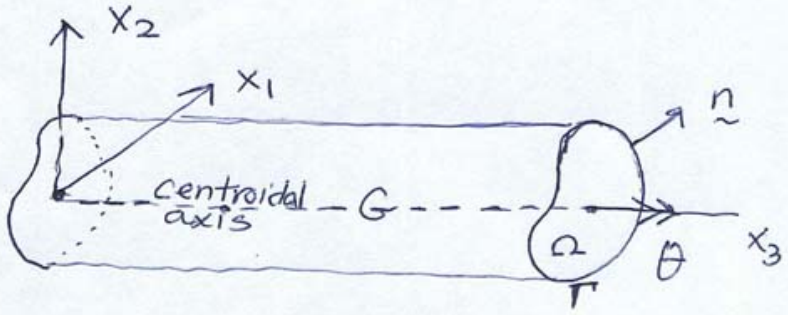
# Torsion of a uniform shaft

## Warping function formulation




$$G\theta \nabla^2 u = 0 \quad \text{in } \Omega$$

$$q = \frac{\partial u}{\partial n} = G\theta (x_2 n_1 - x_1 n_2) \quad \text{on } \Gamma$$

where $u$ is warping function & $G$ is shear modulus and $\theta$ is the angle of twist/unit length

The shear stresses are related to $q_1$ and $q_2$ by:

$$q_1 = \tau_{13} + G\theta x_2$$

$$q_2 = \tau_{23} - G\theta x_1$$

where $q_1 = \frac{\partial u}{\partial x_1}$ & $q_2 = \frac{\partial u}{\partial x_2}$

Stress function formulation

$$\nabla^2 u = -2G\theta \quad \text{in } \Omega$$

$$u = 0 \quad \text{on } \Gamma$$

where $u$ is the stress function.

Shear stresses are given by:

$$\tau_{13} = \frac{\partial u}{\partial x_2}, \quad \tau_{23} = -\frac{\partial u}{\partial x_1}$$

For details, see the book by A.C. Ugural & S.K Fenster, "Advanced Strength and Applied Elasticity".