

```

nb = 40; nd = 1; a0 = 1.;
nb1 = nb / 4; dd = 1. / nb1;
xe = ye = Table[0, {i, 1, nb}];
Do[xe[[i]] = dd i, {i, 1, nb / 4}];
Do[xe[[i + nb1]] = 1, {i, 1, nb / 4}];
Do[xe[[i + 2 nb1]] = 1 - dd i, {i, 1, nb / 4}];
Do[xe[[i + 3 nb1]] = 0, {i, 1, nb / 4}];
Do[ye[[i]] = 0, {i, 1, nb / 4}];
Do[ye[[i + nb1]] = dd i, {i, 1, nb / 4}];
Do[ye[[i + 2 nb1]] = 1, {i, 1, nb / 4}];
Do[ye[[i + 3 nb1]] = 1 - dd i, {i, 1, nb / 4}];
xd = {x}; yd = {y};
tbc = vbc = Table[0, {i, 1, nb}];
Do[tbc[[i]] = 2, {i, 1, nb / 4}];
Do[tbc[[i + nb1]] = 1, {i, 1, nb / 4}];
Do[tbc[[i + 2 nb1]] = 2, {i, 1, nb / 4}];
Do[tbc[[i + 3 nb1]] = 1, {i, 1, nb / 4}];
Do[vbc[[i]] = 0, {i, 1, nb}];
Do[vbc[[i + nb1]] = 1., {i, 1, nb1}];
xm = ym = Table[0, {i, 1, nb}];
jb = If[j == 1, nb, j - 1];
Do[xm[[j]] = (xe[[j]] + xe[[jb]]) / 2;
  ym[[j]] = (ye[[j]] + ye[[jb]]) / 2, {j, 1, nb}];
dat1 = Table[{xe[[i]], ye[[i]]}, {i, 1, nb}];
dat2 = Table[{xm[[i]], ym[[i]]}, {i, 1, nb}];
dat3 = Table[{xd[[i]], yd[[i]]}, {i, 1, nd}];
p1 = ListPlot[dat1, PlotStyle → PointSize[0.02],
  PlotMarkers → "/", Joined → True];
p2 = ListPlot[dat2, PlotStyle → PointSize[0.02], PlotMarkers → "●"];
p3 = ListPlot[dat3, PlotStyle → PointSize[0.02], PlotMarkers → "/"];
Show[p1, p2, p3]
um = Array[u, {nb}]; qm = Array[q, {nb}];
Do[If[tbc[[i]] == 1, um[[i]] = vbc[[i]], qm[[i]] = vbc[[i]]], {i, 1, nb}];
boundarydat = Table[{i, xe[[i]], ye[[i]],
  xm[[i]], ym[[i]], tbc[[i]], vbc[[i]]}, {i, 1, nb}];
TableForm[boundarydat, TableHeadings -> {None,
  {"Element No. ", "XE", "YE", "XM", "YM", "BC-Type", "BC-Value"}}]
Hb = Gb = Table[0, {i, 1, nb}, {j, 1, nb}];
Hd = Gd = Hd1 = Hd2 = Gd1 = Gd2 = Table[0, {i, 1, nd}, {j, 1, nb}];
r = Sqrt[(x - xi) ^ 2 + (y - yi) ^ 2];

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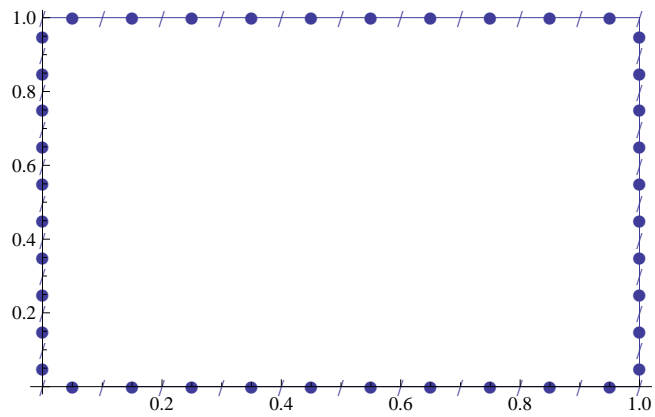
us =  $\frac{\text{Log}[1/r]}{2 \pi a_0}$ ;
qs = a0 * (D[us, x] * n1 + D[us, y] * n2);
usxi = D[us, xi];
usyi = D[us, yi];
qsxi = D[qs, xi];
qsyi = D[qs, yi];
<< NumericalDifferentialEquationAnalysis`;
np = 10; p = w = Table[Null, {np}];
Do[p[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 1]], {i, 1, np}]
Do[w[[i]] = GaussianQuadratureWeights[np, -1, 1][[i, 2]], {i, 1, np}]
Int[f_, z_] := Sum[(f /. z -> p[[i]]) w[[i]], {i, 1, np}]
Do[xb = 1/2 * (xe[[jb]] * (1 - z) + xe[[j]] * (1 + z));
yb = 1/2 * (ye[[jb]] * (1 - z) + ye[[j]] * (1 + z));
L =  $\sqrt{(\text{xe}[[j]] - \text{xe}[[jb]])^2 + (\text{ye}[[j]] - \text{ye}[[jb]])^2}$ ;
ds = L/2;
nx = (ye[[j]] - ye[[jb]])/L;
ny = (xe[[jb]] - xe[[j]])/L;
Do[Gb[[i, j]] =
  Int[us * ds /. {x -> xb, y -> yb, xi -> xm[[i]], yi -> ym[[i]]}, z];
Hb[[i, j]] = Int[qs * ds /. {x -> xb, y -> yb, n1 -> nx, n2 -> ny,
  xi -> xm[[i]], yi -> ym[[i]]}, z], {i, 1, nb}];
Do[Gd[[i, j]] = Int[us * ds /. {x -> xb, y -> yb, xi -> xd[[i]], yi -> yd[[i]]},
  z];
Hd[[i, j]] = Int[qs * ds /. {x -> xb, y -> yb, n1 -> nx,
  n2 -> ny, xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd1[[i, j]] = Int[usxi * ds /. {x -> xb, y -> yb, xi -> xd[[i]], yi -> yd[[i]]},
  z];
Hd1[[i, j]] = Int[qsxi * ds /. {x -> xb, y -> yb, n1 -> nx,
  n2 -> ny, xi -> xd[[i]], yi -> yd[[i]]}, z];
Gd2[[i, j]] = Int[usyi * ds /. {x -> xb, y -> yb, xi -> xd[[i]], yi -> yd[[i]]},
  z];
Hd2[[i, j]] = Int[qsyi * ds /. {x -> xb, y -> yb, n1 -> nx, n2 -> ny,
  xi -> xd[[i]], yi -> yd[[i]]}, z];, {i, 1, nd}], {j, 1, nb}];
Do[Gb[[i, i]] = L / (2 *  $\pi$  * a0) * (Log[2/L] + 1.), {i, 1, nb}]
Do[Hb[[i, i]] = -Sum[Hb[[i, k]], {k, 1, nb}] + Hb[[i, i]], {i, 1, nb}]
sol = Solve[Hb.um == Gb.qm]; um = um /. sol[[1]]; qm = qm /. sol[[1]];
BoundarySol = Table[{i, xm[[i]], ym[[i]],
  um[[i]] /. sol[[1]], qm[[i]] /. sol[[1]]}, {i, 1, nb}];

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TableForm[BoundarySol, TableHeadings ->
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{None, {"Node No. ", "XM", "YM", "Potential (u)", "Flux (qn)"}}]
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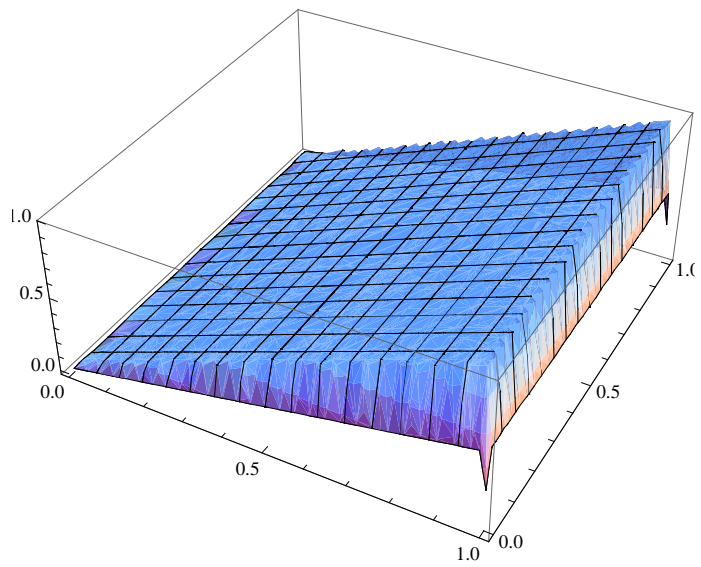
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ud = (-Hd.um + Gd.qm); q1 = (-Hd1.um + Gd1.qm); q2 = (-Hd2.um + Gd2.qm);
```



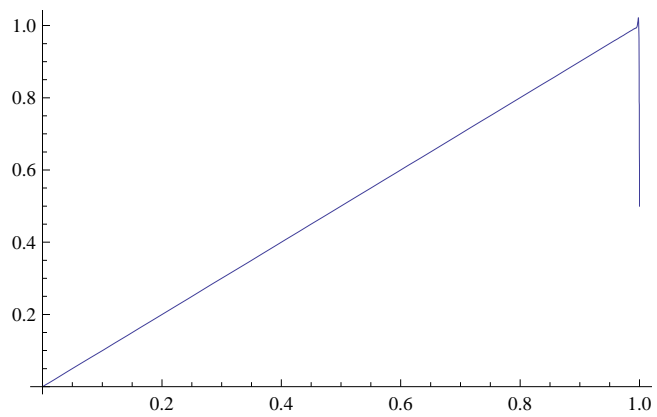
Element No.	XE	YE	XM	YM	BC-Type	BC-Value
1	0.1	0	0.05	0.	2	0
2	0.2	0	0.15	0	2	0
3	0.3	0	0.25	0	2	0
4	0.4	0	0.35	0	2	0
5	0.5	0	0.45	0	2	0
6	0.6	0	0.55	0	2	0
7	0.7	0	0.65	0	2	0
8	0.8	0	0.75	0	2	0
9	0.9	0	0.85	0	2	0
10	1.	0	0.95	0	2	0
11	1	0.1	1.	0.05	1	1.
12	1	0.2	1	0.15	1	1.
13	1	0.3	1	0.25	1	1.
14	1	0.4	1	0.35	1	1.
15	1	0.5	1	0.45	1	1.
16	1	0.6	1	0.55	1	1.
17	1	0.7	1	0.65	1	1.
18	1	0.8	1	0.75	1	1.
19	1	0.9	1	0.85	1	1.
20	1	1.	1	0.95	1	1.
21	0.9	1	0.95	1.	2	0
22	0.8	1	0.85	1	2	0
23	0.7	1	0.75	1	2	0
24	0.6	1	0.65	1	2	0
25	0.5	1	0.55	1	2	0
26	0.4	1	0.45	1	2	0
27	0.3	1	0.35	1	2	0
28	0.2	1	0.25	1	2	0
29	0.1	1	0.15	1	2	0
30	0.	1	0.05	1	2	0
31	0	0.9	0.	0.95	1	0
32	0	0.8	0	0.85	1	0
33	0	0.7	0	0.75	1	0
34	0	0.6	0	0.65	1	0
35	0	0.5	0	0.55	1	0
36	0	0.4	0	0.45	1	0
37	0	0.3	0	0.35	1	0
38	0	0.2	0	0.25	1	0
39	0	0.1	0	0.15	1	0
40	0	0.	0	0.05	1	0

Node No.	XM	YM	Potential (u)	Flux (qn)
1	0.05	0.	0.0471861	0
2	0.15	0	0.148363	0
3	0.25	0	0.248999	0
4	0.35	0	0.349447	0
5	0.45	0	0.449822	0
6	0.55	0	0.550178	0
7	0.65	0	0.650553	0
8	0.75	0	0.751001	0
9	0.85	0	0.851637	0
10	0.95	0	0.952814	0
11	1.	0.05	1.	1.04959
12	1	0.15	1.	0.982446
13	1	0.25	1.	0.995463
14	1	0.35	1.	0.997094
15	1	0.45	1.	0.997813
16	1	0.55	1.	0.997813
17	1	0.65	1.	0.997094
18	1	0.75	1.	0.995463
19	1	0.85	1.	0.982446
20	1	0.95	1.	1.04959
21	0.95	1.	0.952814	0
22	0.85	1	0.851637	0
23	0.75	1	0.751001	0
24	0.65	1	0.650553	0
25	0.55	1	0.550178	0
26	0.45	1	0.449822	0
27	0.35	1	0.349447	0
28	0.25	1	0.248999	0
29	0.15	1	0.148363	0
30	0.05	1	0.0471861	0
31	0.	0.95	0	-1.04959
32	0	0.85	0	-0.982446
33	0	0.75	0	-0.995463
34	0	0.65	0	-0.997094
35	0	0.55	0	-0.997813
36	0	0.45	0	-0.997813
37	0	0.35	0	-0.997094
38	0	0.25	0	-0.995463
39	0	0.15	0	-0.982446
40	0	0.05	0	-1.04959

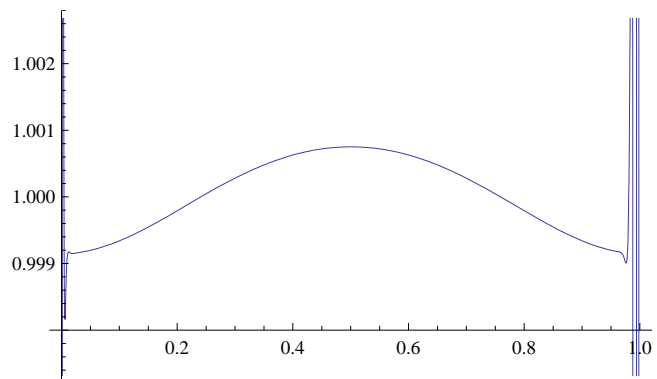
```
Plot3D[ud, {x, 0, 1}, {y, 0, 1}]
```



```
Plot[ud /. y -> 0.5, {x, 0, 1}]
```



```
Plot[q1 /. y -> 0.5, {x, 0, 1}]
```



```
Plot[q1 /. y -> 0.5, {x, 0, 1}, PlotRange -> {0, 1.2}]
```

