CE 203 - 3 [072]

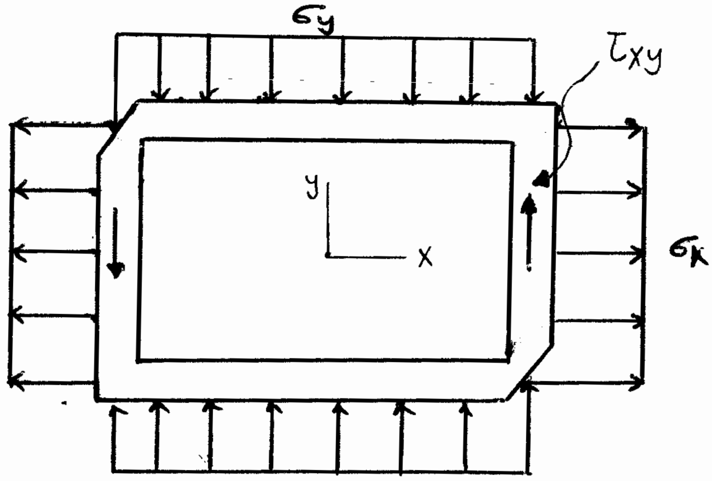
H. W. # 5 - solution

problem 1:-

Given:- A plate with $E = 20\times10^{6}$ psi and $\nu = 0.25$ is subjected to a plane state of stress as shown in fig. If $\sigma_x = 40$ ksi (T), $\sigma_y = 30$ ksi (C), and $\tau_{xy} = 20$ ksi,

Req:- The strains ε_x, ε_y, γ_{xy}, and ε_z.

solution:-



First: $\sigma_z = 0.0$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$= \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$= \frac{40\times10^{3}}{20\times10^{6}} - \frac{0.25}{20\times10^{6}}(-30\times10^{3}+0)$$

$$\varepsilon_x = 2\times10^{-3} + 0.375\times10^{-3}$$

$$\boxed{\varepsilon_x = 2.375\times10^{-3} \text{ in/in}}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$= \frac{-30\times10^{3}}{20\times10^{6}} - \frac{0.25}{20\times10^{6}}(40\times10^{3}+0) = -1.5\times10^{-3} - 0.5\times10^{-3}$$

$$\boxed{\varepsilon_y = -2.0\times10^{-3} \text{ in/in}}$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad ; \quad G = \frac{E}{2(1+\nu)}$$

$$G = \frac{20\times10^{6}}{2(1+0.25)} \implies G = 8\times10^{6}\ \text{psi}$$

$$\gamma_{xy} = \frac{1}{8\times10^{6}}(20\times10^{3})$$

$$\boxed{\gamma_{xy} = 2.5\times10^{-3}\ \text{rad}}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$= 0 - \frac{0.25}{20\times10^{6}}(40\times10^{3} - 30\times10^{3})$$

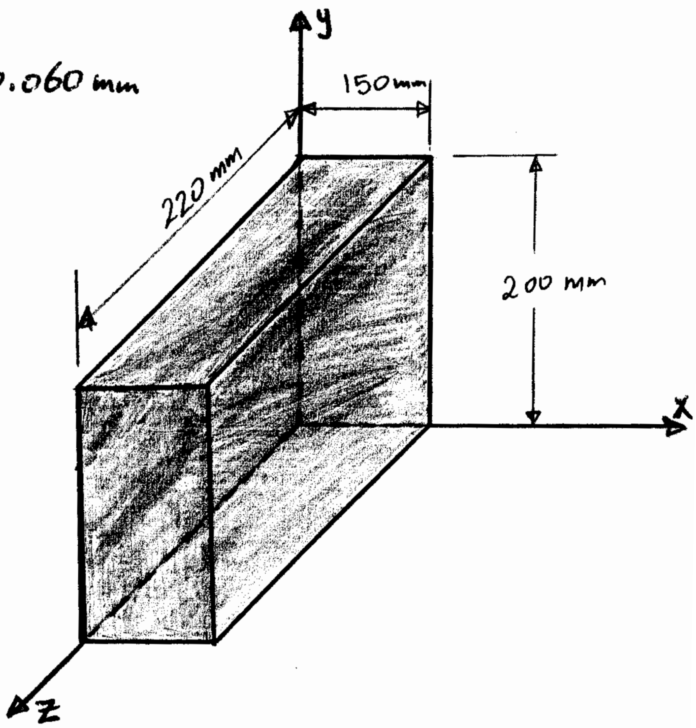
$$\boxed{\varepsilon_z = -1.25\times10^{-4}\ \text{in/in}}$$

Problem 2:-

Given :- The x, y, and z dimensions of the block shown in fig. become 150.060 mm, 199.800 mm, and 220.040 mm, respectively, due to σ_x, σ_y, and σ_z that are applied uniformly to its faces. Knowing that $E = 250$ Gpa and $\nu = 0.35$.

Req. :- (a) The normal strains ε_x, ε_y, and ε_z.
(b) The corresponding stresses σ_x, σ_y and σ_z.

Solution:-



a) $\delta_x = 150.060 - 150 = 0.060$ mm

$$\varepsilon_x = \frac{\delta_x}{l_{x_0}} = \frac{0.060}{150}$$

$$\boxed{\varepsilon_x = 4.0 \times 10^{-4} \text{ mm/mm}}$$

$$\delta_y = 199.80 - 200 = -0.20$$

$$\varepsilon_y = \frac{\delta_y}{l_{y_0}} = -\frac{0.20}{200}$$

$$\boxed{\varepsilon_y = -1.0 \times 10^{-3} \text{ mm/mm}}$$

$$\delta_z = 220.040 - 220 = 0.040$$

$$\varepsilon_z = \frac{\delta_z}{l_{z_0}} = \frac{0.040}{220}$$

$$\boxed{\varepsilon_z = 1.8182 \times 10^{-4} \text{ mm/mm}}$$

b) $$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y+\varepsilon_z)\right]$$

$$\sigma_x = \frac{250\times10^{3}}{(1+0.35)(1-2\times0.35)}\left[(1-0.35)(4\times10^{-4})+(0.35)(-1\times10^{-3}+1.8182\times10^{-4})\right]$$

$$\boxed{\sigma_x = -16.27 \text{ Mpa "c"}}$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_x+\varepsilon_z)\right]$$

$$\sigma_y = \frac{250\times10^{3}}{(1+0.35)(1-2\times0.35)}\left[(1-0.35)(-1\times10^{-3})+(0.35)(4\times10^{-4}+1.8182\times10^{-4})\right]$$

$$\boxed{\sigma_y = -275.5 \text{ Mpa "c"}}$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x+\varepsilon_y)\right]$$

$$\sigma_z = \frac{250\times10^{3}}{(1+0.35)(1-2\times0.35)}\left[(1-0.35)(1.8182\times10^{-4})+(0.35)(4\times10^{-4}-1\times10^{-3})\right]$$

$$\boxed{\sigma_z = -56.68 \text{ Mpa "c"}}$$

Problem 3:-

Given :- The material of the block shown in fig. has $E = 10,000$ ksi and $\nu = 0.33$.

Req. :- a) The normal strains in the x, y, and z directions.
b) The new dimensions of the block.
c) The normal strain in the diagonal AB.

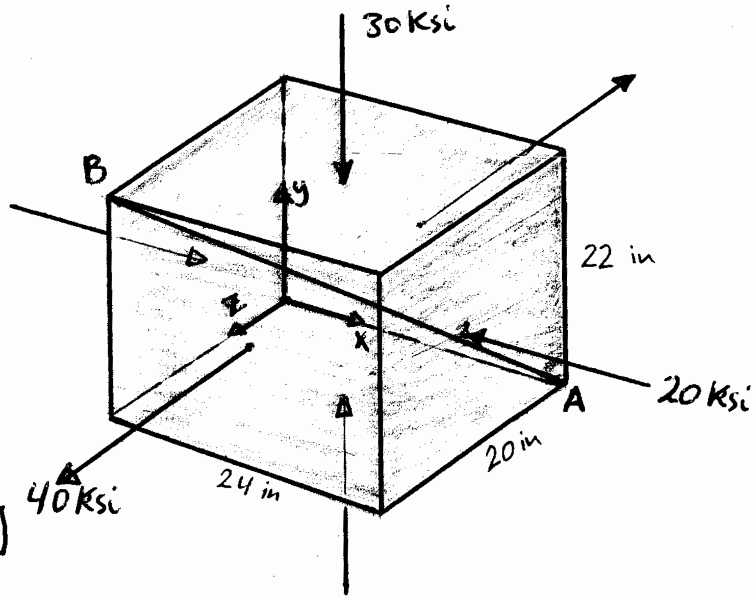
Solution:-

First we have to put the signs of stresses due to there directions.

$\sigma_x = -20$ ksi

$\sigma_y = -30$ ksi

$\sigma_z = +40$ ksi



a) $\varepsilon_x = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$

$\varepsilon_x = \frac{1}{10\,000}\left[-20 - 0.33(-30+40)\right]$

$\boxed{\varepsilon_x = -2.33 \times 10^{-3} \text{ in/in}}$

$\varepsilon_y = \frac{1}{10\,000}\left[-30 - 0.33(-20+40)\right]$

$\boxed{\varepsilon_y = -3.66 \times 10^{-3} \text{ in/in}}$

$\varepsilon_z = \frac{1}{10\,000}\left[40 - 0.33(-20-30)\right]$

$\boxed{\varepsilon_z = 5.65 \times 10^{-3} \text{ in/in}}$

b) $\varepsilon = \frac{\delta}{l_o} = \frac{l_f - l_o}{l_o}$

$l_f - l_o = \varepsilon l_o$

$l_f = l_o + \varepsilon l_o$ (General)

$x_f = x_o + \varepsilon_x x_o$
$= 24 + (-2.33 \times 10^{-3})(24) \Rightarrow \boxed{x_f = 23.944 \text{ in}}$

$y_f = y_o + \varepsilon_y y_o$
$= 22 + (-3.66 \times 10^{-3})(22) \Rightarrow \boxed{y_f = 21.919 \text{ in}}$

$z_f = z_o + \varepsilon_z z_o$
$= 20 + (5.65 \times 10^{-3})(20) \Rightarrow \boxed{z_f = 20.113 \text{ in}}$

c) First:
Find the initial length of AB.

$AB^{(i)} = \sqrt{(20)^2 + (24)^2 + (22)^2} = 38.2099 \text{ in}$

Second:
Find the final length of AB.

$AB^{(f)} = \sqrt{(23.944)^2 + (21.919)^2 + (20.113)^2} = 38.1876 \text{ in}$

$\therefore \Delta AB = AB^{(f)} - AB^{(i)} = 38.1876 - 38.2099$
$\Delta AB = -0.0223 \text{ in}$

$\varepsilon_{AB} = \frac{\Delta AB}{AB^{(i)}} = -\frac{0.0223}{38.2099}$

$\boxed{\varepsilon_{AB} = -5.836 \times 10^{-4} \text{ in/in}}$

Problem 4:-

Given :- A cylindrical pressure vessel with a thickness of 20 mm is to be made from a material whose ultimate tensile strength is 1600 Mpa. If the cylinder is to be capable of containing a gas at a pressure of 2 Mpa. A safty factor = 4.

Req :- a) The maximum radius for the tank.
b) Re work part (a) but if sphere. What conclusion can you make?

Solution :-

a) Allowable tensile strength $= \dfrac{\text{ultimate}}{\text{F.S.}}$

$$= \frac{1600}{4} = 400 \text{ Mpa.}$$

$$\sigma_{max} = \frac{Pr}{t} \equiv \sigma_{allow} = 400 \text{ Mpa.}$$

$$400 = \frac{2(r)}{20 \times 10^{-3}} \Rightarrow \boxed{r_{max} = 4 \text{ m}}$$

b) $\sigma_{max} \equiv \sigma_{allow} = 400$ Mpa.

$$\sigma_{max} = \frac{Pr}{2t} \Rightarrow 400 = \frac{2(r)}{2(20) \times 10^{-3}}$$

$$\boxed{r_{max} = 8 \text{ m}}$$

Conclusion :-

spheres are more efficient.
spheres, not sylinders, should be used as internal pressure vesseles.

Problem 5:-

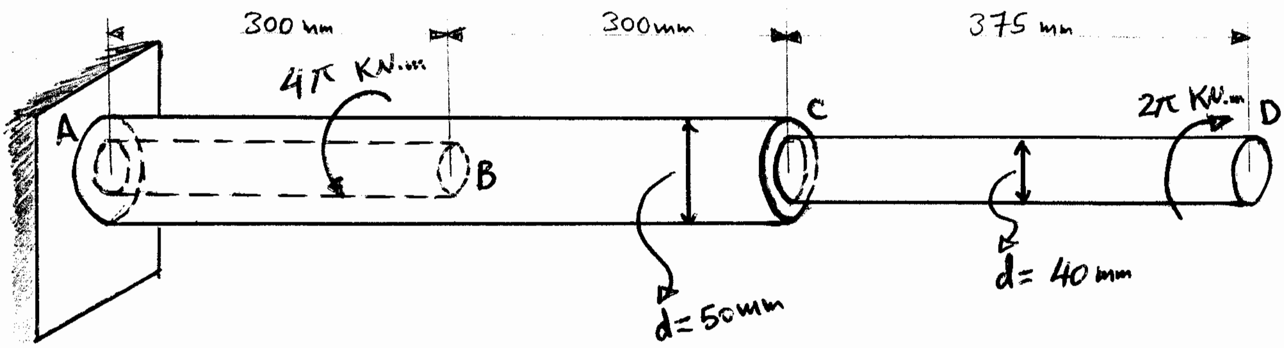
Given:- The torsion step-shaft shown in fig. ($G = 28$ Gpa) has a (25-mm) diameter concentric hole.

Req:- a) Sketch the shear stress distribution for cross section in the portion of the shaft containing the hole.

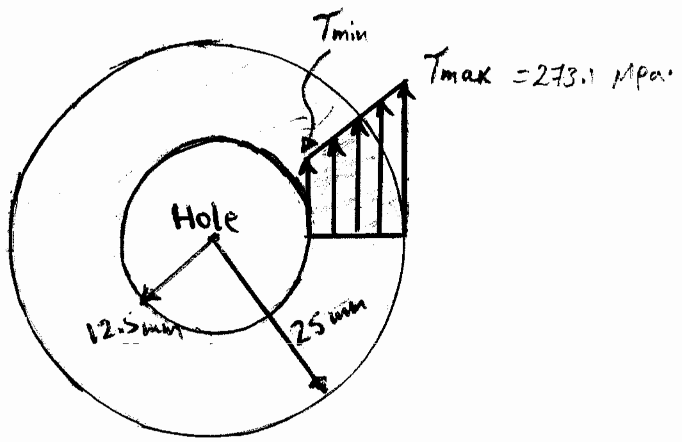
b) The value and Location of the max. shear stress.

c) The angle of twist at the free end of the shaft.

Solution:-

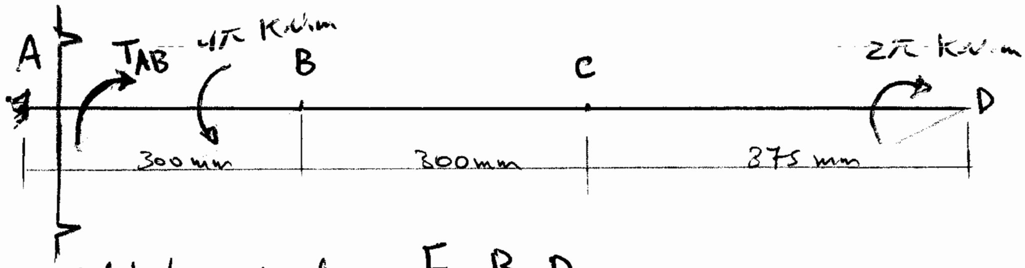


a)



sketch containing the hole, with shear stress distribution.

b)

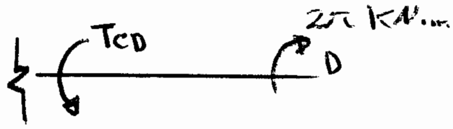


Cut between A and B F.B.D.

Equilibrium:-

$$T_{AB} - 4\pi + 2\pi = 0 \Rightarrow T_{AB} = 2\pi \text{ KN.m}$$

$$T_{CD} - 2\pi = 0 \Rightarrow T_{CD} = 2\pi \text{ KN.m}$$



$$T_{BC} - 2\pi = 0 \Rightarrow T_{BC} = 2\pi \text{ KN.m}$$

$$\tau = \frac{T r}{J}, \quad \tau_{max} = \frac{T\, r_{max}}{J}$$

$$\tau_{AB} = \frac{T_{AB}\, r}{J} = \frac{2\pi \times 10^3 (0.025)}{\frac{\pi}{2}(0.025^4 - 0.0125^4)} = 273.1 \text{ Mpa}$$

τ_{BC} : It is clear that (T_{BC}) does not control (i.e., τ_{max} will not be in BC). Why and how?!

$$\tau_{CD} = \frac{T_{CD}\, r}{J} = \frac{2\pi \times 10^3 (0.02)}{\frac{\pi}{2}(0.02^4)} = 500 \text{ Mpa}$$

$$\therefore \tau_{max} = \tau_{CD}$$

$$\boxed{\tau_{max} = 500 \text{ Mpa}}$$

at the outside radius of portion CD.

c) $\Phi = \Sigma(\Phi_{AB} + \Phi_{BC} + \Phi_{CD})$

$$\Phi = \left(\frac{TL}{JG}\right)_{AB} + \left(\frac{TL}{JG}\right)_{BC} + \left(\frac{TL}{JG}\right)_{CD}$$

$$\Phi = +\frac{2\pi(0.3)\times10^{3}}{\frac{\pi}{2}(0.025^{4}-0.0125^{4})\times28\times10^{9}} - \frac{2\pi(0.3)\times10^{3}}{\frac{\pi}{2}(0.025^{4})\times28\times10^{9}}$$

$$- \frac{2\pi(0.375)\times10^{3}}{\frac{\pi}{2}(0.02^{4})\times28\times10^{9}}$$

$$\Phi = 0.117 - 0.1097 - 0.3348$$

$$\boxed{\Phi = -0.328 \text{ rad.}}$$