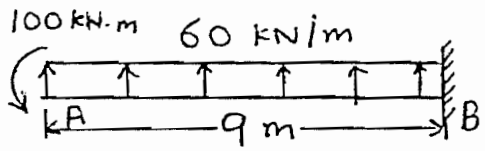
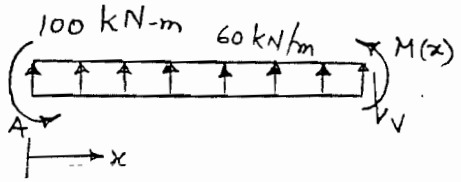
STRUCTURAL MECHANICS I
CE 203
Last HW

PROBLEM #1



Given:
The beam shown

Required:
Algebric equation for the slope and deflection. The value and location of the maximum deflection.



Solution:

$$M(x) = -100 + 60\frac{x^2}{2} = -100 + 30x^2$$

$$EI\theta = \int M\,dx = \int (-100 + 30x^2)\,dx = -100x + 10x^3 + C_1$$

$$EIy = \int EI\theta\,dx = \int (-100x + 10x^3 + C_1)\,dx = -50x^2 + \frac{5}{2}x^4 + C_1x + C_2$$

B.C.'s: $\theta(9) = 0 \Rightarrow -100(9) + 10(9)^3 + C_1 = 0 \Rightarrow C_1 = -6390$

$$y(9) = 0 \Rightarrow -50(9)^2 + \frac{5}{2}(9)^4 - 6390(9) + C_2 = 0 \Rightarrow C_2 = 45157.5$$

$$\therefore \boxed{\theta(x) = \frac{1}{EI}(10x^3 - 100x - 6390)}$$ Ans

$$\boxed{y(x) = \frac{1}{EI}\left(\frac{5}{2}x^4 - 50x^2 - 6390x + 45157.5\right)}$$ Ans

It is clear that y_{max} is at free end A ($x = 0$).

$$\therefore \boxed{y_{max} = \frac{45157.5}{EI}}$$ Ans

PROBLEM #2

Given:

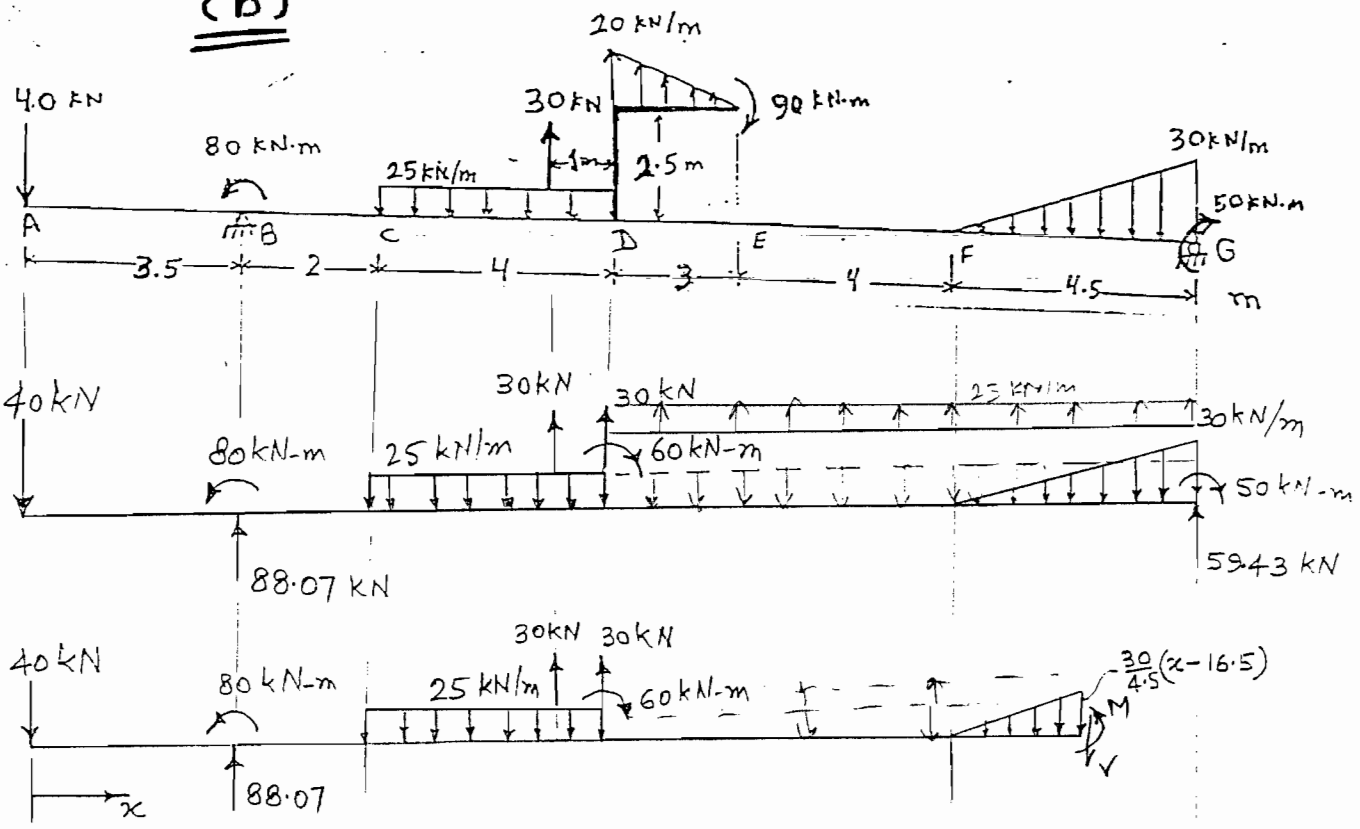
Two beams as shown below

Required:

Single equation for the moment and all boundary conditions.

Solution:

(b)

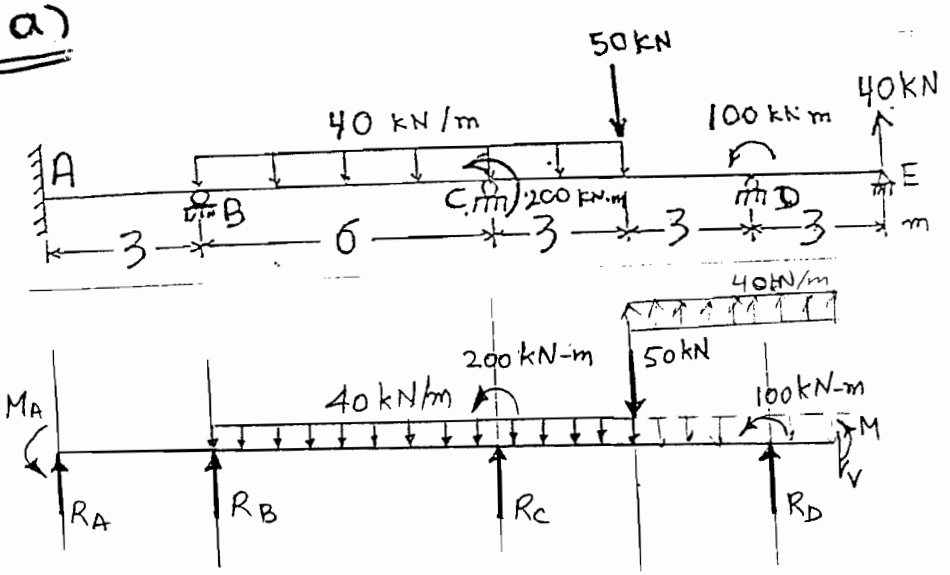


$$M(x) = -40\langle x-0\rangle^{1} - 80\langle x-3.5\rangle^{0} + 88.07\langle x-3.5\rangle^{1} - \frac{25}{2}\left\{\langle x-5.5\rangle^{2} - \langle x-9.5\rangle^{2}\right\} + 30\langle x-8.5\rangle^{1}$$
$$+ 30\langle x-9.5\rangle^{1} + 60\langle x-9.5\rangle^{0} - \frac{1}{6}\frac{30}{4.5}\langle x-16.5\rangle^{3}$$

Ans

B.C.'s: $Y(3.5) = 0$; $Y(21) = 0$ Ans

(a)



$$M(x) = -M_A\langle x-0\rangle^0 + R_A\langle x-0\rangle^1 + R_B\langle x-3\rangle^1 - 20\{\langle x-3\rangle^2 - \langle x-12\rangle^2\}$$
$$-200\langle x-9\rangle^0 + R_C\langle x-9\rangle^1 - 50\langle x-12\rangle^1 - 100\langle x-15\rangle^0 + R_D\langle x-15\rangle^1$$

Ans

B.C.'s: $\theta(0)=0$ $\quad y(0)=0$

$y(3)=0$ $\quad y(9)=0$

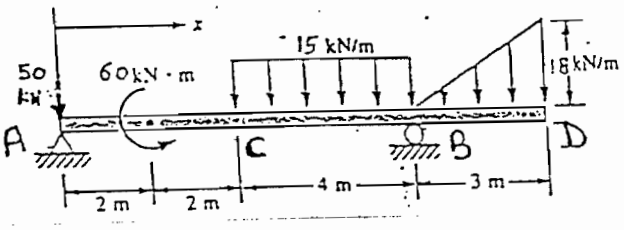
$y(15)=0$ $\quad y(18)=0$

Ans

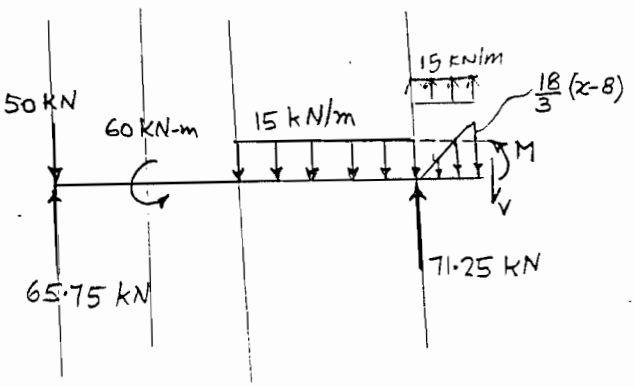
In addition 2 equations of equilibrium will allow determination of all unknowns.

PROBLEM #3

Given:
The beam shown.



Required:
Slope and deflection equation and value of slope and deflection at point c.



Solution:

$$M(x) = 15.75\langle x-0\rangle^{1} - 60\langle x-2\rangle^{0} - \frac{15}{2}\left\{\langle x-4\rangle^{2} - \langle x-8\rangle^{2}\right\} + 71.25\langle x-8\rangle^{1} - \frac{1}{6}\frac{18}{3}\langle x-8\rangle^{3}$$

$$EI\theta = \int M\,dx = \frac{15.75}{2}\langle x-0\rangle^{2} - 60\langle x-2\rangle^{1} - \frac{5}{2}\left\{\langle x-4\rangle^{3} - \langle x-8\rangle^{3}\right\} + \frac{71.25}{2}\langle x-8\rangle^{2} - \frac{1}{4}\langle x-8\rangle^{4} + C_1$$

$$EIy = \int EI\theta\,dx = \frac{15.75}{6}\langle x-0\rangle^{3} - 30\langle x-2\rangle^{2} - \frac{5}{8}\left\{\langle x-4\rangle^{4} - \langle x-8\rangle^{4}\right\} + \frac{71.25}{6}\langle x-8\rangle^{3} - \frac{1}{20}\langle x-8\rangle^{5} + C_1 x + C_2$$

B.C.'s: $y(0)=0 \Rightarrow C_2 = 0$

$$y(8)=0 \Rightarrow \frac{15.75}{6}8^3 - 30(8-2)^2 - \frac{5}{8}(8-4)^4 + 8C_1 = 0 \Rightarrow C_1 = -13$$

$$\therefore \boxed{\theta(x) = \frac{1}{EI}\left[7.875\langle x-0\rangle^{2} - 60\langle x-2\rangle^{1} - 2.5\left\{\langle x-4\rangle^{3} - \langle x-8\rangle^{3}\right\} + 35.6\langle x-8\rangle^{2} - 0.25\langle x-8\rangle^{4} - 13\right]} \quad \text{Ans}$$

$$\boxed{y(x) = \frac{1}{EI}\left[2.625\langle x-0\rangle^{3} - 30\langle x-2\rangle^{2} - 0.625\left\{\langle x-4\rangle^{4} - \langle x-8\rangle^{4}\right\} + 11.875\langle x-8\rangle^{3} - 0.05\langle x-8\rangle^{5} - 13x\right]} \quad \text{Ans}$$

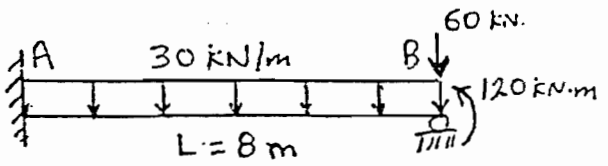
$$\theta_c = \theta(4) = \frac{1}{EI}\left[7.875(4)^2 - 60(4-2) - 13\right] \Rightarrow \boxed{\theta_c = -\frac{7}{EI}} \quad \text{Ans}$$

$$y_c = y(4) = \frac{1}{EI}\left[2.625(4)^3 - 30(4-2)^2 - 13(4)\right] \Rightarrow \boxed{y_c = -\frac{4}{EI}} \quad \text{Ans}$$

PROBLEM #4

Given:

The beam shown

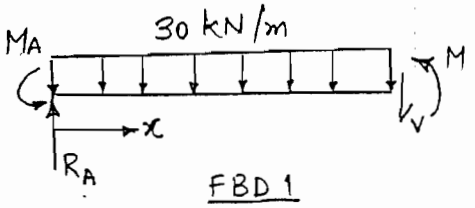


Required:

Reactions of the beam

Solution:

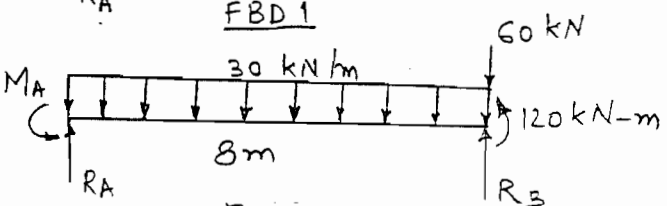
Consider FBD1



FBD 1

$M(x) = -M_A + R_A x - 15x^2$

$EI\theta = \int M\,dx = -M_A x + \frac{R_A}{2}x^2 - 5x^3 + C_1$



FBD2

$\theta(0) = 0 \Rightarrow C_1 = 0 \quad \therefore EI\theta = -M_A x + \frac{R_A}{2}x^2 - 5x^3$

$EIy = \int EI\theta\,dx = -\frac{M_A}{2}x^2 + \frac{R_A}{6}x^3 - \frac{5}{4}x^4 + C_2$

$y(0) = 0 \Rightarrow C_2 = 0 \quad \therefore EIy = -\frac{M_A}{2}x^2 + \frac{R_A}{6}x^3 - \frac{5}{4}x^4$

$y(8) = 0 \Rightarrow -32\,M_A + 85.33\,R_A - 5120 = 0$ —— (1)

$\curvearrowleft \Sigma M_A = 0 \Rightarrow M_A + 8R_B + 120 - 8(60) - 30\frac{8^2}{2} = 0$

$\Rightarrow M_A + 8R_B - 1320 = 0$ —— (2)

$+\uparrow \Sigma F_y = 0 \Rightarrow R_A + R_B - 60 - 30(8) = 0$

$\Rightarrow R_A + R_B - 300 = 0$ — (3)

Solving equations (1), (2) and (3) simultaneously we get,

$$M_A = 300 \text{ kN-m} \;;\; R_A = 172.5 \text{ kN} \;;\; R_B = 127.5 \text{ kN}$$

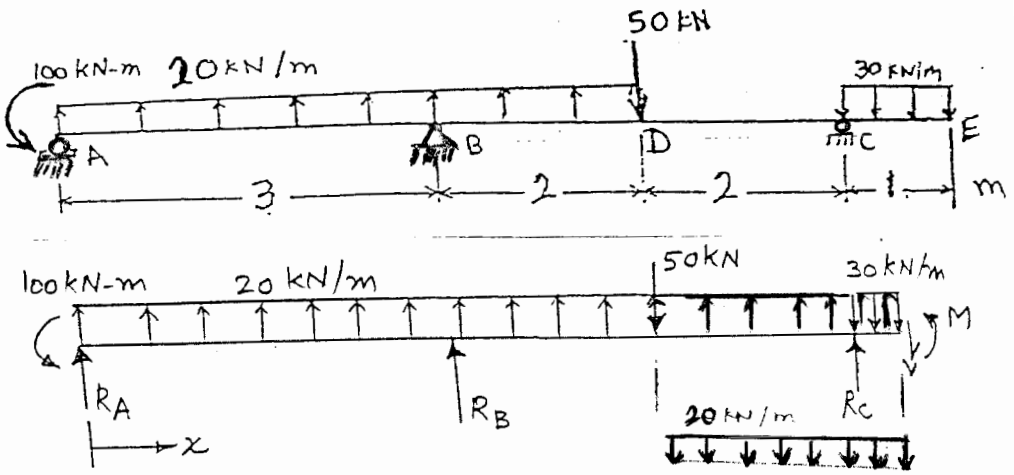
Ans

PROBLEM #5

Given:
The beam shown.

Required:
Reactions and slope and deflection at point D

Solution:



$$M(x) = -100\langle x-0\rangle^0 + R_A\langle x-0\rangle^1 + 10\{\langle x-0\rangle^2 - \langle x-5\rangle^2\} + R_B\langle x-3\rangle^1 - 50\langle x-5\rangle^1 + R_C\langle x-7\rangle^1 - 15\langle x-7$$

$$EI\theta = -100\langle x-0\rangle^1 + \frac{R_A}{2}\langle x-0\rangle^2 + \frac{10}{3}\{\langle x-0\rangle^3 - \langle x-5\rangle^3\} + \frac{R_B}{2}\langle x-3\rangle^2 - 25\langle x-5\rangle^2 + \frac{R_C}{2}\langle x-7\rangle^2 - 5\langle x-7\rangle^3 + C_1$$

$$EIy = -50\langle x-0\rangle^2 + \frac{R_A}{6}\langle x-0\rangle^3 + \frac{5}{6}\{\langle x-0\rangle^4 - \langle x-5\rangle^4\} + \frac{R_B}{6}\langle x-3\rangle^3 - \frac{25}{3}\langle x-5\rangle^3 + \frac{R_C}{6}\langle x-7\rangle^3 - \frac{5}{4}\langle x-7\rangle^4 + C_1x + C_2$$

$y(0)=0 \Rightarrow C_2 = 0$

$y(3)=0 \Rightarrow -50(3)^2 + \frac{R_A}{6}(3)^3 + \frac{5}{6}(3)^4 + 3C_1 = 0 \Rightarrow 3C_1 + 4.5R_A = 382.5$ — ①

$y(7)=0 \Rightarrow -50(7)^2 + \frac{R_A}{6}(7)^3 + \frac{5}{6}\{7^4 - 2^4\} + \frac{R_B}{6}(4)^3 - \frac{25}{3}(2)^3 + 7C_1 = 0$

$\Rightarrow 7C_1 + 57.167R_A + 10.667R_B = 529.167$ — ②

$\circlearrowleft + \sum M_A = 0 \Rightarrow 3R_B + 7R_C + 100 + 20\frac{5^2}{2} - 50(5) - 30(7.5) = 0$

$\Rightarrow 3R_B + 7R_C = 125$ — ③

$+\uparrow \sum F_y = 0 \Rightarrow R_A + R_B + R_C + 20(5) - 50 - 30(1) = 0 \Rightarrow R_A + R_B + R_C = -20$ — ④

Solving equations ①, ②, ③ & ④ simultaneously.

$C_1 = 109.11$; $R_A = 12.26$ kN ; $R_B = -87.7$ kN ; $R_C = 55.4$ kN

$$\boxed{R_A = 12.26\text{ kN}\uparrow \;;\; R_B = 87.7\text{ kN}\downarrow \;;\; R_C = 55.4\text{ kN}\uparrow}$$ Ans

$$\theta_D = \theta(5) = \frac{1}{EI}\left[-100(5) + \frac{12.26}{2}5^2 + \frac{10}{3}5^3 - \frac{87.7}{2}(2)^2 + 109.11\right] \Rightarrow \boxed{\theta_D = \frac{3.63}{EI}}$$ Ans

$$y_D = y(5) = \frac{1}{EI}\left[-50(5)^2 + \frac{12.26}{6}(5)^3 + \frac{5}{6}(5)^4 - \frac{87.7}{6}(2)^3 + 109.11(5)\right] \Rightarrow \boxed{y_D = -\frac{45.13}{EI}}$$ Ans