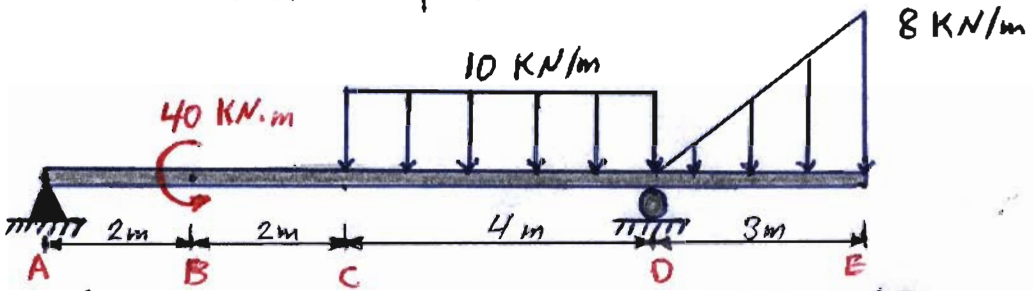
CE 203-3 [072]

H.W. # 9 - solution

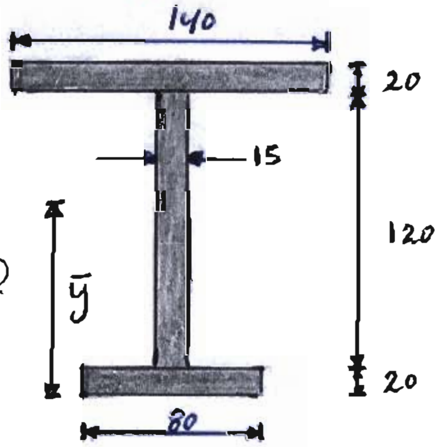
Problem 1 :-

Given :- The beam shown in Fig.

Req :- The values and Locations of the maximum tensile and compressive stresses.



Solution :-



cross section
units in (mm)

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i \bar{y}_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{y} = \frac{(80\times20)(10) + (15\times120)(80) + (20\times140)(150)}{(80\times20) + (15\times120) + (20\times140)}$$

$$\bar{y} = 93.55 \text{ mm}$$

$$I = \sum_{i=1}^{n} (\bar{I}_i + A_i d_i^2)$$

$$\bar{I} = \frac{1}{12} b h^3$$

$$I = \Big[\frac{1}{12}(80)(20)^3 + (80\times20)(93.55-10)^2 + \frac{1}{12}(15)(120)^3 + (15\times120) * (93.55-80)^2 + \frac{1}{12}(140)(20)^3 + (140\times20)(150-93.55)^2\Big]$$

$$I = 11.2223\times10^6 + 2.4905\times10^6 + 9.0158\times10^6$$

$$\therefore I = 22.729\times10^6 \text{ mm}^4$$

* Determine maximum positive & negative Moments, using BMD.

* calculate the reaction from (F B D)

$\circlearrowleft \sum M_A = 0$;

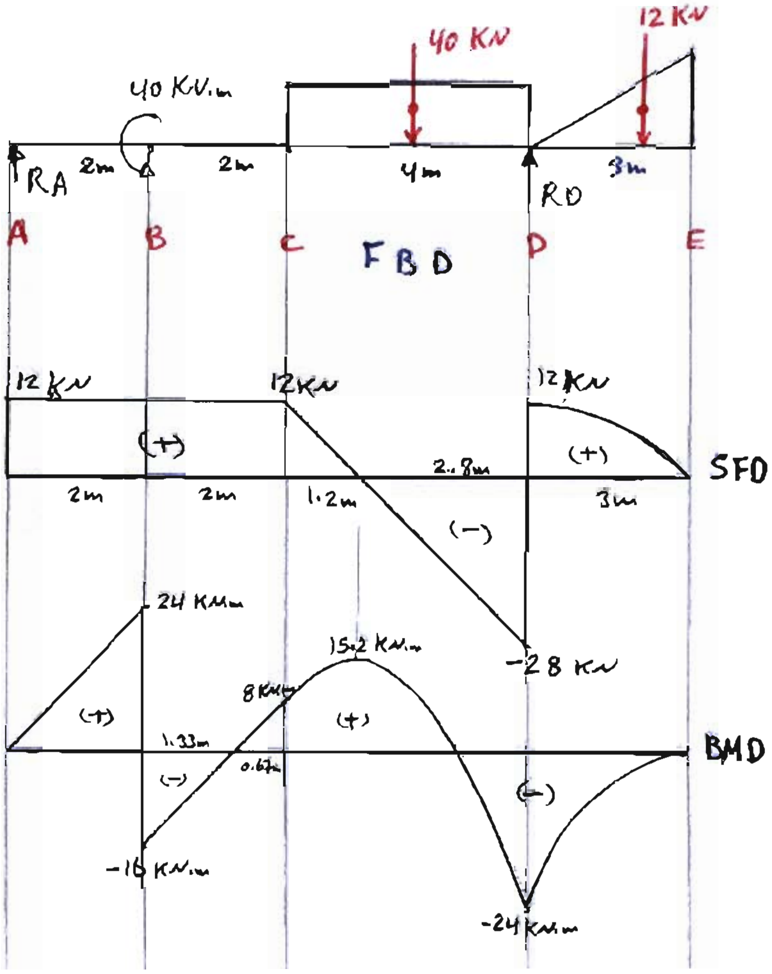
$40 - 40 \times 6 + R_B(8) - 12(10) = 0$

$R_B = 40 \text{ KN} \uparrow$

$+\uparrow \sum F_y = 0$;

$R_A - 40 + 40 - 12 = 0$

$R_A = 12 \text{ KN} \uparrow$



$\therefore$ max +ve $M = 24$ KN.m

max −ve $M = -24$ KN.m

Locate y^+ and y^-

$y^+ = 160 - 93.55$

$= 66.45$ mm (top)

$y^- = 93.55$ mm (bottom)

$$\sigma = -\frac{My}{I}$$

(at point B) :-

$$\sigma_{max}^{T} = -\frac{24 \times 10^{3}(-93.55) \times 10^{3}}{22.729 \times 10^{6}} = 98.78 \text{ Mpa (T) at bottom}$$

$$\sigma_{max}^{c} = -\frac{24 \times 10^{6}(66.45)}{22.729 \times 10^{6}} = 70.166 \text{ Mpa (C) at top}$$

((Note that we can tell this point does not control ; Why?!))

(at point D)

$$\sigma_{max}^{T} = -\frac{-24(66.45)\times 10^{6}}{22.729\times 10^{6}} = 70.166 \text{ Mpa } (T) \text{ at top.}$$

$$\sigma_{max}^{C} = -\frac{-24(-93.55)\times 10^{6}}{22.729\times 10^{6}} = 98.78 \text{ Mpa } (C) \text{ at bottom}$$

((Note that we can tell this point does not control; why?!))

[top of D which is T]

$$\therefore \boxed{\sigma_{max}^{T} = 98.78 \text{ Mpa @ bottom of point B}}$$

$$\boxed{\sigma_{max}^{C} = 98.78 \text{ Mpa @ bottom of point D}}$$

Problem 2:-

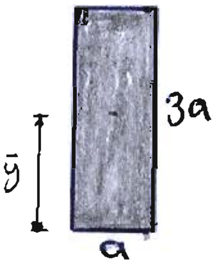
Given :- For the beam shown in fig.

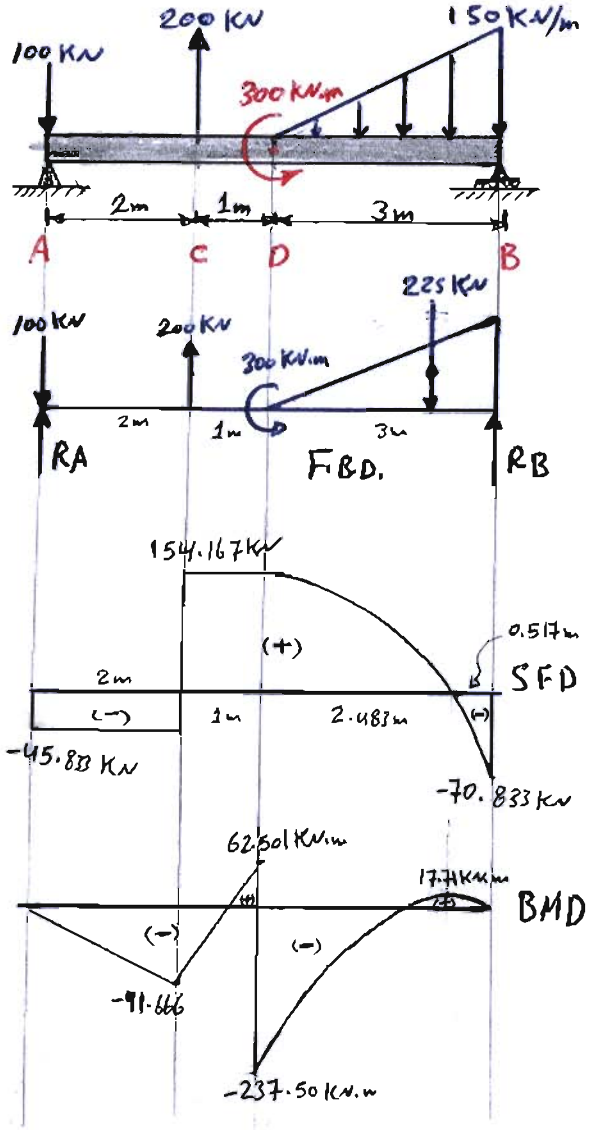
$\sigma^{T}_{max} = 80\ Mpa.$

$\sigma^{C}_{max} = 60\ Mpa.$

Req :- The minimum value of the cross sectional dimesion (a).

solution:-





$\bar{y} = \frac{3a}{2}$

$\bar{y} = 1.5a$

$I = \frac{1}{12} b h^3$

$I = \frac{1}{12} (a)(3a)^3$

$\therefore I = 2.25 a^4$

* Determination of moments.
  Finding reactions from F.B.D.

$\circlearrowleft + \Sigma M_A = 0;$

$200(2) + 300 - 225(5) + RB(6) = 0$

$RB = 70.833\ KN \uparrow$

$+\uparrow \Sigma F_y = 0;$

$RA - 100 + 200 - 225 + 70.833 = 0$

$RA = 54.167\ KN \uparrow$

max +ve $M = 62.501\ KN.m$

max -ve $M = -237.50\ KN.m$

* $y_{max,min} = \pm 1.5a$

Since the cress section is symmetric, then we know that the smaller (σ_{allow}) controls.

$\sigma_{allow}^{c} = 60$ Mpa controls,

We need to check one point only "not four".

It is the point of $|M_{max}|$ at the compression side.

$$\sigma_{max}^{c} = -\frac{My}{I}$$

$$-60 \times 10^{6} = -\frac{-273.5 \times 10^{3}\,(-1.5a)}{2.25\,a^{4}}$$

$$a^{3} = 2.63889 * 10^{-3} \; m^{3}$$

$$a_{min} = 138.2 * 10^{-3} \; m$$

$$\boxed{a_{min} = 138.2 \; mm}$$

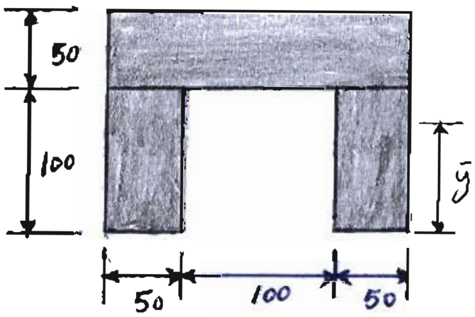
Problem 3:-

Given :- For the beam shown in Fig.

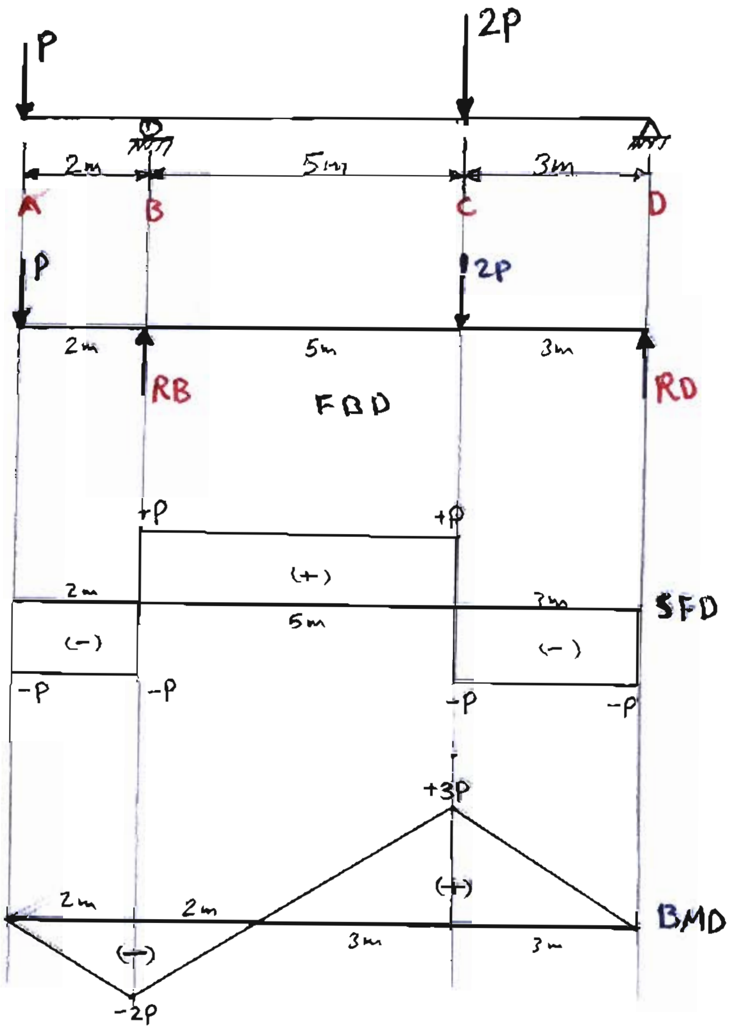
$\sigma^{T}_{allow} = 80 \text{ Mpa}$

$\sigma^{c}_{allow} = 50 \text{ Mpa}$

Req :- The maximum allowable Load (P) which can be applied.



" All dimensions in mm "



Solution :-

(From FBD)

$\circlearrowright \Sigma M_B = 0 ;$

$P \times 2 - 2P \times 5 + R_D \times 8 = 0$

$R_D = P \uparrow$

$+\uparrow \Sigma F_y = 0 ;$

$-P + R_B - 2P + P = 0$

$R_B = 2P \uparrow$

max. +ve $M = 3P$; max -ve $M = -2P$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1} A_i} = \frac{2[(50\times100)\times50] + [(50\times200)\times125]}{2(50\times100) + (50\times200)}$$

$\bar{y} = 87.5 \text{ mm} = 0.0875 \text{ m}$

$$I = \sum_{i=1}^{4} (\bar{I}_i + A_i d_i^2)$$

$$I = \left[\tfrac{1}{12}(200)(50)^3 + (200\times 50)(125-87.5)^2\right] + 2\left[\tfrac{1}{12}(50)(100)^3 + (50\times 100)(87.5-50)^2\right]$$

$$I = 16.1458\times10^6 + 22.3958\times10^6$$

$$\therefore I = 38.542\times10^6 \ mm^4 = 38.542\times10^{-6}\ m^4$$

$$y^+_{max} = 150 - 87.5 = 62.5 \text{ mm} \quad \text{Top}$$

$$y^-_{max} = 87.5 \text{ mm} \quad \text{bottom}$$

(at point c):- $\qquad \sigma = -\dfrac{My}{I}$

$$\sigma^T_{allow} = 80\times10^6 = -\frac{3P(-87.5)\times10^{-3}}{38.542\times10^{-6}}$$

$$\therefore P_1 = 11.746 \text{ KN}$$

$$\sigma^C_{allow} = -50\times10^6 = -\frac{3P(62.5)\times10^{-3}}{38.542\times10^{-6}}$$

$$\therefore P_2 = 10.278 \text{ KN}$$

(at point B):-

$$\sigma^C_{allow} = -50\times10^6 = -\frac{-2P(-87.5)\times10^{-3}}{38.542\times10^{-6}} \Rightarrow P_3 = 11.012 \text{ KN}$$

$$\sigma^T_{allow} = 80\times10^6 = -\frac{-2P(62.5)\times10^{-3}}{38.542\times10^{-6}} \Rightarrow P_4 = 24.667 \text{ KN}$$

"Note that we can eliminate some of these points by inspection before any calculations"

$$\therefore P_{max} = P_{min}(P_1, P_2, P_3, \text{and } P_4)$$

$$\therefore \boxed{P_{max} = 10.278 \text{ KN}}$$

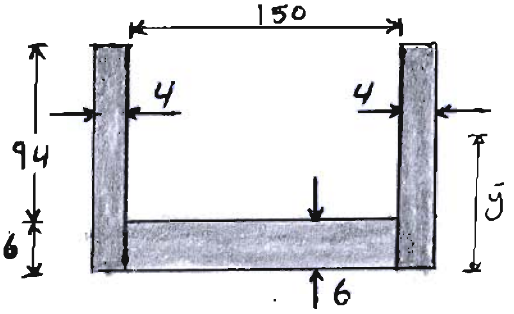
Problem 4 :-

Given :- The beam shown in Fig.

$\sigma^{T}_{allow} = 100$ Mpa.

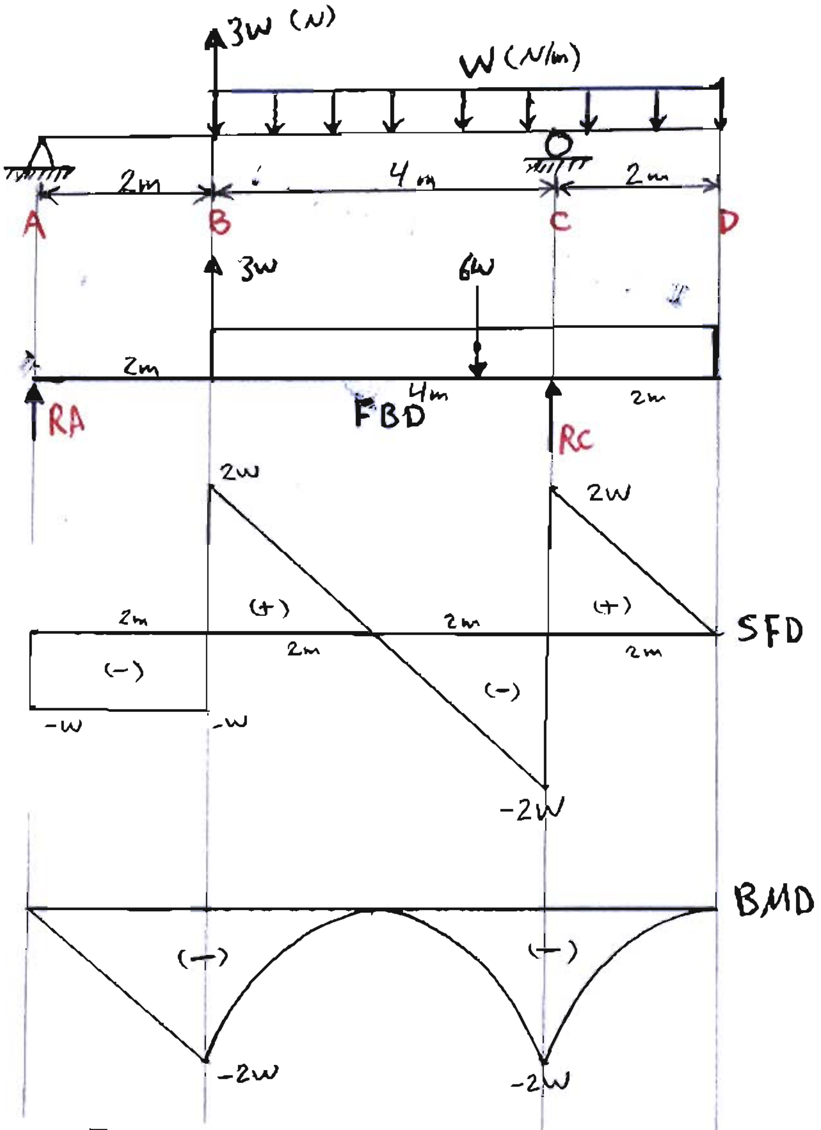
$\sigma^{c}_{allow} = 80$ Mpa.

Req. :- The maximum Load (W) that can be applied.



cross section

Units in mm



Solution :-

From FBD ;

$\circlearrowleft + \Sigma M_A = 0$;

$3W(2) - 6W(5) + R_C(6) = 0$

$R_C = 4W \uparrow$; $R_A = -W$

$R_A = W \downarrow$

max -ve $M = -2W$

$$\bar{y} = \frac{\sum_{i=1} A_i y_i}{\sum_{i=1} A_i}$$

$$\bar{y} = \frac{(6\times150)(3) + 2[(100\times4)(50)]}{(6\times150) + (100\times4)} \Rightarrow \bar{y} = 25.12 \text{ mm}$$

$$I = [\tfrac{1}{12}(150)(6)^3 + (150\times6)(25.12-3)^2] + 2[\tfrac{1}{12}(4)(100)^3 + (4\times100)(50-25.12)^2]$$

$$I = (0.44306 + 1.162)\times10^6$$

$$I = 1.605\times10^{-6} \text{ m}^4$$

$y^{+}_{max} = 100 - 25.12 = 74.88\ mm$ Top

$y^{-}_{max} = 25.12\ mm$ (bottom)

(at points B and C) :-

$$\sigma = -\frac{My}{I}$$

$$\sigma^{T}_{allow} = 100\times10^{6} \equiv -\frac{-2w\,(74.88)\times10^{-3}}{1.605\times10^{-6}}$$

$\therefore\ w = 1.072\ KN/m$

$$\sigma^{C}_{allow} = -80\times10^{6} \equiv -\frac{-2w\,(-25.12)\times10^{-3}}{1.605\times10^{-6}}$$

$\therefore\ w = 2.556\ KN/m$

$$\therefore\ \boxed{w_{max} = 1.072\ KN/m}$$

Problem 5 :-

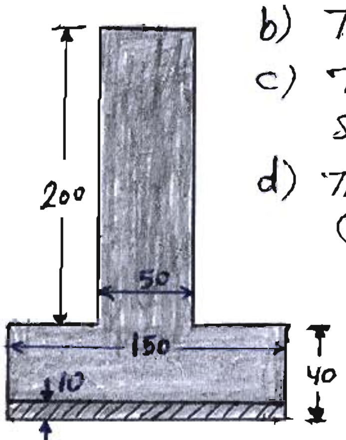
Given :- At section l-l in the beam shown in Fig.

Req :- a) The total normal force acting on the shaded area of the flange.

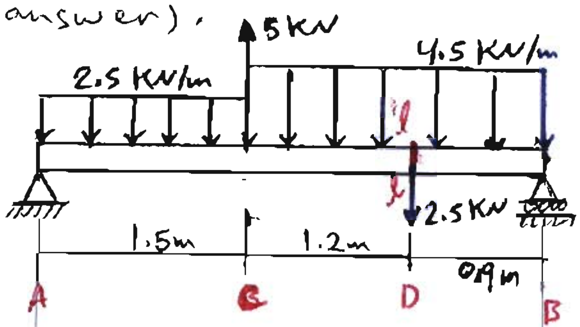
b) The total normal force acting on the web

c) The total normal force acting on the cross-section (coment on the answer).

d) The moment of the forces about the N.A. (coment on the answer).



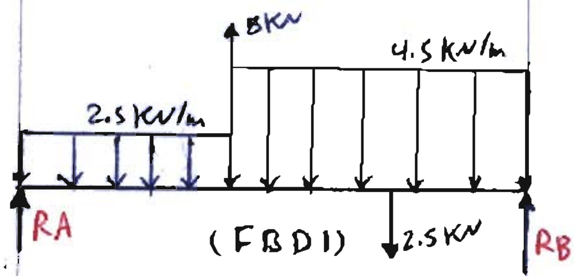
X- section
(Units in mm)



Solution :-

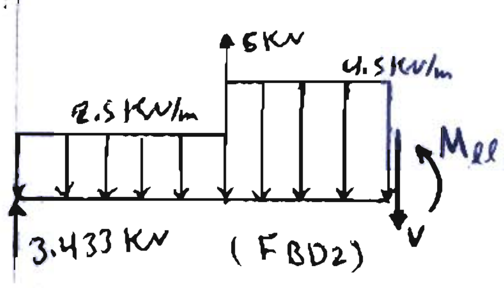
* From (FBD1);

$\circlearrowleft + \sum M_B = 0$;



$-RA(3.6) + 2.5 \times 1.5 \times (2.1 + \frac{1.5}{2})$
$- 5 \times 2.1 + 2.5 \times 0.9 + 4.5 \times 2.1 \times \frac{2.1}{2} = 0$

$\therefore RA = 3.433$ KN



* From (FBD2)

$\circlearrowleft + \sum M = 0$;

$M_{ll} + 4.5 * 1.2 * \frac{1.2}{2} - 5 * 1.2 + 2.5 * 1.5 * (1.2 + \frac{1.5}{2}) - 3.433 \times 2.7 = 0$

$\therefore M_{ll} = 4.717$ KN.m

$$\bar{y} = \frac{(150 \times 40 \times 20) + 50 \times 200 \times 140}{(150 \times 40) + (50 \times 200)}$$

$\bar{y} = 95$ mm

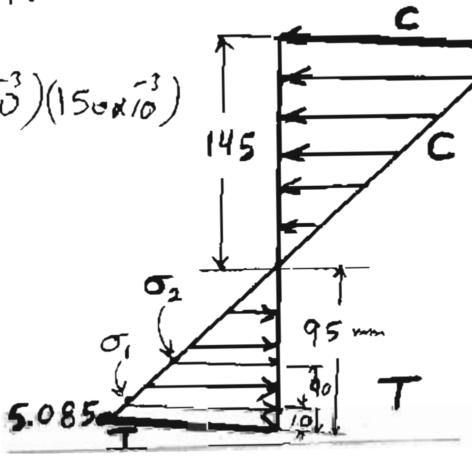
$$I = \left[\tfrac{1}{12}(150)(40)^3 + (150\times 40)(95-20)^2\right] + \left[\tfrac{1}{12}(50)(200)^3 + (50\times 200)(140-95)^2\right]$$

$$I = 88.13 \times 10^{6}\ mm^4$$

$$I = 88.13 \times 10^{-6}\ m^4$$

$$\sigma_{11}^{top} = \frac{-(4.717\times10^{3})(240-95)\times10^{-3}}{88.13\times10^{-6}} = -7.761\times10^{6}\ N/m^2$$

$$\sigma_{11}^{bottom} = \frac{-(4.717\times10^{3})(-95)\times10^{-3}}{88.13\times10^{-6}} = 5.085\times10^{6}\ N/m^2$$



$$\sigma_1 = -\frac{4.717(-85)}{88.13} = 4.55\ MPa$$

$$\sigma_2 = -\frac{4.717(55)}{88.13} = 2.994\ MPa$$

@ top of flange

a) $$F_{shaded} = \left(\frac{5.085+4.55}{2}\right)\times10^{6}(10\times10^{-3})(150\times10^{-3})$$

$$F_{shaded} = 7.225\times10^{3}\ N$$

$$\boxed{F_{shaded} = 7.225\ KN\ (T)}$$

b) $$F_{web} = \left(\frac{(-7.761\times10^{6})(145\times10^{-3})}{2} + \frac{(2.944\times10^{6})(200-145)\times10^{-3}}{2}\right)(50\times10^{-3})$$

$$F_{web} = -24.09\times10^{3}\ N$$

$$\boxed{F_{web} = 24.1\ KN\ (C)}$$

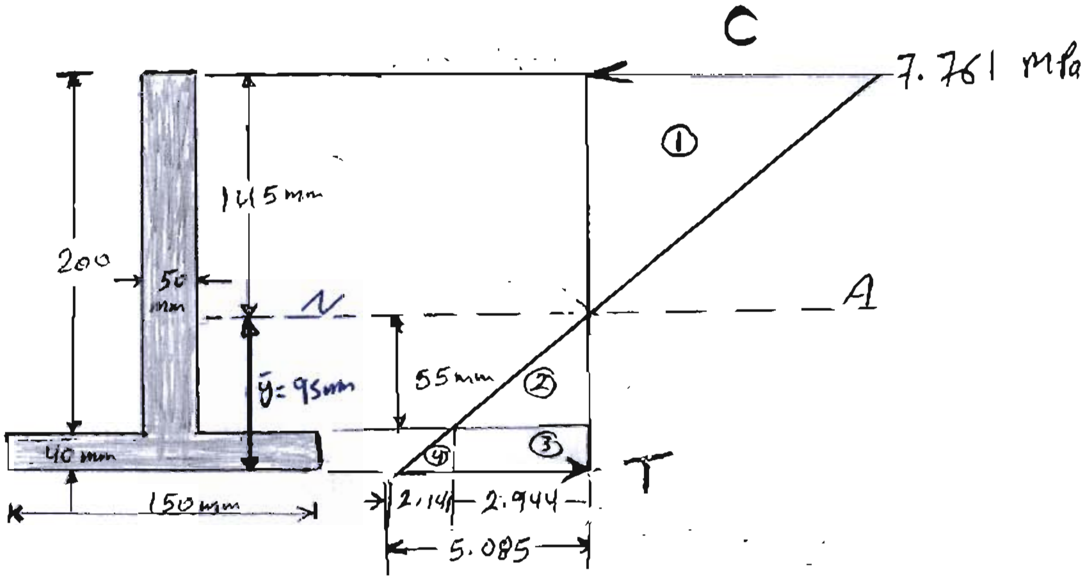
c) $$F_{total} = F_{web} + \left[\frac{(2.944+5.085)\times10^{6}}{2}\right](40\times10^{-3})(150\times10^{-3})$$

$$F_{total} = -24.09\times10^{3} + 24.09\times10^{3}$$

$$\therefore \boxed{F_{total} = 0.0}$$

∴ The total normal force or axial force on a section under pure bending is zero.

d) To find moment of forces about N.A.
We will find moment of four areas shown about N.A.



$$M = \left[\frac{(7.761\times10^{6})(145\times10^{-3})}{2} * \frac{2}{3}(145\times10^{-3}) + \frac{(2.944\times10^{6})(55\times10^{-3})}{2} * \frac{2}{3}(55\times10^{-3})\right] * (50\times10^{-3})$$

$$+\left[(2.944\times10^{6})(40\times10^{-3})\left(\frac{40}{2}+55\right)\times10^{-3} + \frac{(2.141\times10^{6})(40\times10^{-3})}{2} * \left(\frac{2}{3}*40+55\right)\times10^{-3}\right] * (150\times10^{-3})$$

$$\therefore \quad M = 4.717\times10^{3}\ N.m$$

$$\therefore \quad \boxed{M = 4.717\ KN.m}$$

The moment of normal stresses about N.A. is equal to the moment on the section (calculated above).