CE 203-3 [072]

H.W. #7 - Solution

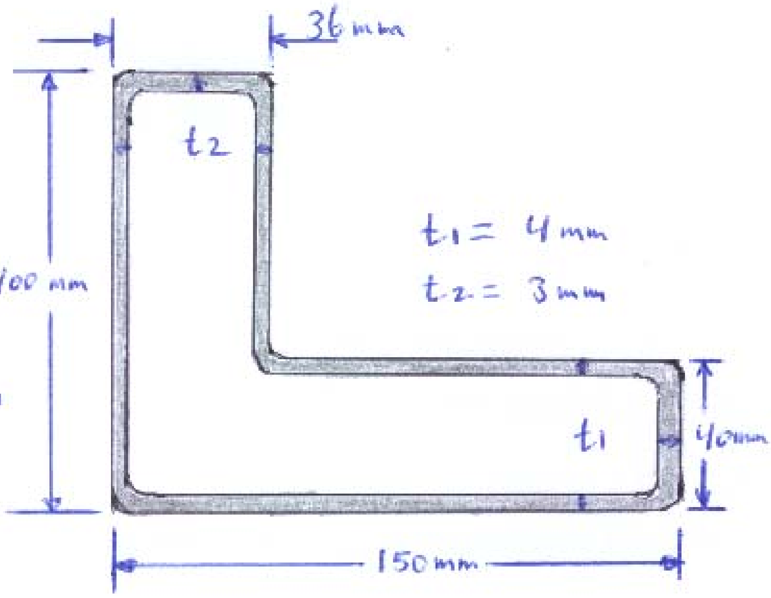
Problem 1:-

Given :- A torque (T) is applied to a shaft with the cross section shown.
$L = 4m$, $\tau_{(allow)} = 50$ Mpa, $\Phi_{max} = 0.003$ rad, and $G = 40$ Gpa.

Req. :- The maximum torque (T).

Solution :-

The section is a thin-walled closed section.



$A_m = (33 \times 96.5) + (113.5 \times 36)$

$A_m = 7270.5 \ mm^2$

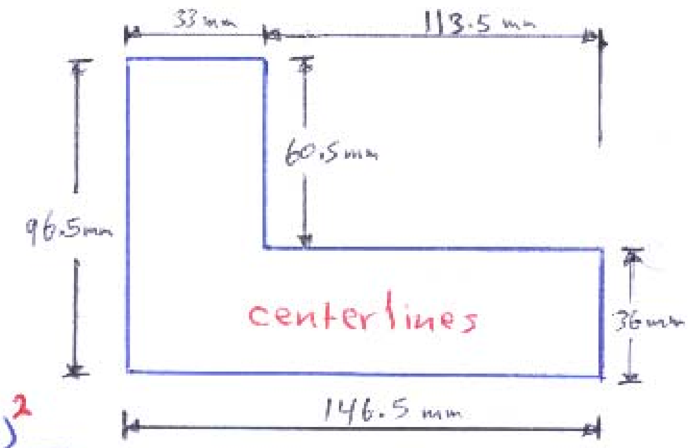
$$\tau = \frac{T}{2t A_m} \Rightarrow T = 2\tau t A_m$$

* For smallest $t = 3mm$

$T = 2(50)(3)(7270.5)$

$T = 2.181 \times 10^6 \ N.mm$

$T = 2.181 \ KN.m$



$$\Phi = \frac{TL}{4GA_m^2} * \sum_{i=1}^{n} \frac{s_i}{t_i}$$

$$T = \frac{4 \times \Phi \times G \times A_m^2}{L \sum_{i=1}^{n} \frac{s_i}{t_i}}$$

$$T = \frac{4(0.003)(40 \times 10^3)(7270.5)^2}{(4 \times 10^3)\left(\frac{146.5}{4} + \frac{113.5}{4} + \frac{33}{3} + \frac{96.5}{3} + \frac{60.5}{3} + \frac{36}{4}\right)}$$

$T = 46.2 \times 10^6 \ N.m = 46.2 \ KN.m$

T_{max} is the smaller $\Rightarrow$ $\boxed{T_{(max)} = 2.18 \ KN.m}$

Problem 2:-

Given :- A shaft of I cross section is subjected to a torque (T) as shown.

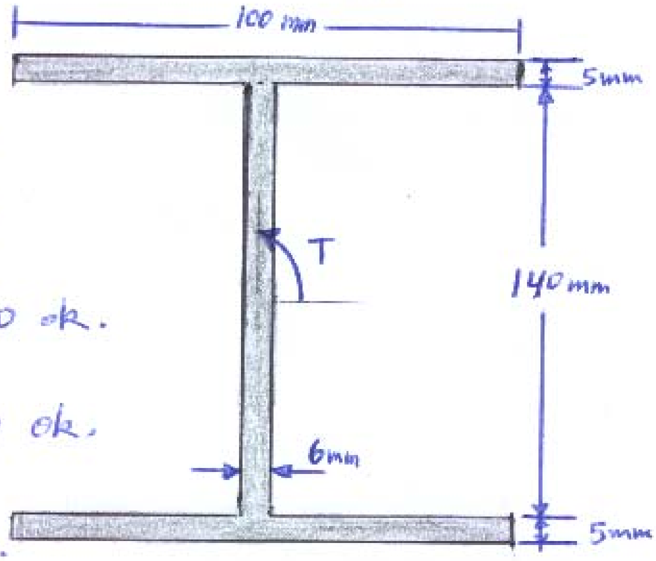
$G = 28$ Gpa.

Req. :- The max torque (T), if

(i) $\tau_{(yield)} = 200$ Mpa; F.S. = 2

(ii) $\Delta\Phi_{\substack{allow \\ ends}} \leq 6$ degree over its (2m) length.

Solution:-



First, check if the section is narrow rectangular section.

- Flange :- $\frac{b}{a} = \frac{100}{5} = 20 > 10$ ok.

- Web :- $\frac{b}{a} = \frac{140}{6} = 23.33 > 10$ ok.

∴ Apply narrow rectangular theory.

(i) $\tau_{all.} = \frac{\tau_y}{F.S.} = \frac{200}{2} = 100$ Mpa.

$$J = \sum_{i=1}^{3} J_i = \sum_{i=1}^{3} \left(\tfrac{1}{3} b h^3\right)_i = \tfrac{1}{3}(2 * 100 \times 5^3 + 140 \times 6^3)$$

$J = 18413 \ mm^4$

$$\tau_{max} = \frac{Th}{J} \leq \tau_{all.} \Rightarrow T \leq \frac{\tau_{all.} \times J}{h}$$

$$T_{max} = \frac{\tau_{all.} \times J}{h} = \frac{100 \times 18413}{6} = 306883 \ N.mm$$

∴ $T_{max} = 306.883$ N.m

(ii) $$\frac{d\Phi}{dz} = \frac{T}{JG} \leq \left(\frac{d\Phi}{dz}\right)_{allow.} = \frac{\Delta\Phi_{ends}}{L}$$

$$\therefore \frac{T}{JG} = \frac{\Delta\Phi_{ends}}{L} \Rightarrow T = \frac{JG\Delta\Phi_{ends}}{L}$$

$$\Delta\Phi_{ends\,(all)} = 6 \text{ degree.}$$

$$= \frac{6\times\pi}{180} = 0.10472 \text{ rad.}$$

$$T_{(max)} = \frac{(18413)(28\times10^3)(0.10472)}{2\times10^3} = 26994.9 \text{ N.mm}$$

$$\therefore \boxed{T^{ii}_{(max)} = 26.995 \text{ N.m}}$$

Note that $T_{max} = T_{min}(T_i, T_{ii})$

$$\Rightarrow \boxed{T_{max} = 27.0 \text{ N.m}}$$

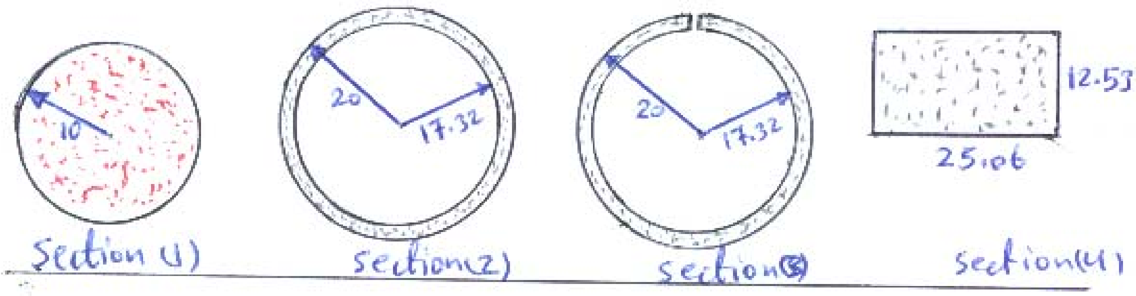
Problem 3 :-

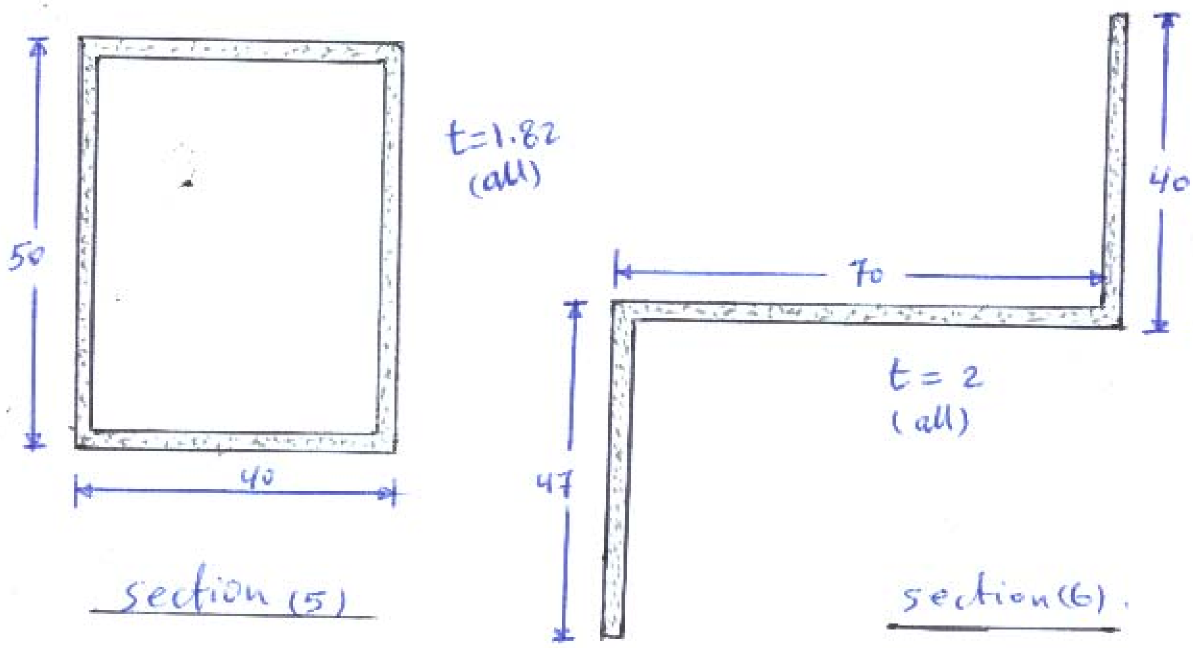
Given :- The length, the applied torque, and the material are the same regardless of the section chosen. Note that the material areas of the sections are all the same.

Req. :- a) Order the sections in terms of their efficiency in shear, from best to worst, prove your order!

b) Order the sections in terms of their efficiency in rotation (angle of twist); from best to worst, show why they are in this order!

c) What conclusions/recommendations can you make?





Solution:-

a) <u>section 1:-</u>

$$\tau_{max} = \frac{T r_{max}}{J} = \frac{(10)\,T}{\frac{\pi}{2}(10)^4} = 6.3662 \times 10^{-4}\,T$$

<u>section 2:-</u>

(i) hollow circular:

$$\tau_{max} = \frac{20\,T}{\frac{\pi}{2}(20^4 - 17.32^4)} = 1.8186 \times 10^{-4}\,T$$

(ii) thin-walled closed:

$$\tau = \frac{T}{2tA_m} = \frac{T}{2(2.68)\left[\pi\left(\frac{20+17.32}{2}\right)^2\right]} = 1.7055 \times 10^{-4}\,T$$

<u>Section 3:-</u>

$$\frac{b}{h} = \frac{2\pi r_c}{h}, \quad h = (\text{thickness}) = 20 - 17.32 = 2.68$$

$$r_c = \frac{20 + 17.32}{2} = 18.66$$

$$\frac{b}{h} = \frac{2\pi(18.66)}{2.68} = 43.75 > 10$$

we use narrow rectangular theory.

$$\tau_{max} = \frac{Th}{J} = \frac{(2.68)\,T}{(\frac{1}{3})(2\pi \times 18.66)(2.68)^3} = 35.62 \times 10^{-4}\,T$$

<u>section 4:-</u> (solid non circular section)

$$\frac{b}{a} = \frac{25.06}{12.53} = 2.0, \text{ so use Table 4.2}$$

$$\alpha = 0.246$$

$$\tau_{max} = \frac{T}{\alpha d a^2} = \frac{T}{(0.246)(25.06)(12.53)^2} = 10.33 \times 10^{-4}\,T$$

Section 5:- (Thin-walled closed section)

$$\tau_{max} = \frac{T}{2t\,A_m}$$

$A_m = (50-t)(40-t) = (50-1.82)(40-1.82)$
$A_m = 1839.51$

$$\tau_{max} = \frac{T}{2(1.82)(1839.51)} = 1.4935 \times 10^{-4}\,T$$

Section 6:-

$$\frac{b_{min}}{h} = \frac{40}{2} = 20 > 10$$

use narrow rectangular section.

$$J = \sum_{i=1}^{3} J_i = \sum_{i=1}^{3} \left(\tfrac{1}{3} b . h^3\right) = \tfrac{1}{3}[47 + 70 + 40] * (2)^3$$

$J = 418.67$.

$$\tau_{max} = \frac{Th}{J} = \frac{T(2)}{418.67} = 47.77 \times 10^{-4}\,T$$

∴ The ordering from best to worst is:-

section ⑤, section ②, section ①, section ④; section ③, section ⑥.

Note that section ② was solved by two different theories. However, hollow circular section theory is more accurate than thin-walled closed section theory as it involves no approximation in the theory (derivation)

b) section 1:-

$$\frac{d\phi}{dz} = \frac{T}{JG} = \frac{T}{\frac{\pi}{2}(10)^4 G} = 6.366 \times 10^{-5} \frac{T}{G}$$

section 2:-

(i) $$\frac{d\phi}{dz} = \frac{T}{\frac{\pi}{2}(20^4 - 17.32^4)G} = 0.9093 \times 10^{-5} \frac{T}{G}$$

(ii) $$\frac{d\phi}{dz} = (T/4GA_m^2)\frac{s}{t} = 0.91401 \times 10^{-5} \frac{T}{G}$$

section 3:-

$$\frac{d\phi}{dz} = \frac{T}{\frac{1}{3}(2\pi \times 18.66)(2.68)^3 \times G} = 132.9 \times 10^{-5} \frac{T}{G}$$

section 4:-

$$\frac{d\phi}{dz} = \frac{T}{\beta b a^3 G} \quad ; \frac{b}{a} = 2 \text{, from Table 4.2}$$

$\beta = 0.229$.

$$\frac{d\phi}{dz} = \frac{T}{(0.229)(25.06)(12.53)^3 G} = 8.858 \times 10^{-5} \frac{T}{G}$$

section 5:- Thin-walled closed section.

$$\frac{d\phi}{dz} = \frac{T \sum \frac{s_i}{t_i}}{4G A_m^2}$$

$$\sum_i \frac{s_i}{t_i} = \sum \frac{s_i}{t} = \frac{2(40-1.82) + 2(50-1.82)}{1.82} = 94.90$$

$$\frac{d\phi}{dz} = \frac{(T)(94.90)}{4G(1839.51)^2} = 0.7011 \times 10^{-5} \frac{T}{G}$$

section 6:-

$$\frac{d\phi}{dz} = \frac{T}{JG} = \frac{T}{(418.67)G} = 238.85 \times 10^{-5} \frac{T}{G}$$

$\therefore$ The best section is the one with minimum $\frac{d\phi}{dz}$

The ordering from best to worst is the same as for shear i.e.

section ⑤, section ②, section ①, section ④

section ③, section ⑥.

C) * We can notice that the order of section interms of their efficiencies in shear and rotation are the same.

* Also we can see from above that thin-walled sections including circular are most efficient, followed by solid sections, we can see that the circular shape is more efficient, this can be seen if the 20-radius of section (2) increased to 40 and 50 (as in section (5)). The worst sections are the thin-walled (narrow) open sections; they are weak in torsion.