CE 203-3 [072]

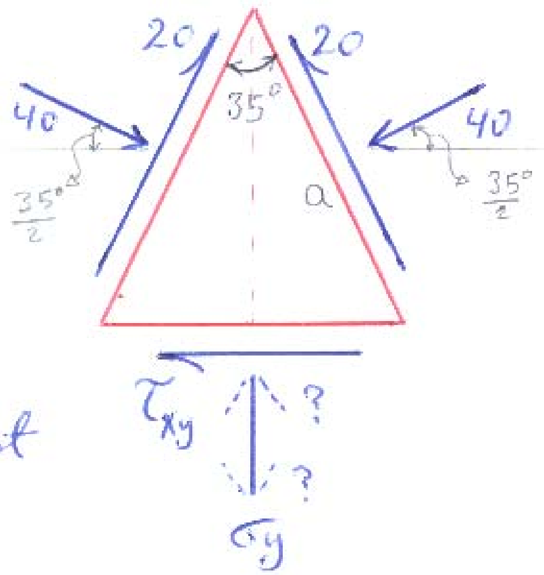
H.W. #12 Solution

Problem 1:-

Given:- For the state of stress shown in Fig.

Req:- σ_y , τ_{xy} , σ_{max} , σ_{min} , and τ_{max}

Solution:-



all in mpa

Considering the equilibrium of the element, the inclined side of the triangle has a value = a.

Note that equilibrium of forces not strains is taken

$\uparrow \Sigma F_V = 0$

$$-\sigma_y \left(2a \sin \frac{35}{2}\right) - 2 \times 40 \times a \sin \frac{35}{2} + 2 \times 20 \times a \cos \frac{35}{2} = 0$$

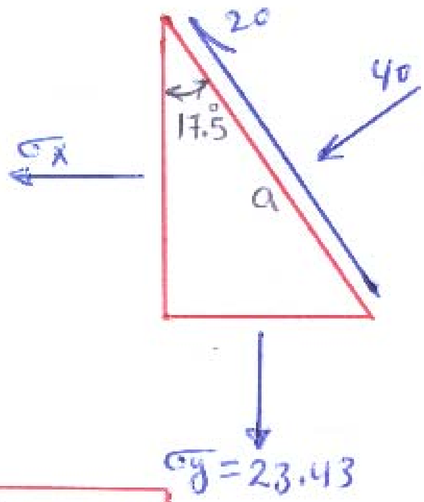
$$\Rightarrow \sigma_y = -40 + 20 \frac{\cos 17.5}{\sin 17.5} \Rightarrow \boxed{\sigma_y = 23.43 \text{ Mpa (tension)}}$$

$$\Sigma F_H = 0, \Rightarrow \boxed{\tau_{xy} = 0}$$

Since $\tau_{xy} = 0$, $\Rightarrow$ σ_y is a principal stress.

We get σ_x which is the other principal stress by considering the equilibrium of the following element

$\overset{+}{\leftarrow} \Sigma F_H = 0 ;$



$$\sigma_x (a \cos 17.5) + 20 a \sin 17.5 + 40 a \cos 17.5 = 0$$

$$\Rightarrow \sigma_x = -40 - 20 \frac{\sin 17.5}{\cos 17.5}$$

$$\therefore \sigma_x = -46.31 \text{ Mpa}$$

$$\therefore \boxed{\sigma_{max} = 23.43 \text{ Mpa}} , \boxed{\sigma_{min} = -46.31 \text{ Mpa}}$$

$$\tau_{\substack{max \\ min}} = \pm \left(\frac{\sigma_{max} - \sigma_{min}}{2} \right) = \pm \left(\frac{23.43 + 46.31}{2} \right) \ldots$$

$$\tau_{\substack{max \\ min}} = \pm 34.87 \text{ Mpa.}$$

$$\therefore \boxed{\tau_{max} = 34.87 \text{ Mpa.}}$$

$$\boxed{\tau_{min} = -34.87 \text{ Mpa.}}$$

Problem 2:-

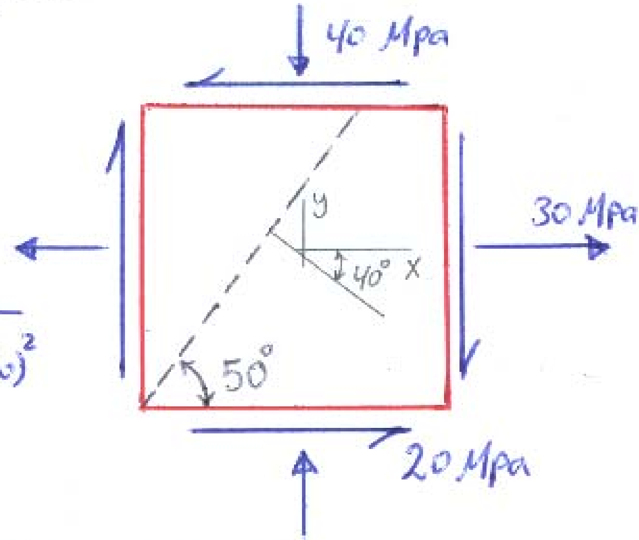
Given:- For the state of stress shown in fig.
Use the equations

Req:- a) The magnitude and direction of the principal normal stresses; show then on a properly oriented element.

b) The magnitude and direction of the maximum (principal) shear stresses; show then on a properly oriented element.

c) The normal and shear stresses on the plane indicated by the dashed line; show then on a properly oriented element.

solution :-



a) $$\sigma_{\substack{max \\ min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{30 - 40}{2} \pm \sqrt{\left(\frac{30+40}{2}\right)^2 + (-20)^2}$$

$$= -5 \pm 40.3$$

$\Rightarrow$ $\boxed{\sigma_{max} = 35.3 \text{ Mpa}}$; $\boxed{\sigma_{min} = -45.3 \text{ Mpa}}$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-20)}{30+40} = -0.5714$$

$$2\theta_{p_1} = 330.26° \, (-29.74°)$$

$$2\theta_{p_2} = 150.25°$$

$\therefore$ $\boxed{\theta_{p_1} = 165°}$; $\boxed{\theta_{p_2} = 75.1°}$

* To check θ_{p1} corresponds to which of the σ_{max} or σ_{min}

$$\sigma_\xi = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta_{p1} + \tau_{xy} \sin 2\theta_{p1}$$

$$= \frac{30 - 40}{2} + \frac{30 + 40}{2} \cos(330.26) + (-20) \sin(330.26)$$

$$= 35.3 = \sigma_{max} \quad \text{ok.}$$

b) $$\tau_{\substack{max \\ min}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\sqrt{\left(\frac{30+40}{2}\right)^2 + (-20)^2}$$

$$= \pm 40.3 \text{ Mpa}$$

$\therefore$ $\boxed{\tau_{max} = 40.3 \text{ Mpa}}$; $\boxed{\tau_{min} = -40.3 \text{ Mpa.}}$

For this case of τ_{max} and τ_{min}

$$\sigma_\xi = \sigma_\eta = \frac{\sigma_x + \sigma_y}{2} = \frac{30 - 40}{2} = -5 \text{ Mpa.}$$

$$\tan 2\theta_s = -\frac{1}{\tan 2\theta_p} = -\frac{1}{(-0.5714)} = 1.75$$

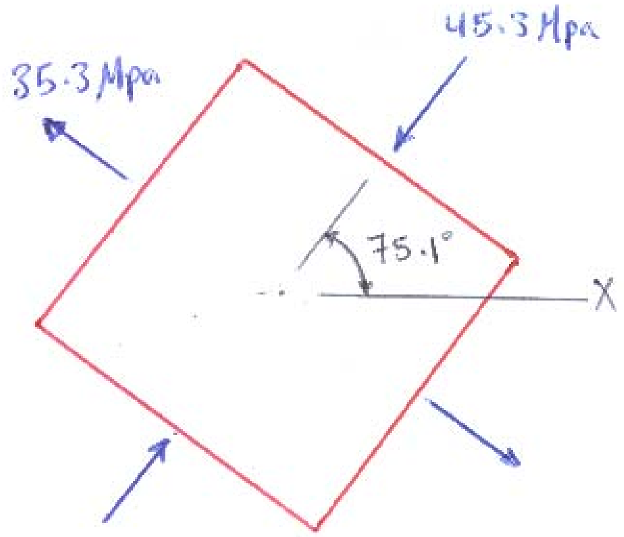
$\therefore$ $\boxed{\theta_{s1} = 30.13^\circ}$; $\boxed{\theta_{s2} = 120.1^\circ}$

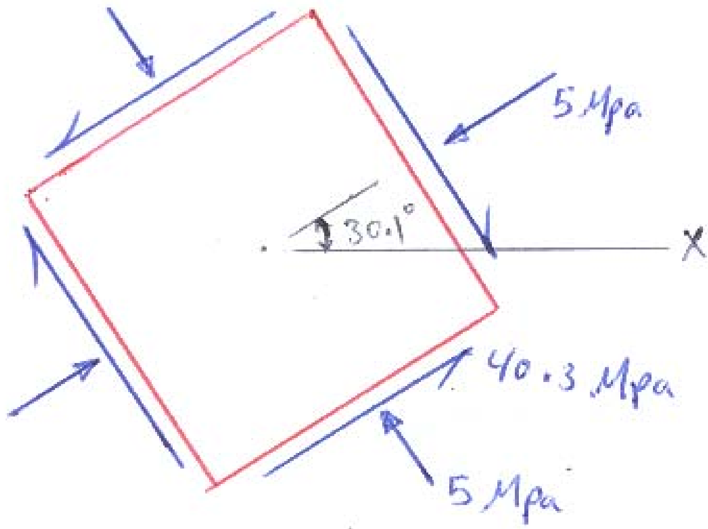
* To check θ_{s1} corresponds to τ_{max} or τ_{min}

$$\tau_{\xi\eta} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta_{s1} + \tau_{xy} \cos \theta_{s1}$$

$$= -\frac{30 + 40}{2} \sin(30.13) + (-20) \cos(30.13)$$

$$= -40.3 \text{ Mpa} \quad (\tau_{min})$$





a) Principal normal stresses

b) principal shear stresses

C) At 50° plane (−40° from x-axis)

$$\sigma_\xi(-40^\circ) = \frac{30-40}{2} + \frac{30+40}{2}\cos(2\times-40^\circ) + (-20)\sin(2\times-40^\circ)$$

$$\therefore \sigma_\xi = 20.77 \text{ Mpa.}$$

$$\sigma_\eta(-40^\circ) = \frac{30-40}{2} - \frac{30+40}{2}\cos(2\times-40^\circ) - (-20)\sin(2\times-40^\circ)$$

$$\therefore \sigma_\eta = -30.77 \text{ Mpa.}$$

$$\tau_{\xi\eta}(-40^\circ) = -\frac{30+40}{2}\sin(2\times-40^\circ) + (-20)\cos(2\times-40^\circ)$$

$$\therefore \tau_{\xi\eta} = 31.00 \text{ Mpa}$$

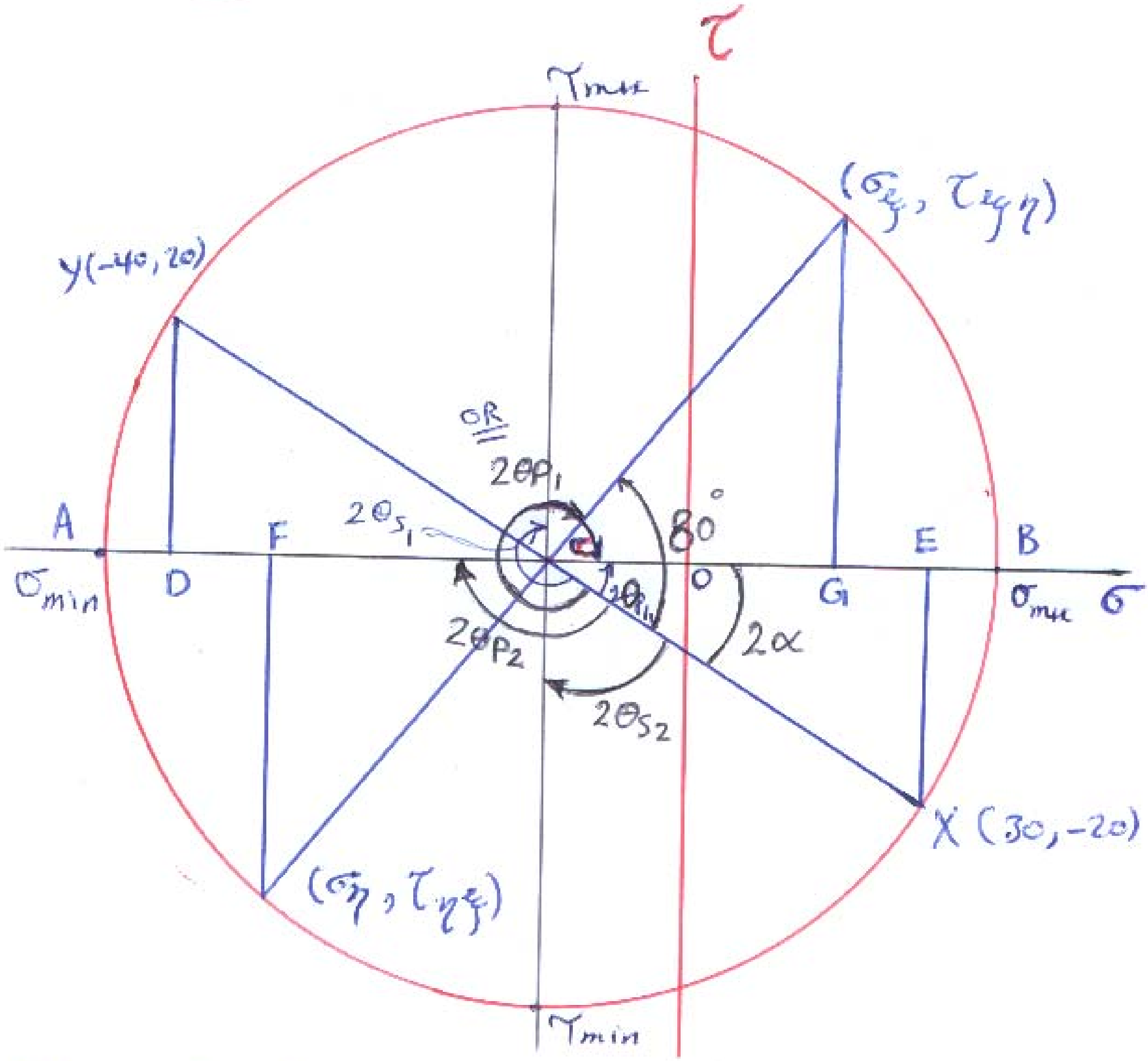
$$\therefore \boxed{\sigma_\xi = 20.8 \text{ Mpa} \;;\; \sigma_\eta = 30.8 \text{ Mpa} \;;\; \tau_{\xi\eta} = 31.0 \text{ Mpa}}$$

Problem 3:-

Given:- The same state of stress as given in problem 2

Req :- Use Mohr's circle to find everything (a, b, & c) as required in problem 2

Solution:-



a) $OC = \dfrac{OE + OD}{2} = \dfrac{30 - 40}{2} = -5$

$\tan 2\alpha = \dfrac{XE}{CE} = \dfrac{|XE|}{|OC| + |OE|} = \dfrac{20}{5 + 30} = 0.5714$

$\Rightarrow 2\alpha = 29.74°$

$2\theta_{p1} = 360° - 2\alpha = 330.2° \Rightarrow \theta_{p1} = 165°$

$\therefore$ $\boxed{\theta_{P_1} = 165^\circ \text{ ccw } (\circlearrowleft) \text{ meaured from x-axis to axis of } \sigma_{max}}$

$2\theta_{P_2} = 180^\circ - 2\alpha \Rightarrow \boxed{\theta_{P_2} = 75.1^\circ \text{ ccw } (\circlearrowleft) \text{ from x-axis}}$

$R = CX = \frac{XE}{\sin 2\alpha} = \frac{20}{\sin(29.74)} = 40.3 \text{ Mpa.}$

$\sigma_{max} = R - OC = 40.3 - 5 \Rightarrow \boxed{\sigma_{max} = 35.3 \text{ Mpa.}}$

$\sigma_{min} = -(R + OC) = -(40.3 + 5) \Rightarrow \boxed{\sigma_{min} = -45.3 \text{ Mpa}}$

b) $\tau_{\substack{max \\ min}} = \pm R \Rightarrow \boxed{\begin{array}{l} \tau_{max} = 40.3 \text{ Mpa} \\ \tau_{min} = -40.3 \text{ Mpa} \end{array}}$

$2\theta_{S_1} = 2\theta_{P_1} - 90^\circ \Rightarrow \theta_{S_1} = 120.1^\circ$ ccw from x-axis

$2\theta_{S_2} = 2\theta_{P_2} - 90^\circ \Rightarrow \theta_{S_2} = 30.13^\circ$ ccw from x-axis

c) $\sigma_{\xi} = CG - OC = 40.3 \cos(80 - 20C) - 5$

$\boxed{\sigma_{\xi} = 20.77 \text{ Mpa}}$

$\sigma_{\eta} = -(OC + CD) = -\{5 + 40.3 \cos(80 - 2\alpha)\}$

$\boxed{\sigma_{\eta} = -30.77 \text{ Mpa}}$

$\tau_{\xi\eta} = \pm R \sin(80 - 2\alpha) = \tau_{\xi\eta} = \pm 40.3 \sin(80 - 2\alpha)$

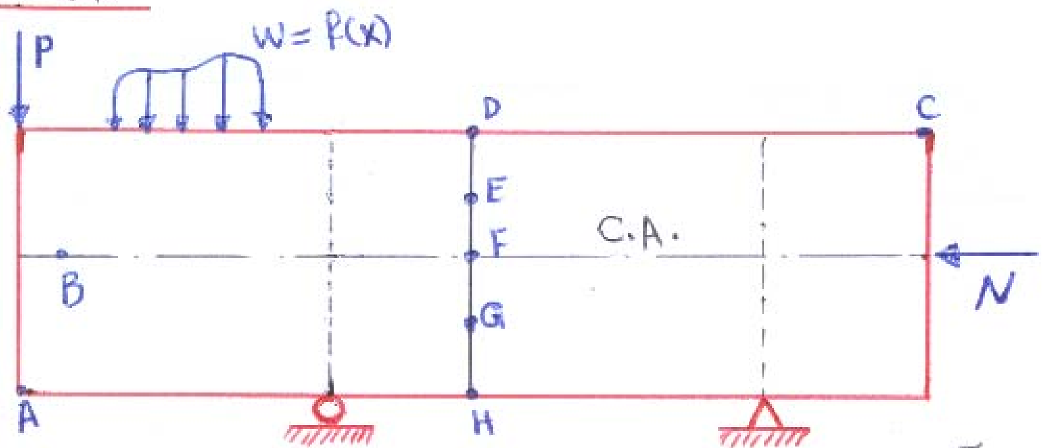
$\boxed{\tau_{\xi\eta} = 31.0 \text{ Mpa.}}$

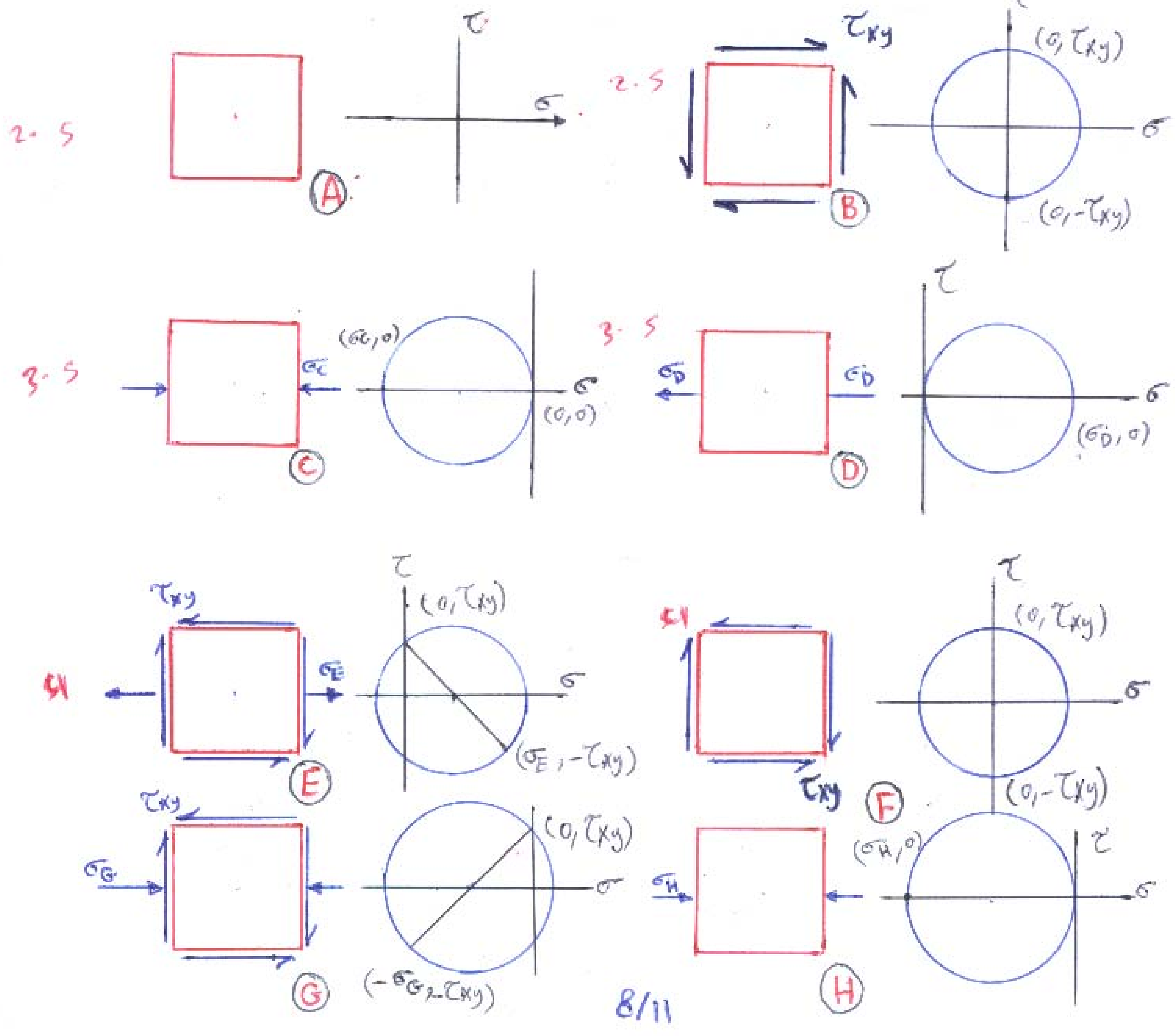
Problem 4:-

Given:- The beam shown in Fig.

Req:- Qualitatively, sketch the state of stress and Mohr's circle for points A to H

Solution:-





$$\tau_{C.A.} = \frac{VQ}{Ib} = \frac{26\times10^3\left(100\times\frac{125^2}{2} + 2\times50\times\frac{25^2}{2}\right)}{131.3\times10^6\ (200)} = 0.804\ Mpa$$

$$\tau_i = \frac{26\times10^3\left(100\times100\times\left(\frac{100}{2}+25\right)\right)}{131.3\times10^6\ (100)} = 1.485\ Mpa.$$

At point **C** maximum normal stress is at top and bottom

Top:- $\sigma_x = 53.18$ Mpa

$\sigma_y = 0$

$\tau_{xy} = 0$

For this using Mohr's circle

$\therefore\ \sigma_{max} = 53.18$ Mpa ; $\sigma_{min} = 0$

$\tau_{max} = 26.59$ Mpa ; $\tau_{min} = -26.59$ Mpa.

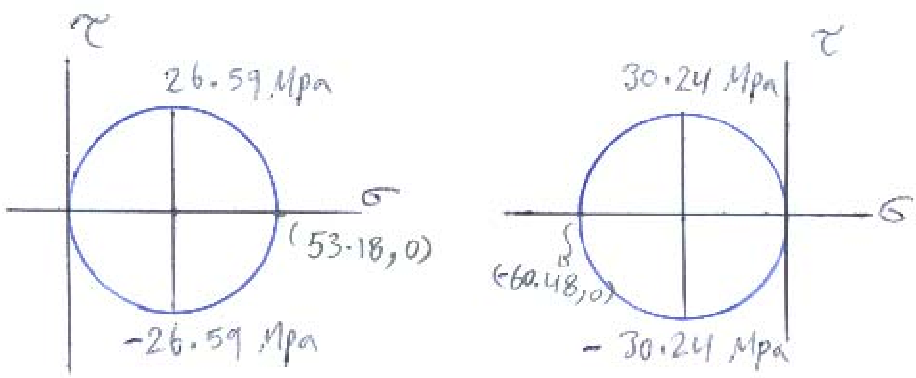
Bottom:- $\sigma_x = -60.48$ Mpa

$\sigma_y = 0$

$\tau_{xy} = 0$

$\therefore\ \sigma_{max} = 0$; $\sigma_{min} = -60.48$ Mpa.

$\tau_{max} = 30.24$ Mpa ; $\tau_{min} = -30.24$ Mpa.



$V_{max} = 32\ KN$ @ B.

$M = 32\ KN.m = 32\times10^6\ N.mm$ @ B.

$$\sigma = -\sigma_N \pm \sigma_M = -\frac{100\times10^3}{30\times10^3} \pm \frac{32\times10^6\, y}{131.3\times10^6}$$

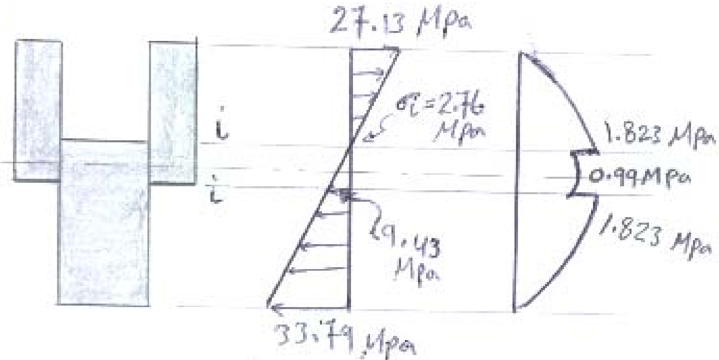
$$\Rightarrow \sigma = -3.333 \pm 0.244\, y$$
$$= -3.333 \pm 30.46$$

$\sigma_{Top} = -3.333 + 30.46 = 27.13$ Mpa

$\sigma_{bottom} = -3.333 - 30.46 = -33.79$ Mpa.

$$\tau_{C.A.} = \frac{32\times10^3\,(100\times125^2/2 + 2\times50\times25^2/2)}{131.3\times10^6\ (200)}$$
$$= 0.99 \text{ Mpa.}$$

$$\tau_i = \frac{32\times10^3\,(100\times100\times(100/2 + 25))}{131.3\times10^6\ (100)}$$
$$= 1.823 \text{ Mpa}$$



stress distribution at B

τ is maximum at lower i-face at B.

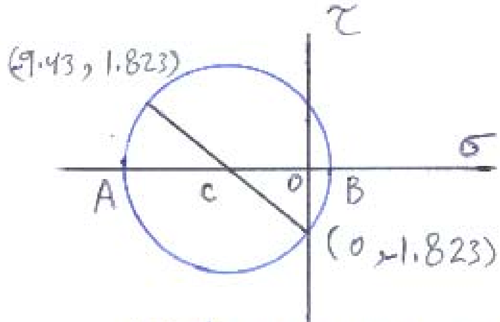
$\sigma_x = -9.43$ Mpa , $\sigma_y = 0$; $\tau_{xy} = 1.83$ Mpa.

$\Rightarrow \sigma_{max} = 0.340$ Mpa , $\sigma_{min} = -9.77$ Mpa.

$\tau_{max} = 5.06$ Mpa , $\tau_{min} = -5.06$ Mpa.

$\therefore$

$$\boxed{\begin{aligned} \sigma^{T}_{max} &= 53.18 \text{ Mpa} \\ \sigma^{C}_{max} &= 60.48 \text{ Mpa} \\ \tau_{max} &= 30.24 \text{ Mpa} \end{aligned}}$$



$$|OC| = \frac{9.43+0}{2} = 4.72 \text{ Mpa}$$

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$$R = \sqrt{(9.43-4.72)^2 + 1.823^2}$$
$$= 5.06 \text{ Mpa.}$$