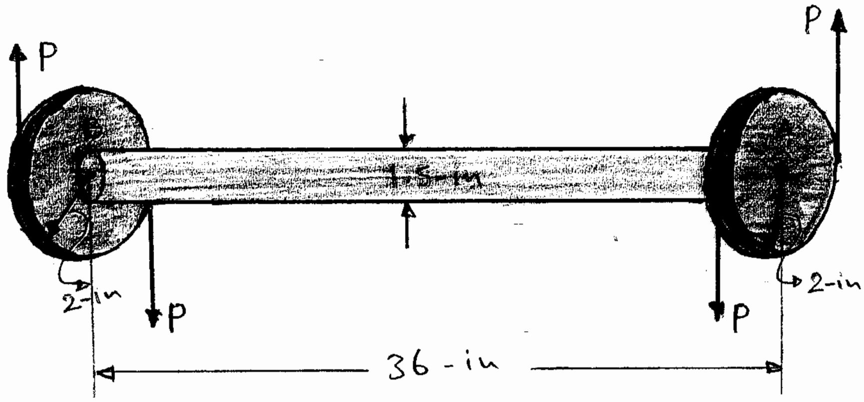
CE 203 -3[072]
H.W.# 6-Solution

Problem 1:-

Given :- Pulleys are securely attached to 1.5-in diameter aluminium ($G = 4 \times 10^6$ psi) rod as shown in Fig. Four vertical forces of Magnitude (P) cause the pulley at A to rotate 5 degree relative to the pulley at B.

Req.:- The value of (P).
The maximum shear stress in the rod.

Solution:-



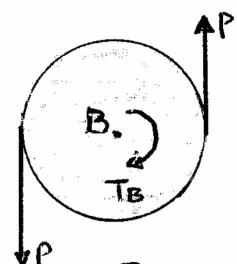
$\Sigma M_B = 0$

$-T_B + 2P(2) = 0$

$T_B = 4P$

$\Phi_A = 5$ degree

$\Phi_A = \frac{5 \times \pi}{180} = 0.0872665$ rad.



F.B.D.

$\Phi_A = \frac{T L}{J G}$

$J = \frac{\pi}{2} \left(\frac{1.5}{2}\right)^4.$

$$0.0872665 = \frac{(4P)(36)}{\frac{\pi}{2}(0.75)^4 (4 \times 10^6)}$$

$\therefore P = 1204.79$ Ib

$$\boxed{P = 1.205 \text{ kip}}$$

$$\tau_{max} = \frac{T c}{J} = \frac{(4 \times 1204.79) \times (0.75)}{\frac{\pi}{2}(0.75)^4}$$

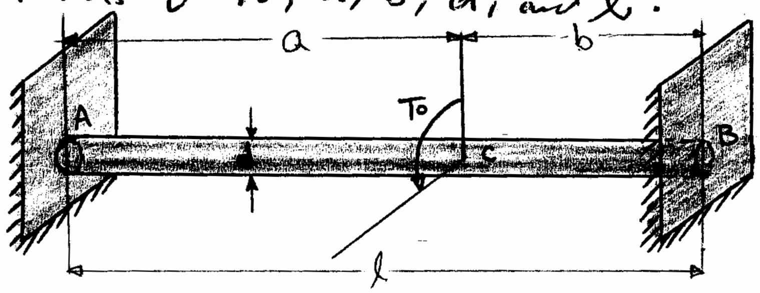
$$\boxed{\tau_{max} = 7.272 \text{ ksi}}$$

Problem 2:-

Given :- The shaft shown in Fig. has support A and B.

Req :- - Formulas for the reactions of the rigid support at A and B on the shaft.

- Formula for the max shear stress. Express your results in terms of T_o, a, b, d, and ℓ.

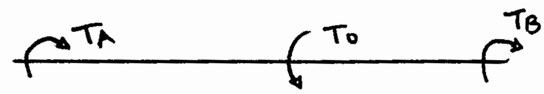


Solution:-

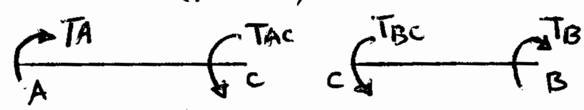
From (F.B.D.1)

Take $\Sigma M = 0$

$$T_o - T_A - T_B = 0 \quad \text{----(1)}.$$



(F.B.D.1)



(F.B.D.2)

Use the material behavior from (F.B.D.2), we know that the internal torque in segment AC, $(T_{AC}) = T_A$ and in segment BC, $(T_{BC}) = -T_B$.

Since the end supports are fixed, therefore.

$$\Phi_{A/B} = 0 \quad \text{-----} (2).$$

$$\Phi_{A/B} = \Sigma \Phi = 0 \quad ; \quad \frac{T_{AC}\, a}{JG} + \frac{T_{BC}\, b}{JG} = 0$$

$$\Rightarrow \quad \frac{T_A\, a}{JG} - \frac{T_B\, b}{JG} = 0 \quad \Rightarrow \quad T_A = \frac{T_B\, b}{a} \quad \text{----(3)}$$

solving eq. (3) with eq. (1). where $\ell = (a+b)$.

$$T_o - \frac{T_B\, b}{a} - T_B = 0 \quad \Rightarrow \quad T_o = \frac{T_B\, b + T_B\, a}{a}$$

$$a\, T_o = T_B(b+a) \quad \Rightarrow \quad a\, T_o = T_B\, \ell$$

$$\therefore \boxed{T_B = \frac{(T_o)(a)}{\ell}} \quad \text{and} \quad \boxed{T_A = \frac{(T_o)(b)}{\ell}}$$

$$\tau_{max} = \frac{T c}{J} \quad \text{-----} \; (4).$$

$$c = \frac{d}{2}$$

$$J = \frac{\pi}{2} \left(\frac{d}{2} \right)^4$$

$T = T_A$ or $T = T_B$; subtitution in eq (4).

$$\tau_{max} = \frac{(T_A)\,(d/2)}{\frac{\pi}{2}\,(d/2)^{\cancel{4}\,3}} = \frac{(T_o\, b)/l}{\frac{\pi}{2}\,(d/2)^3}$$

$$\boxed{\tau_{max} = \frac{16}{\pi} \frac{b T_o}{l d^3}}$$

or

when we use $T = T_B$

$$\tau_{max} = \frac{(T_B)\,\cancel{(d/2)}}{\frac{\pi}{2}\,(d/2)^{\cancel{4}\,3}} = \frac{(T_o\, a)/l}{\frac{\pi}{2}\,(d/2)^3}$$

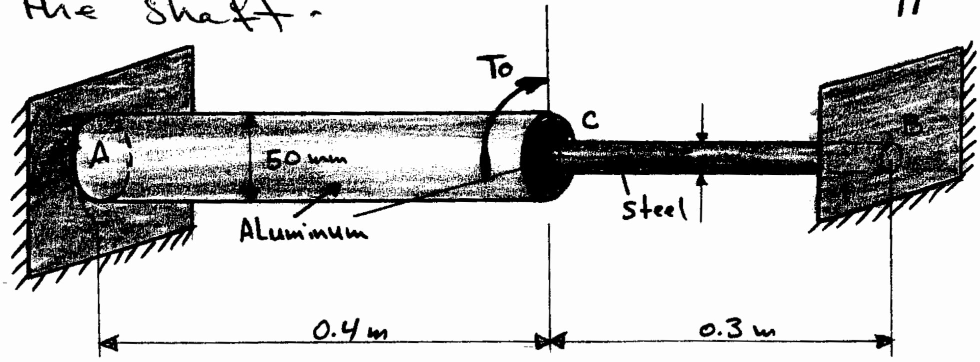
$$\therefore \boxed{\tau_{max} = \frac{16}{\pi} \frac{a T_o}{l d^3}}$$

Problem 3:-

Given :- The allowable shear stress for either the aluminum or steel portions of step-shaft shown is (55 Mpa).

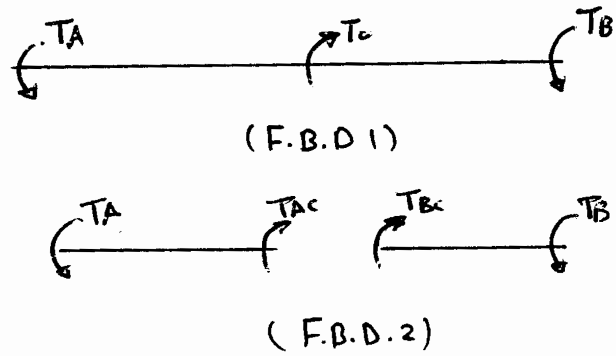
$G_{AL} = 28$ GPa , $G_{st} = 84$ Gpa.

Req :- The maximum permissible To that can be applied to the shaft.



solution:-

Equillibrium:-



$T_A + T_B - T_o = 0$ ----(1)

$T_{AC} = T_A$ (internal torque)

$T_{BC} = -T_B$ (internal torque)

Geometry and compatibility:-

$$\Phi = \frac{TL}{JG}$$

$$\Phi_{A/B} = \Sigma \Phi_i = 0$$

$$= \frac{T_{AC}(0.4)}{\frac{\pi}{2}(0.025)^4 \times 28\times10^9} - \frac{T_{BC}(0.3)}{\frac{\pi}{2}(0.0125)^4 \times 84\times10^9} = 0$$

$\therefore$ $36571.429\, T_{AC} = 146285.714\, T_{BC}$

$\therefore$ $T_{AC} = 4\, T_{BC}$ and $T_{BC} = 0.25\, T_{AC}$ ----(2)

subtitution in eq (1). interm of T_{BC}.

$4T_{BC} + T_{BC} - T_o = 0 \Rightarrow T_o = 5\, T_{BC}$

Assume T_{max} is in shaft BC [steel]

$$\tau_{max} \equiv \tau_{allow} = \frac{T_{BC}\,(r_{max})}{J}$$

$$\therefore \quad T_{BC} = \frac{(\tau_{allow})(J)}{r_{max}} = \frac{(55)(10^6)(\frac{\pi}{2})(0.0125)^4}{(0.0125)}$$

$T_{BC} = 168.74$ N.m

subtitute T_{BC} in T_o

$T_o = 5\,T_{BC} \Rightarrow T_o = 5(168.74)$

$T_o = 843.7$ N.m

$\therefore \quad T_o = 1.25\,T_{AC}$; from eq (1) and (2).

$T_{AC} = 0.8\,T_o \Rightarrow T_{AC} = 0.8(843.7)$

$T_{AC} = 674.96$ N.m.

Check τ_{max} in segment AC [Aluminum]

$$\tau_{max\,(AC)} = \frac{T_{AC}\;r_{max}}{J} = \frac{(674.96)(0.025)}{\frac{\pi}{2}(0.025)^4}$$

$\tau_{max\,(AC)} = 27.5$ Mpa.

" $27.5 < 55$ "

which means that steel will control.

$\Rightarrow (\tau_{max})$ will be in segment BC [steel]

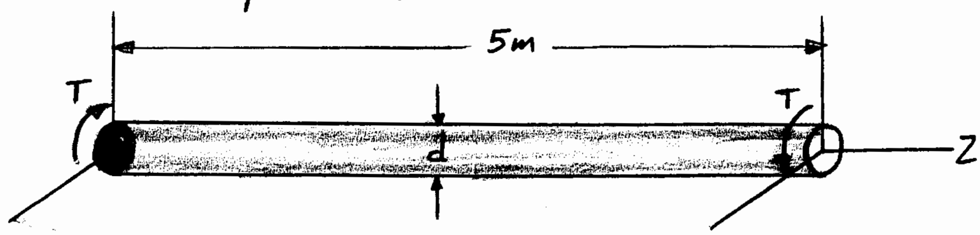
$\therefore \quad \boxed{T_{o(max)} = 843.7 \text{ N.m}}$

Problem 4 :-

Given :- The steel shaft is required to transmit 20π HP at 5.5 Hz.
If the allowable angle of twist per meter of shaft is not to exceed 4.5 degree, and if the allowable shear stress is not to exceed 84 Mpa,

Req :- The minimum permissible diameter (d).

solution :-



1 HP = 745.7 watt

20π HP = ?? power $\Rightarrow$ $\therefore$ power = 46.8537 K watt

power = 46.8537 KN.m/s

$$\omega = 2\pi f = 2\pi(5.5) \Rightarrow \omega = 34.56 \text{ rad/s}$$

$$P = T\omega \Rightarrow T = \frac{P}{\omega}$$

$$T = \frac{46.8537 \text{ KN.m/s}}{34.56 \text{ rad/s}} \Rightarrow T = 1.3558 \text{ KN.m}$$

$$T = 1355.8 \text{ N.m}$$

$$\tau = \frac{Tr}{J} \quad \text{where} \quad \tau_{max} = \frac{T\, r_{max}}{J} \equiv \tau_{allow}$$

$$r_{max} = \frac{\tau_{max} \times J}{T} = \frac{r_{max} \times T}{\tau_{max}} = \frac{\pi}{2}(r_{max})^{4^{3}}$$

$$\therefore \quad r_{max}^{3} = \frac{2T}{\pi\,\tau_{max}} \Rightarrow r_{max} = \sqrt[3]{\frac{2(1355.8)}{\pi(84\times10^{6})}}$$

$$r_{max} = 0.02174 \text{ m}$$

$\therefore \quad r_{max} = 21.74 \text{ mm}$

$d_1 = 2 r_{max} \quad \Rightarrow \quad d_1 = 2(21.74)$

$\underline{d_1 = 43.48 \text{ mm}}$

$\Phi_{1m} = $ "4.5 deg/m"

$\Phi_{5m} = 5(4.5) \quad \Rightarrow \quad \Phi_{5m} = 22.5 \text{ deg}.$

$\therefore \Phi = 0.392699081 \text{ rad}$

$\Phi = \dfrac{TL}{GJ} \quad \Rightarrow \quad 0.392699081 = \dfrac{(1355.8)(5)}{(84 \times 10^9)(J)}$

$\therefore J = 0.2055 \times 10^{-6}$

$\dfrac{\pi}{2}(r)^4 = 0.2055 \times 10^{-6}$

$r = 0.019019 \text{ m}$

$r = 19.019 \text{ mm}$

$d_2 = 2 \times r = 2(19.019)$

$\underline{d_2 = 38.037 \text{ mm}}$

$\therefore$ d_{min} is the bigger of d_1, d_2

$$\boxed{d_{min} = 43.48 \text{ mm}}$$

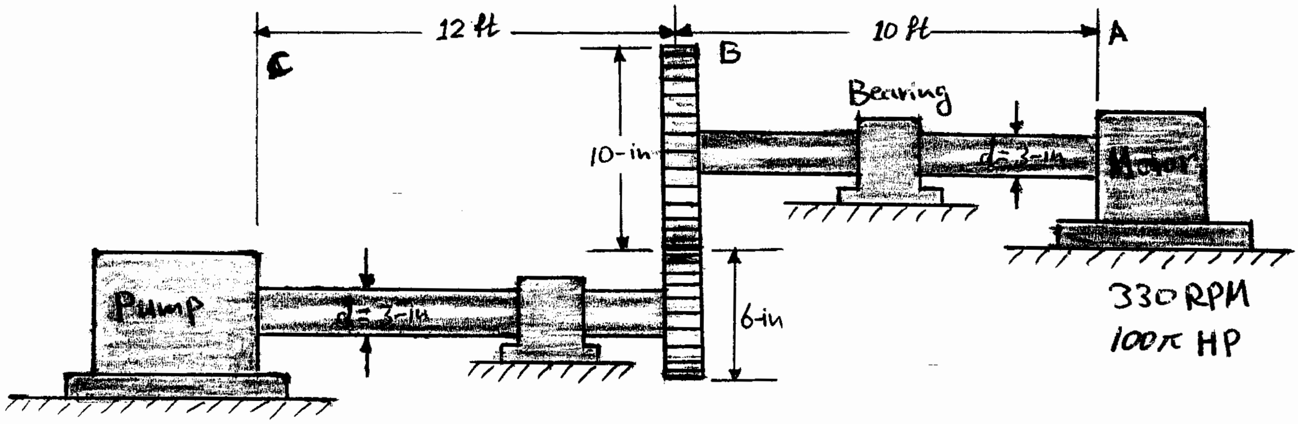
Problem 5:-

Given :- A pump is connected to an electric motor through steel shafting as shown in Fig. An offset necessary to connect the pump to its power source is provided by the gear arrangement as shown. If the motor delivers (100π HP) at 330 rpm at its shaft.

$G_{st} = 12 \times 10^6$ psi

Req. : a) The maximum shear stress in the shaft.

b) The relative rotation of sections A and C



Solution:-

a) $$HP_{AB} = \frac{nT}{63\,000} \Rightarrow (100\pi) = \frac{(330)\,T_{AB}}{63\,000}$$

$$\therefore T_{AB} \simeq \frac{(100\pi)(63\,000)}{330} \Rightarrow T_{AB} = 59976 \text{ in-Ib}$$

$$\therefore T_{AB\,(approximate)} = 59976 \text{ in-Ib}$$

$$T_{AB\,(exact)} = \frac{P}{2\pi n} = \frac{(550)(100\pi)(12)(60)}{2\pi(330)}$$

$$\therefore T_{AB\,(exact)} = 60\,000 \text{ in-Ib}$$

$$\tau_{AB\,(max)} = \frac{T_{AB(exact)} * C}{J} = \frac{(60\,000)(1.5)}{\frac{\pi}{2}(1.5)^4}$$

$\tau_{AB\,(max)} = 11\,318 \text{ psi}$

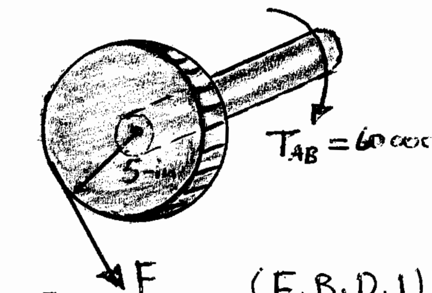
$\underline{\tau_{AB\,(max)} = 11.318 \text{ Ksi}}$

* In (F.B.D.1)

$\Sigma M_z = \Sigma T = 0$

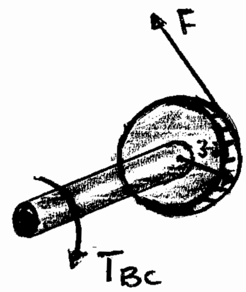
$60000 - (5)F = 0$

$F = 12\,000 \text{ Ib}$



(F.B.D.1)

* In (F.B.D.2)



(F.B.D.2)

$\Sigma M_z = \Sigma T = 0$

$T_{BC} - 12000(3) = 0$

$\therefore T_{BC} = 36\,000 \text{ in-Ib}$

$$\tau_{BC\,(max)} = \frac{T_{BC} \times C_{max}}{J} = \frac{(36\,000)(1.5)}{\frac{\pi}{2}(1.5)^4}$$

$\tau_{BC\,(max)} = 6791 \text{ psi}$

$\underline{\tau_{BC\,(max)} = 6.791 \text{ Ksi}}$

$\therefore$ τ_{max} is at segment (AB)

$$\boxed{\tau_{max} = 11.318 \text{ Ksi}}$$

b) $\Phi_{A/C} = \Phi_{AB} + \Phi_{\substack{gear\ (10\text{-}in) \\ @\ B}}$

$$\Phi_{A/B} = \left(\frac{TL}{GJ}\right)_{AB} = \frac{(60\,000)(10 \times 12)}{\frac{\pi}{2}(1.5)^4(12 \times 10^6)}$$

$$\Phi_{A/B} \simeq 0.07545 \text{ rad.}$$

* For the two gears (10-in and 6-in)

$$\frac{\Phi_{10\text{-}in}}{\Phi_{6\text{-}in}} = \frac{6}{10} = \frac{3}{5}$$

$$\therefore \Phi_{10\text{-}in} = \left(\frac{3}{5}\right) * \Phi_{6\text{-}in}$$

$$\Phi_{6\text{-}in} = \left(\frac{TL}{GJ}\right)_{BC} = \Phi_{BC} = \frac{(36000)(12)(12)}{\frac{\pi}{2}(1.5)^4(12 \times 10^6)}$$

$$\Phi_{6\text{-}in} = 0.0543249 \text{ rad.}$$

$$\therefore \Phi_{10\text{-}in} = \left(\frac{3}{5}\right)(0.0543249)$$

$$\Phi_{10\text{-}in} = 0.032595 \text{ rad.}$$

$$\therefore \Phi_{A/C} = 0.07545 + 0.032595$$

$$\Phi_{A/C} = 0.108045 \text{ rad}$$

$$\boxed{\Phi_{A/C} = 6.191^\circ}$$