Introduction to Mathematica

I.1 Numerical Computations

Mathematica can be used as a calculator:

- ### Addition:

```
2 + 3
```

```
5
```

- ### Subtraction:

```
2 - 3
```

```
-1
```

- ### Multiplication:

Use a star between multiplied numbers

```
2 * 3
```

```
6
```

You can replace the star by a blank:

```
2 × 3
```

```
6
```

- ### Division:

```
2 / 3
```

$\frac{2}{3}$

```
2. / 3
```

```
0.666667
```

```
N[2 / 3]
```

```
0.666667
```

- ### Powers

```
2^5
```

```
32
```

```
2^-5
```

$\frac{1}{32}$

```
5^(1/3)
```

$5^{1/3}$

To find the numerical value, use the command "N"

```
N[5^(1/3)]
```

1.70998

```
Sqrt[2]
```

$\sqrt{2}$

```
N[Sqrt[2]]
```

1.41421

Sequence of Operations

```
1 + 2 * 3
```

7

```
2. + 3 / 4
```

2.75

```
(2. + 3) / 4
```

1.25

```
(2. + 3) / 5 + 1
```

2.

```
2 * 3^2 + 1
```

19

```
32^1 / 5
```

$\frac{32}{5}$

```
32^(1/5)
```

2

Defining Variables

```
x = 5
```

5

```
y = 12
```

12

```
x
```

5

```
x + y
```

17

```
x / y
```

$\frac{5}{12}$

```
N[%]
```

0.416667

To clear the value assigned, use ".":

```
x =.
```

```
y =.
```

```
x
```

x

```
y
```

y

If you do not want to have some of the output to be printed, you can end the input by ' ; '

```
x = 1; y = 2;
```

```
f = (x + y) ^2
```

9

```
g = f^2
```

81

```
x =.; y =.
```

I.2 Symbolic Computations

```
f = 1 + x^2
```

$1 + x^2$

```
g = f^2
```

$(1 + x^2)^2$

```
(1 + x^2)^2
```

$(1+x^2)^2$

```
f /. x → 3
```

10

```
f
```

$1+x^2$

■ Functions

Let us clear the assignment given to f earlier:

```
f =.
```

To define a function:

```
f[x_] := 1 + x^2
```

```
f[3]
```

10

```
f[2]
```

5

```
f[a]
```

$1+a^2$

```
f[a + 3]
```

$1+(3+a)^2$

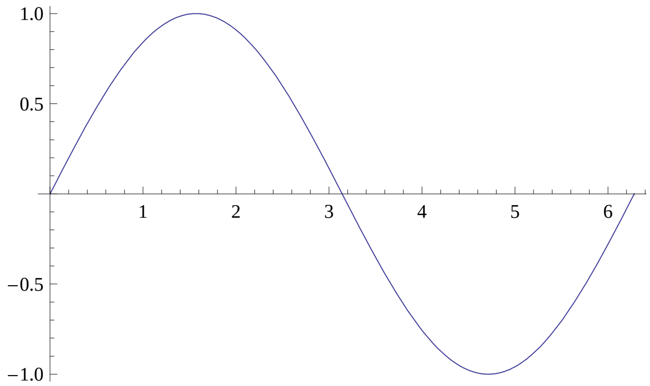
```
f[a + b + c]
```

$1+(a+b+c)^2$

I.3 Graphics

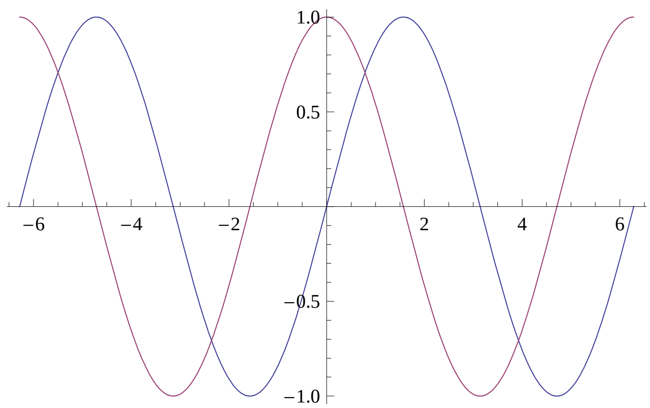
To plot one function along x-axis:

```
Plot[Sin[x], {x, 0, 2 Pi}]
```

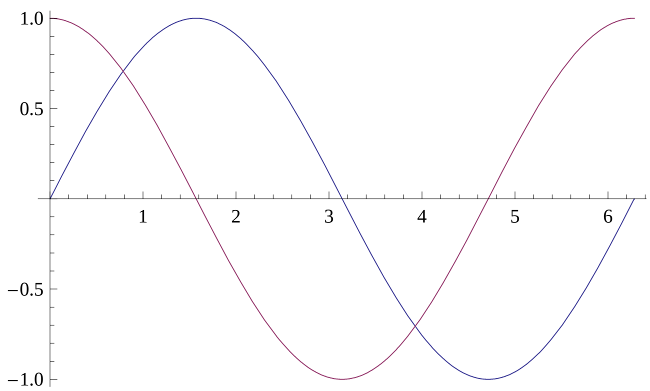


To plot more than one function:

```
Plot[{Sin[x], Sin[ x + Pi / 2]}, {x, -2 Pi, 2 Pi}]
```

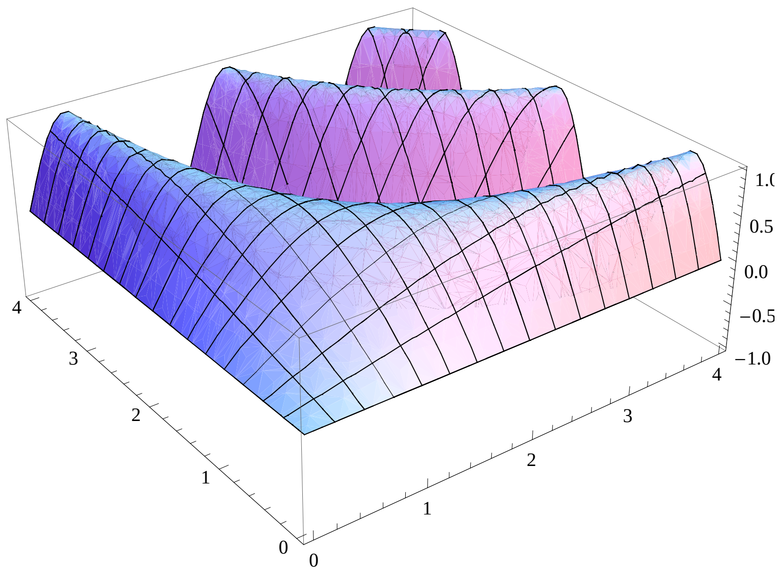



```
Plot[{Sin[x], Cos[x]}, {x, 0, 2 Pi}]
```



To do 3-D plot:

```
Plot3D[Sin[x * y], {x, 0, 4}, {y, 0, 4}]
```



Excercise1

Define the function $g = x^2 - y^2$. Evaluate the function at $x = 2$ and $y = 3$ without destroying the definition. Then, Plot the function for x ranges from −1 to 1 and y from −1 to 1.

1.4 Lists

```
x = Table[2 * i, {i, 1, 6}]
```

{2, 4, 6, 8, 10, 12}

```
x[[3]]
```

6

```
x =.
```

```
m = Table[i * j, {i, 1, 3}, {j, 1, 3}]
```

{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}

```
m =.
```

```
v1 = {1, -2, 5}; v2 = {3, 0, 1};
v1 + v2
```

{4, -2, 6}

```
v1 - v2
```

{-2, -2, 4}

```
v1.v2
```

8

```
Dot[v1, v2]
```

8

```
Cross[v1, v2]
```

{-2, 14, 6}

```
m1 = {{3, 6, 4}, {5, 2, 7}, {2, 1, 5}}
```

{{3, 6, 4}, {5, 2, 7}, {2, 1, 5}}

```
m1[[1]]
```

{3, 6, 4}

```
m1[[2]]
```

{5, 2, 7}

```
m1[[1, 2]]
```

6

```
m2 = 2 m1
```

{{6, 12, 8}, {10, 4, 14}, {4, 2, 10}}

```
m3 = m1 + m2
```

{{9, 18, 12}, {15, 6, 21}, {6, 3, 15}}

```
Dot[m1, m2]
```

{{94, 68, 148}, {78, 82, 138}, {42, 38, 80}}

```
m1.m2
```

{{94, 68, 148}, {78, 82, 138}, {42, 38, 80}}

```
Transpose[m1]
```

{{3, 5, 2}, {6, 2, 1}, {4, 7, 5}}

```
Inverse[m1]
```

$$\left\{\left\{-\frac{3}{53}, \frac{26}{53}, -\frac{34}{53}\right\}, \left\{\frac{11}{53}, -\frac{7}{53}, \frac{1}{53}\right\}, \left\{-\frac{1}{53}, -\frac{9}{53}, \frac{24}{53}\right\}\right\}$$

```
MatrixForm[%]
```

$$\begin{pmatrix} -\frac{3}{53} & \frac{26}{53} & -\frac{34}{53} \\ \frac{11}{53} & -\frac{7}{53} & \frac{1}{53} \\ -\frac{1}{53} & -\frac{9}{53} & \frac{24}{53} \end{pmatrix}$$

```
%.m1
```

{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

```
MatrixForm[%]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Det[m1]
```

-53

```
m1.v1
```

{11, 36, 25}

Excercise2

Define two 3 x3 matrices (a and b). Determine their dot product c = a.b. Determine the inverse of c and show that the $c.c^{-1}$ = the identity matrix.

1.5 Differentiation

```
D[x^5, x]
```

$5\,x^4$

```
D[x^5, {x, 2}]
```

$20\,x^3$

```
D[x^5, {x, 6}]
```

0

```
D[x^n, {x, 3}]
```

$(-2+n)\,(-1+n)\,n\,x^{-3+n}$

$$\mathbf{D}\left[\left(\frac{\mathrm{Tan}[x]+x}{\mathrm{Sin}[x]+x^2}+\mathrm{Log}\left[x^2+\mathrm{Cos}[x]\right]\right)^3,\ x\right]$$

$$3\left(\frac{2\,x-\mathrm{Sin}[x]}{x^2+\mathrm{Cos}[x]}+\frac{1+\mathrm{Sec}[x]^2}{x^2+\mathrm{Sin}[x]}-\frac{(2\,x+\mathrm{Cos}[x])\,(x+\mathrm{Tan}[x])}{\left(x^2+\mathrm{Sin}[x]\right)^2}\right)\left(\mathrm{Log}\left[x^2+\mathrm{Cos}[x]\right]+\frac{x+\mathrm{Tan}[x]}{x^2+\mathrm{Sin}[x]}\right)^2$$

1.6 Integration

```
Integrate[x^2, x]
```

$$\frac{x^3}{3}$$

```
Integrate[x^2, {x, a, b}]
```

$$-\frac{a^3}{3}+\frac{b^3}{3}$$

```
Integrate[x^2, {x, 0, 1}]
```

$$\frac{1}{3}$$

```
Integrate[x^n, x]
```

$$\frac{x^{1+n}}{1+n}$$

```
Integrate[1/(x^4 - a^4), x]
```

$$-\frac{\text{ArcTan}\left[\frac{x}{a}\right]}{2\,a^3}+\frac{\text{Log}[a-x]}{4\,a^3}-\frac{\text{Log}[a+x]}{4\,a^3}$$

$$\text{Integrate}\left[\frac{\text{Exp}[x]}{x}, \{x, 1, 2\}\right]$$

−ExpIntegralEi[1] + ExpIntegralEi[2]

$$\text{NIntegrate}\left[\frac{\text{Exp}[x]}{x}, \{x, 1, 2\}\right]$$

3.05912

$$\int x\text{^}2 \, dx$$

$$\frac{x^3}{3}$$

$$\int_0^1 x\text{^}2 \, dx$$

$$\frac{1}{3}$$

$$\int_0^1 \int_0^1 x\, y \, dx \, dy$$

$$\frac{1}{4}$$

Excercise3

1 – Compute $\int_0^1 x^x \, dx$ (Answer = 0.783431)

2 – Compute $\int_0^1 \int_0^1 (x^2 + y^2) \, dx \, dx$ (Answer = 2 / 3)

1.7 Algebraic Equations

Example 1

Suppose we want to solve the following system of linear equations:

$$\begin{pmatrix} 3 & 5.2 & 10 \\ 5 & 10 & 19 \\ 1 & 3 & 5 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6.7 \\ -4.4 \end{pmatrix}$$

We will solve the above eqs using three different commands:

- **Using Solve:**

```
eq1 = 3 * x1 + 5.2 * x2 + 10 * x3 == -1
```

```
3 x1 + 5.2 x2 + 10 x3 == -1
```

```
eq2 = 5 * x1 + 10 * x2 + 19 * x3 == 6.7
```

```
5 x1 + 10 x2 + 19 x3 == 6.7
```

```
eq3 = x1 + 3 * x2 + 5 * x3 == -4.4
```

```
x1 + 3 x2 + 5 x3 == -4.4
```

```
sol = Solve[{eq1, eq2, eq3}]
```

```
{{x1 → -17.7636, x2 → -31.9545, x3 → 21.8455}}
```

```
x1
```

```
x1
```

```
x2
```

```
x2
```

```
x3
```

```
x3
```

```
x1 = x1 /. sol
```

```
{-17.7636}
```

```
x2 = x2 /. sol
```

{-31.9545}

```
x3 = x3 /. sol
```

{21.8455}

```
x1 =.; x2 =.; x3 =.;
```

■ Using LinearSolve

```
a = {{3, 5.2, 10}, {5, 10, 19}, {1, 3, 5}}
```

{{3, 5.2, 10}, {5, 10, 19}, {1, 3, 5}}

```
b = {-1, 6.7, -4.4}
```

{-1, 6.7, -4.4}

```
sol = LinearSolve[a, b]
```

{-17.7636, -31.9545, 21.8455}

■ You can input the matrices using pallets:

$$a = \begin{pmatrix} 3 & 5.2 & 10 \\ 5 & 10 & 19 \\ 1 & 3 & 5 \end{pmatrix}$$

{{3, 5.2, 10}, {5, 10, 19}, {1, 3, 5}}

$$b = \begin{pmatrix} -1 \\ 6.7 \\ -4.4 \end{pmatrix}$$

{{-1}, {6.7}, {-4.4}}

```
sol = LinearSolve[a, b]
```

{{-17.7636}, {-31.9545}, {21.8455}}

■ If you want to assign names to the solution variables:

```
{x1, x2, x3} = LinearSolve[a, b]
```

{{-17.7636}, {-31.9545}, {21.8455}}

```
x1
```

{-17.7636}

```
x2
```

{-31.9545}

■ Using Inverse

```
sol3 = Inverse[a].b
```

{{-17.7636}, {-31.9545}, {21.8455}}

Example2: Solve the following equations:

$$-a\,x_1 + b\,x_2 - x_3 = a + c$$
$$-b\,x_1 - a\,x_2 + c\,x_3 = 10$$
$$c\,x_1 + a\,x_2 + 2\,x_3 = c$$

```
a =.; b =.; c =.; x1 =.; x2 =.; x3 =.;
```

```
eq1 = -a x1 + b x2 - x3 == a + c
```

-a x1 + b x2 - x3 == a + c

```
eq2 = -b x1 - a x2 + c x3 == 10
```

-b x1 - a x2 + c x3 == 10

```
eq3 = c x1 + a x2 + 2 x3 == c
```

c x1 + a x2 + 2 x3 == c

```
Solve[{eq1, eq2, eq3}, {x1, x2, x3}]
```

$$\left\{\left\{x1 \to -\frac{10\,a + 2\,a^2 + 20\,b + 3\,a\,c + a^2\,c + a\,c^2 - b\,c^2}{2\,a^2 + a\,b + 2\,b^2 - a\,c + a^2\,c + b\,c^2},\ x2 \to -\frac{20\,a - 2\,a\,b - 10\,c - 3\,b\,c - 2\,a\,c^2 - c^3}{2\,a^2 + a\,b + 2\,b^2 - a\,c + a^2\,c + b\,c^2},\ x3 \to -\frac{-10\,a^2 + a^2\,b - 2\,a^2\,c - 10\,b\,c + a\,b\,c - b^2\,c - a\,c^2}{2\,a^2 + a\,b + 2\,b^2 - a\,c + a^2\,c + b\,c^2}\right\}\right\}$$

```
Solve[{eq1, eq2, eq3}, {a, b, c}]
```

$$\left\{\left\{b \to -\frac{-10 + 10\,x1^2 - 10\,x2 + 3\,x2\,x3 - x1\,x2\,x3 + 2\,x3^2 + 2\,x1\,x3^2 - x2\,x3^2}{-x1 + x1^3 - x1\,x2 - x2^2 + x1\,x2^2 + x2^2\,x3},\ a \to -\frac{-10\,x2 + 10\,x1\,x2 - 3\,x1\,x3 + x1^2\,x3 + 2\,x2\,x3^2}{-x1 + x1^3 - x1\,x2 - x2^2 + x1\,x2^2 + x2^2\,x3},\ c \to -\frac{-10\,x2^2 + 2\,x1\,x3 + 2\,x1^2\,x3 - x1\,x2\,x3 + 2\,x2^2\,x3}{-x1 + x1^3 - x1\,x2 - x2^2 + x1\,x2^2 + x2^2\,x3}\right\}\right\}$$

Example 3:

Invert the following stress – strain equations,
so that stresses are given in terms of strains

$$\epsilon 11 = \frac{1}{E}[\sigma 11 - \nu\,(\sigma 22 + \sigma 33)]$$

$$\epsilon 22 = \frac{1}{E}[\sigma 22 - \nu\,(\sigma 11 + \sigma 33)]$$

$$\epsilon 11 = \frac{1}{E}[\sigma 33 - \nu\,(\sigma 11 + \sigma 22)]$$

$$\epsilon 12 = \frac{1}{2\,G}\,\sigma 12$$

$$\epsilon 12 = \frac{1}{2\,G}\,\sigma 12$$

$$\epsilon 12 = \frac{1}{2\,G}\,\sigma 12$$

```
eq1 = e11 == (s11 - nu (s22 + s33)) / e;
eq2 = e22 == (s22 - nu (s11 + s33)) / e;
eq3 = e33 == (s33 - nu (s11 + s22)) / e;
```

```
Solve[{eq1, eq2, eq3}, {s11, s22, s33}]
```

$$\left\{\left\{s11 \to -\frac{e\,e11 - e\,e11\,nu + e\,e22\,nu + e\,e33\,nu}{-1 + nu + 2\,nu^2},\right.\right.$$
$$\left.\left. s22 \to -\frac{e\,e22 + e\,e11\,nu - e\,e22\,nu + e\,e33\,nu}{(1 + nu)\,(-1 + 2\,nu)},\ s33 \to -\frac{e\,e33 + e\,e11\,nu + e\,e22\,nu - e\,e33\,nu}{(1 + nu)\,(-1 + 2\,nu)}\right\}\right\}$$

Exercises

Functions

```
f[x_] := x^2
```

```
f[3]
```

9

```
a =.
```

```
f[a]
```

a^2

```
s =.
```

```
f[a + s]
```

$(a + s)^2$

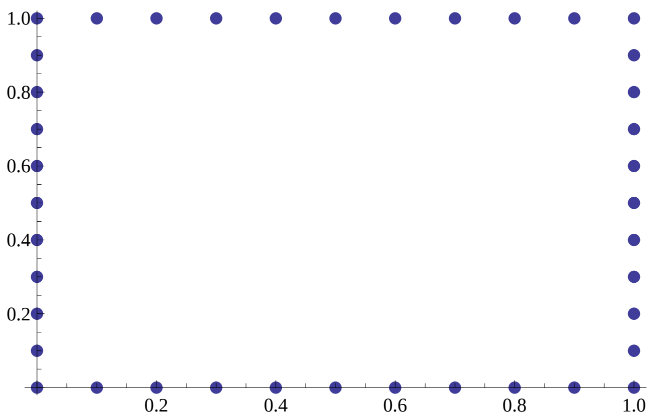
```
f[x_, y_] := x / y;
```

```
f[a, b]
```

$\frac{a}{b}$

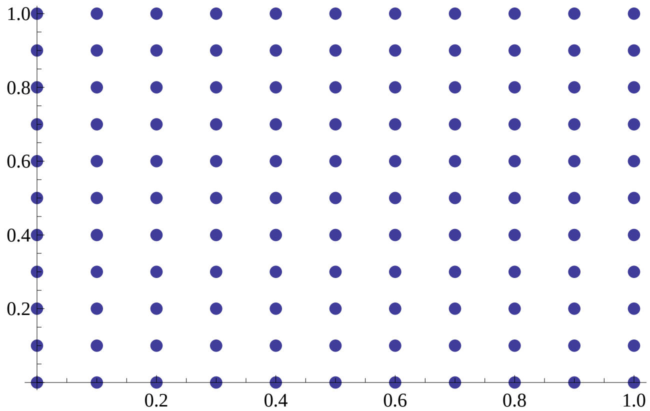
```
nodes[Lx_, Ly_, nx_, ny_] := (xe = Join[Table[i * Lx / nx, {i, 1, nx}],
     Table[Lx, {i, 1, ny}], Table[Lx - i * Lx / nx , {i, 1, nx}], Table[0, {i, 1, ny}]];
   ye = Join[Table[0, {i, 1, nx}], Table[i * Ly / ny, {i, 1, ny}],
     Table[Ly, {i, 1, nx}], Table[Ly - i * Ly / ny , {i, 1, ny}]];
   data1 = Table[{xe[[i]], ye[[i]]}, {i, 1, 2 (nx + ny)}];
   ListPlot[data1, PlotStyle → PointSize[0.02]]);
```

```
nodes[1, 1, 10, 10]
```




```
nodes[Lx_, Ly_, nx_, ny_] := (xe = Join[Table[i * Lx / nx, {i, 1, nx}],
    Table[Lx, {i, 1, ny}], Table[Lx - i * Lx / nx , {i, 1, nx}], Table[0, {i, 1, ny}]];
  ye = Join[Table[0, {i, 1, nx}], Table[i * Ly / ny, {i, 1, ny}], Table[Ly, {i, 1, nx}],
    Table[Ly - i * Ly / ny , {i, 1, ny}]]; xd = Flatten[Table[j * Lx / nx, {i, 1, ny - 1}, {j, 1, nx - 1}]];
  yd = Flatten[Table[i * Ly / ny, {i, 1, ny - 1}, {j, 1, nx - 1}]];
  data1 = Table[{xe[[i]], ye[[i]]}, {i, 1, 2 (nx + ny)}];
  p1 = ListPlot[data1, PlotStyle → PointSize[0.02]];
  data2 = Table[{xd[[i]], yd[[i]]}, {i, 1, (nx - 1) (ny - 1)}];
  p2 = ListPlot[data2, PlotStyle → PointSize[0.02]]; Show[p1, p2])
```

```
nodes[1, 1, 10, 10]
```




```
a
```

a