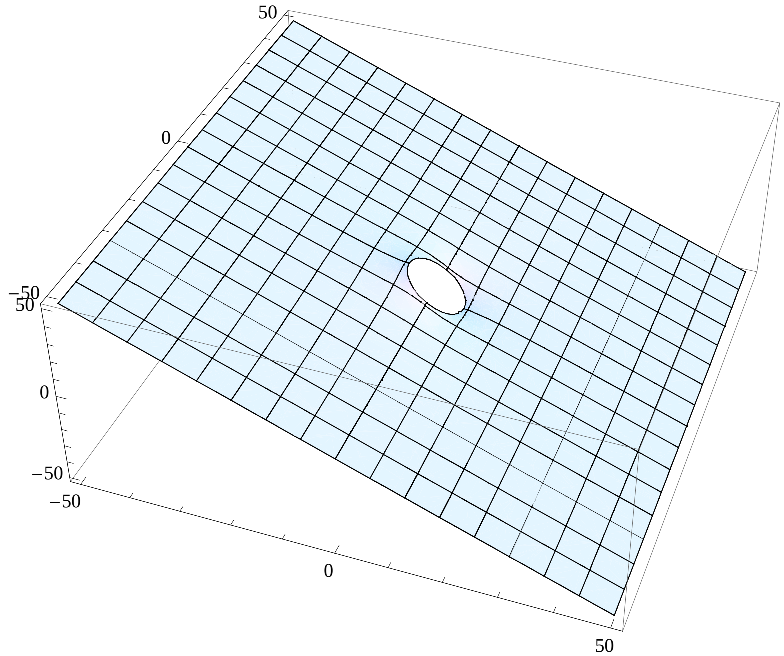
Node No.	XM	YM	Potential (u)	Flux (qn)
1	4.9863	−0.261321	−4.97485	−0.994522
2	4.93167	−0.7811	−4.89294	−0.978148
3	4.82301	−1.29232	−4.75743	−0.951057
4	4.6615	−1.78938	−4.56979	−0.913545
5	4.44893	−2.26684	−4.33208	−0.866025
6	4.18761	−2.71946	−4.04691	−0.809017
7	3.8804	−3.14229	−3.7174	−0.743145
8	3.53069	−3.53069	−3.34716	−0.669131
9	3.14229	−3.8804	−2.94025	−0.587785
10	2.71946	−4.18761	−2.50113	−0.5
11	2.26684	−4.44893	−2.0346	−0.406737
12	1.78938	−4.6615	−1.54578	−0.309017
13	1.29232	−4.82301	−1.04003	−0.207912
14	0.7811	−4.93167	−0.522878	−0.104528
15	0.261321	−4.9863	-1.1618×10^{-15}	-6.12323×10^{-17}
16	−0.261321	−4.9863	0.522878	0.104528
17	−0.7811	−4.93167	1.04003	0.207912
18	−1.29232	−4.82301	1.54578	0.309017
19	−1.78938	−4.6615	2.0346	0.406737
20	−2.26684	−4.44893	2.50113	0.5
21	−2.71946	−4.18761	2.94025	0.587785
22	−3.14229	−3.8804	3.34716	0.669131
23	−3.53069	−3.53069	3.7174	0.743145
24	−3.8804	−3.14229	4.04691	0.809017
25	−4.18761	−2.71946	4.33208	0.866025
26	−4.44893	−2.26684	4.56979	0.913545
27	−4.6615	−1.78938	4.75743	0.951057
28	−4.82301	−1.29232	4.89294	0.978148
29	−4.93167	−0.7811	4.97485	0.994522
30	−4.9863	−0.261321	5.00226	1.
31	−4.9863	0.261321	4.97485	0.994522
32	−4.93167	0.7811	4.89294	0.978148
33	−4.82301	1.29232	4.75743	0.951057
34	−4.6615	1.78938	4.56979	0.913545
35	−4.44893	2.26684	4.33208	0.866025
36	−4.18761	2.71946	4.04691	0.809017
37	−3.8804	3.14229	3.7174	0.743145
38	−3.53069	3.53069	3.34716	0.669131
39	−3.14229	3.8804	2.94025	0.587785
40	−2.71946	4.18761	2.50113	0.5
41	−2.26684	4.44893	2.0346	0.406737
42	−1.78938	4.6615	1.54578	0.309017
43	−1.29232	4.82301	1.04003	0.207912
44	−0.7811	4.93167	0.522878	0.104528
45	−0.261321	4.9863	5.83449×10^{-16}	1.83697×10^{-16}
46	0.261321	4.9863	−0.522878	−0.104528
47	0.7811	4.93167	−1.04003	−0.207912
48	1.29232	4.82301	−1.54578	−0.309017

49	1.78938	4.6615	−2.0346	−0.406737
50	2.26684	4.44893	−2.50113	−0.5
51	2.71946	4.18761	−2.94025	−0.587785
52	3.14229	3.8804	−3.34716	−0.669131
53	3.53069	3.53069	−3.7174	−0.743145
54	3.8804	3.14229	−4.04691	−0.809017
55	4.18761	2.71946	−4.33208	−0.866025
56	4.44893	2.26684	−4.56979	−0.913545
57	4.6615	1.78938	−4.75743	−0.951057
58	4.82301	1.29232	−4.89294	−0.978148
59	4.93167	0.7811	−4.97485	−0.994522
60	4.9863	0.261321	−5.00226	−1.

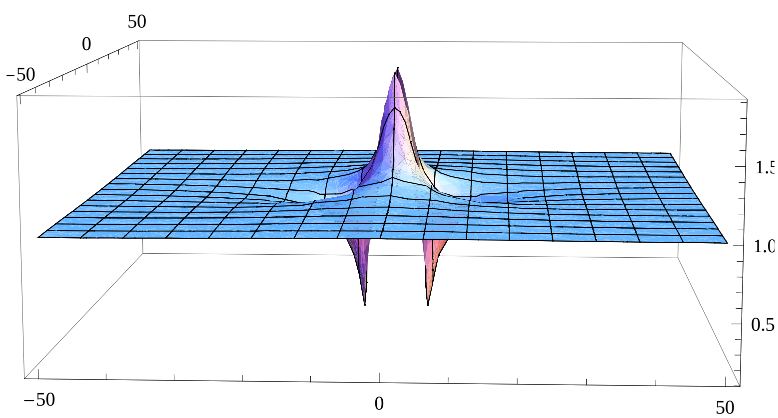
```
un = ud - x;
Plot3D[ue, {x, -50, 50}, {y, -50, 50}, RegionFunction → Function[{x, y}, x^2 + y^2 > 30]]
```



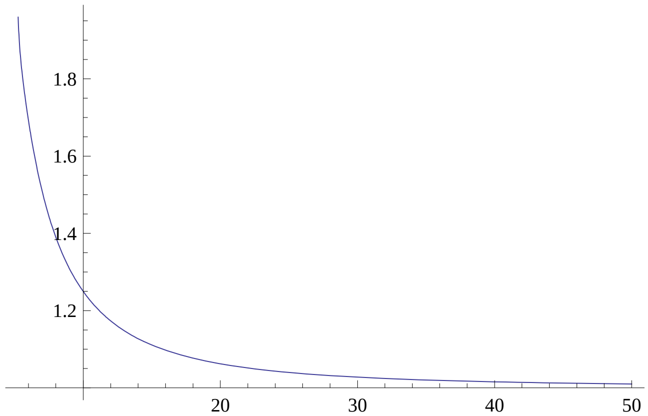
```
Plot3D[un, {x, -50, 50}, {y, -50, 50}, RegionFunction → Function[{x, y}, x^2 + y^2 > 30]]
```



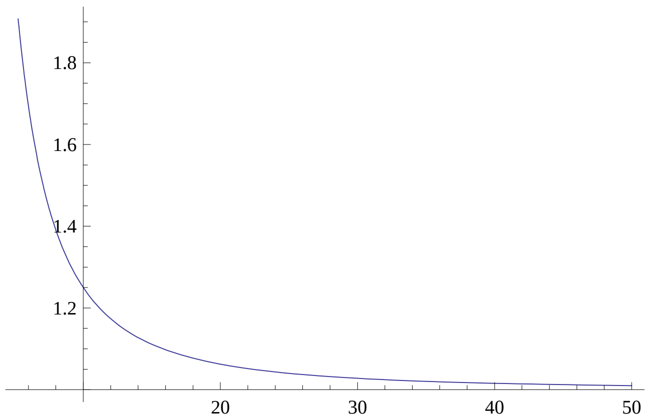

```
Vx = -D[un, x];
Plot3D[Vx, {x, -50, 50}, {y, -50, 50},
 RegionFunction → Function[{x, y}, x^2 + y^2 > 25], PlotRange → All]
```



```
p1 = Plot[Vx /. x → 0, {y, 5.25, 50}, PlotRange → All]
```




```
p2 = Plot[-uex /. x → 0, {y, 5.25, 50}, PlotRange → All]
```




```
Show[p1, p2]
```