CE 203-3 [072]

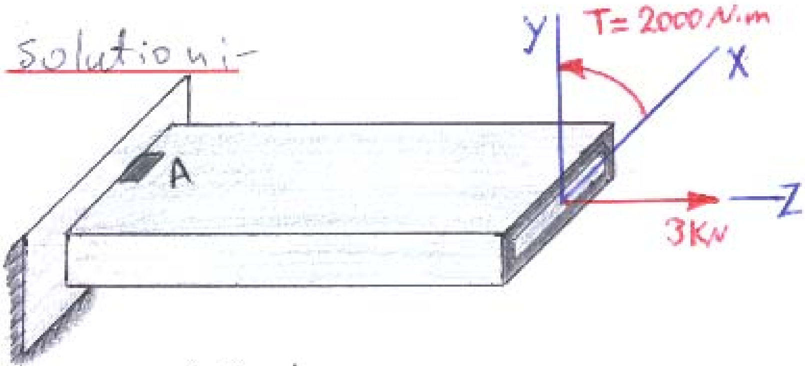
H.W. # 11 solution

Problem 1:-

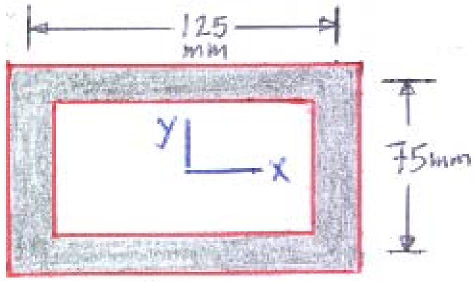
Given :- A thin aluminum box beam is required to resist the axial force and twisting moment shown in fig.

Req :- The normal and shear stresses at point (A) near the wall.

Solution:-



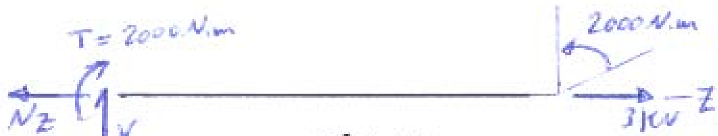
(a) beam



t = 5 mm

(b) X-section

* From (F B D)



(FBD)

$\Sigma F_Z = 0 \xrightarrow{+}$

$3 - N_Z = 0 \Rightarrow N_Z = 3\ KN$

$$\sigma_Z = \pm \frac{N_Z}{A} \mp \frac{M_x y}{I_x}$$

Since $M_x = 0.0 \Rightarrow \frac{M_x y}{I_x} = 0.0$

$$\therefore \sigma_Z = \pm \frac{N_Z}{A} = \frac{3 \times 10^3}{[(130 \times 80) - (120 \times 70)]} \Rightarrow \boxed{\sigma_Z = 1.5\ Mpa\ \text{"T"}}$$

$\tau_{max} = \frac{VQ}{Ib} + \frac{T}{2tA_m}$; $V = 0.0$, then $\frac{VQ}{Ib} = 0.0$.

$A_m = (125 \times 75) = 9375\ mm^2$ 《 thin-walled closed section 》

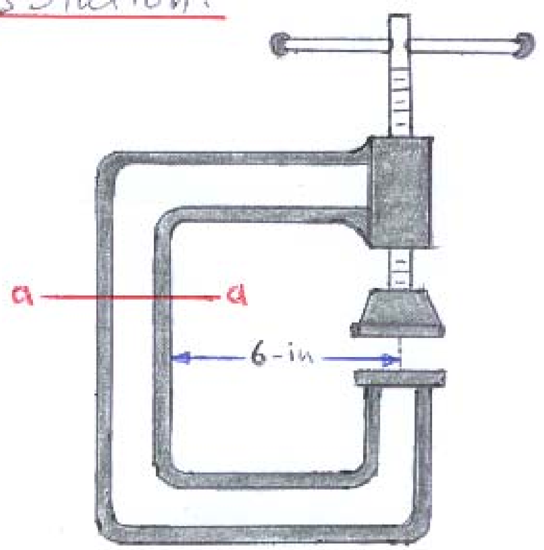
$$\therefore \tau_{max} = 0 + \frac{2000 \times 10^3\ N.mm}{(2 \times 5 \times 9375)\ mm^3} \Rightarrow \boxed{\tau_{max} = 21.33\ Mpa}$$

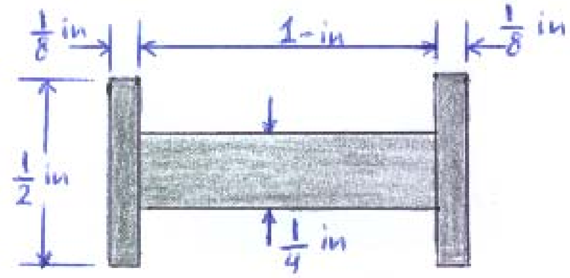
Problem 2 :-

Given :- A steel C clamp has the dimensions shown in Fig.
If the normal stress is not to exceed 12 ksi (T or C)

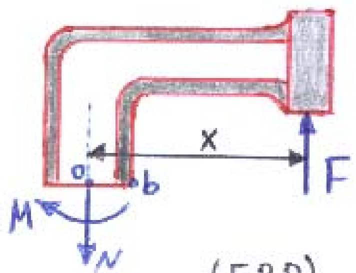
Req :- The max. permissible clamping force.

Solution :-





section aa



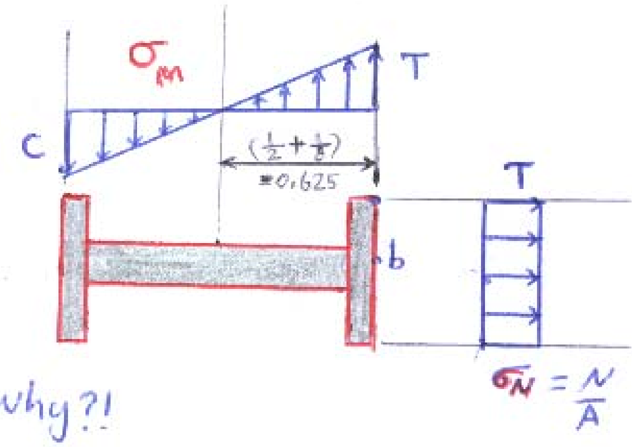
(FBD)

$$X = 6 + \tfrac{1}{8} + \tfrac{1}{2} = 6.625 \text{-in}$$

$$A = 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{8}\right) + (1)\left(\tfrac{1}{4}\right)$$

$$A = 0.375 \ \text{in}^2$$

$$I = 2\left[\tfrac{1}{12}\left(\tfrac{1}{2}\right)\left(\tfrac{1}{8}\right)^3 + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{8}\right)\left(\tfrac{1}{2} + \tfrac{1}{16}\right)^2\right] + \left[\tfrac{1}{12}\left(\tfrac{1}{4}\right)(1)^3\right] + 0 \leftarrow \text{why?!}$$



$$I = 0.060547 \ \text{in}^4$$

* From (FBD) :-

$$+\uparrow \Sigma F = 0 \Rightarrow F - N = 0 \Rightarrow N = F \text{ "T"}$$

$$+\circlearrowleft \Sigma M_o = 0 \Rightarrow -M + F(6.625) \Rightarrow M = 6.625F$$

It is clear that σ_{max} will be at point (b) which is (T). Why?!

$\sigma = \sigma_N + \sigma_M \Rightarrow \underset{\uparrow}{\sigma} \equiv 12\ ksi = 12{,}000\ psi$

$\sigma_{allow.}$

$$12 \times 10^3 = \frac{F}{0.375} + \frac{(6.625\,F)(0.625)}{0.060547}$$

$$12 \times 10^3 = 2.6667\,F + 68.387\,F$$

$$12000 = 71.054\,F_{max}$$

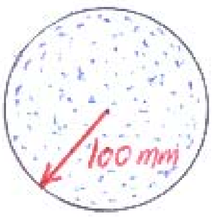
$$\therefore \boxed{F_{\substack{max \\ allow.}} = 168.9 \quad lb}$$

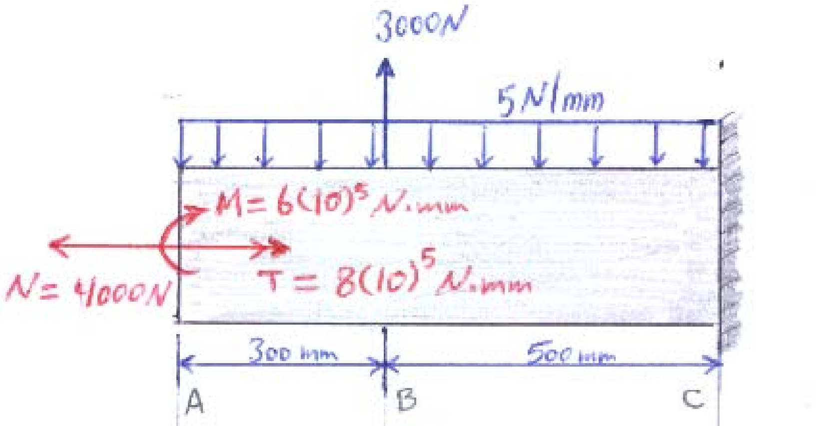
Problem 3 :-

Given :- The beam (Shaft) shown in Fig.

Req :- The values and locations of the max. normal and shear stresses.

Solution :-





* From (FBD) :-

$\Sigma F_x = 0 \; \rightarrow$

$R_x - 4000 = 0 \Rightarrow R_x = N = 4000 \, N$

$+\uparrow \Sigma F_y = 0$

$3000 - 5(800) + R_y = 0$

$R_y = 1000 \, N$

$\circlearrowleft \Sigma M = 0$

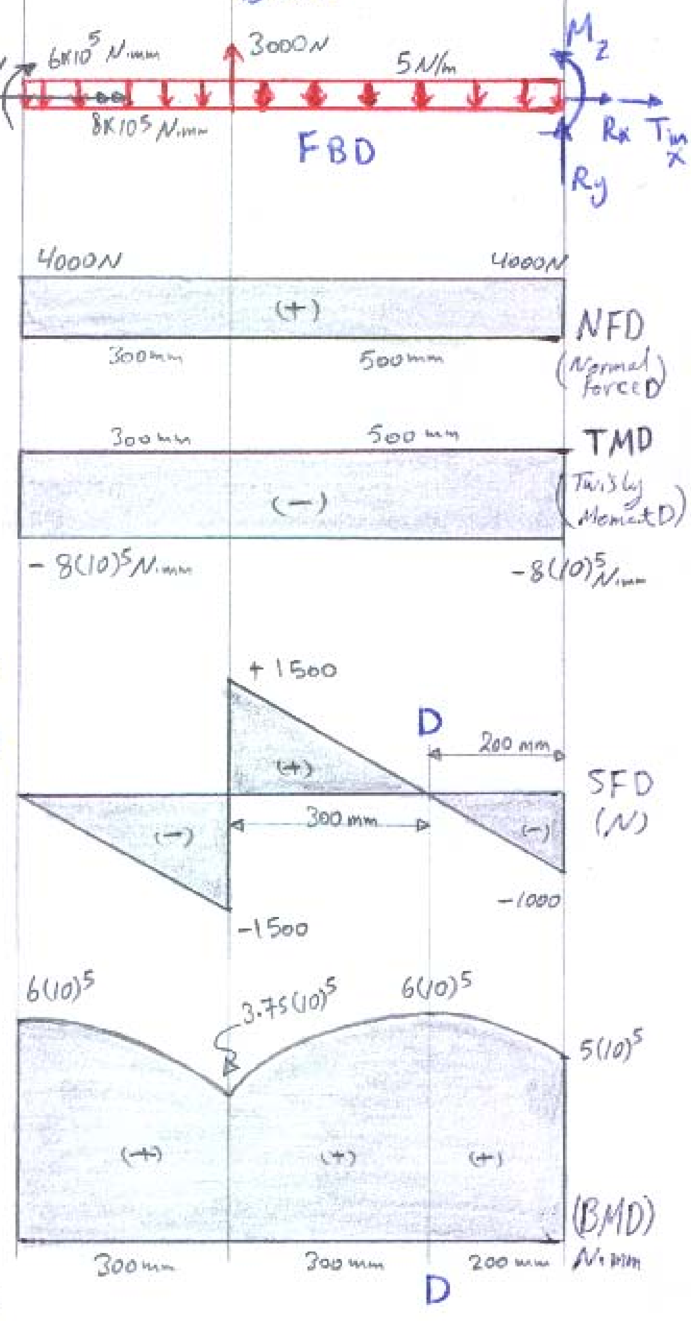
$M_z + 4000(400) - 3000(500) - 6 \times 10^5 = 0$

$M_z = 5 \times 10^5 \, N.mm$

$A = \pi r^2 = \pi (100)^2 = (10)^4 \pi \; mm^2$

$I_z = \frac{\pi}{4} (r)^4 = \frac{\pi}{4} (100)^4 = 2.5(10)^7 \pi \; mm^4$

$J = \frac{\pi}{2} (r)^4 = \frac{\pi}{2} (100)^4 = 5(10)^7 \pi \; mm^4$



From the diagrams, Clearly

$N_{max} = 4000 \, N$

$T_{max} = 8(10)^5 \, N.mm$

$V^{+}_{max} = 1500 \, N$ @ B+

$V^{-}_{max} = 1500 \, N$ @ B−

$M^{+}_{max} = 6(10)^5 \, N.mm$ @ A and D

$M^{-}_{max} = 0$

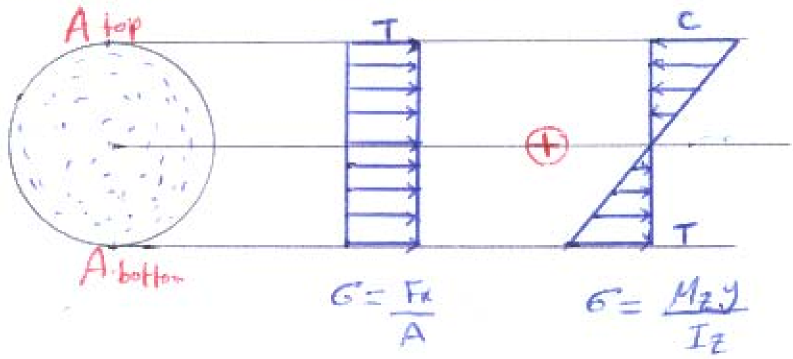
$$\sigma = \pm \frac{N}{A} \pm \frac{M_z y}{I_z}$$

$$\sigma^{T}_{max} = \frac{4000}{(10)^4 \pi} + \frac{6(10)^5(100)}{2.5(10)^7 \pi} = 0.127324 + 0.763944$$

$$\boxed{\sigma^{T}_{max} = 0.8913 \text{ Mpa @ } A_{\text{bottom}}}$$

$$\sigma^{C}_{max} = 0.127324 - 0.763944$$

$$\boxed{\sigma^{C}_{max} = 0.6366 \text{ Mpa @ } A_{\text{top}}}$$

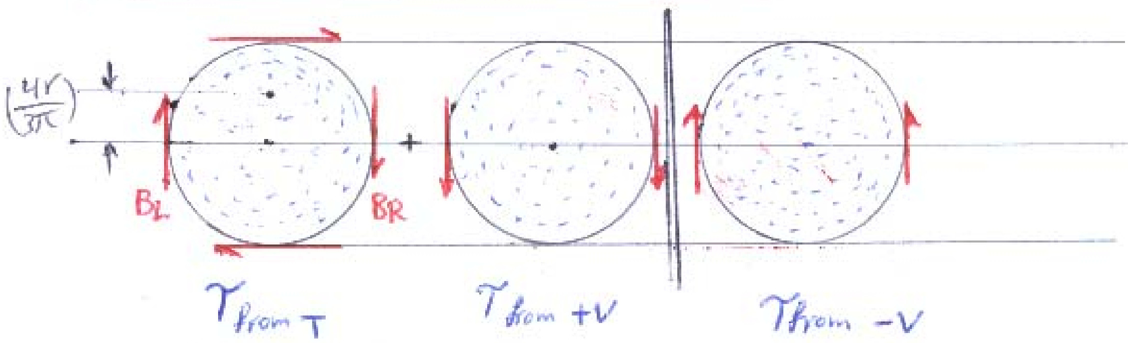


$$\tau_{max} = \frac{VQ}{Ib} + \frac{Tr}{J}$$

$$= \frac{(1500)\left[\left((10)^4 \pi /2\right)\left(\frac{4(100)}{3\pi}\right)\right]}{(2.5(10)^7 \pi)(200)} + \frac{(8 \times 10^5)(100)}{5(10)^7 \pi}$$

$$= 0.063662 + 0.509296$$

$$\therefore \boxed{\tau_{max} = 0.572958 \text{ Mpa @ } B_R^{+} \text{ and } B_L^{-}}$$

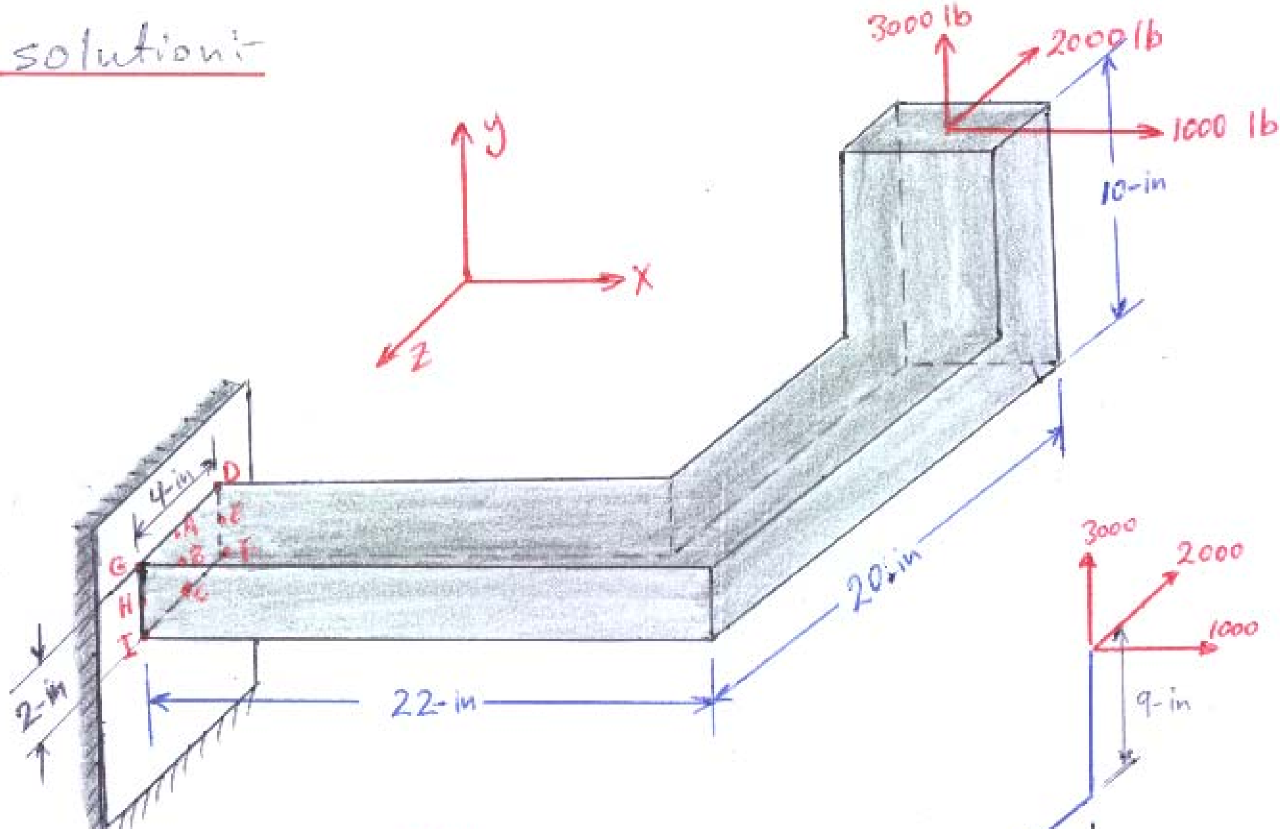


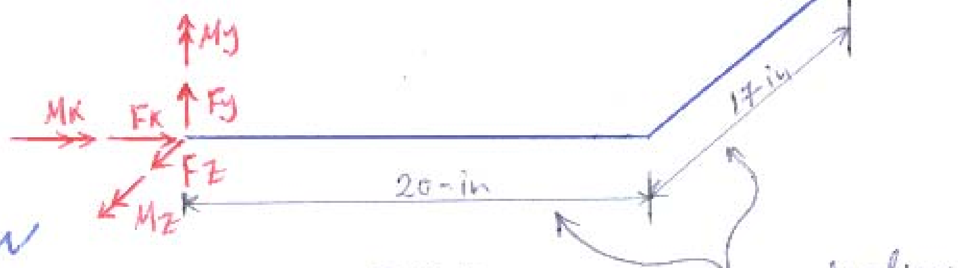
Problem 4:-

Given:- In the Fig. shown

Req:- The values and locations of the max. tensile and compressive (normal) stresses at the fixed end.

solution:-





$F_x = 1000\,N \overset{+}{\leftarrow} = N$

F.B.D

$$M_y = -(2000)(20) + (1000)(17) = -23\,000 \text{ in-lb}$$

$$M_z = -(3000)(20) + (1000)(9) = -51\,000 \text{ in-lb}$$

$$A = (4 \times 2) = 8 \text{ in}^2$$

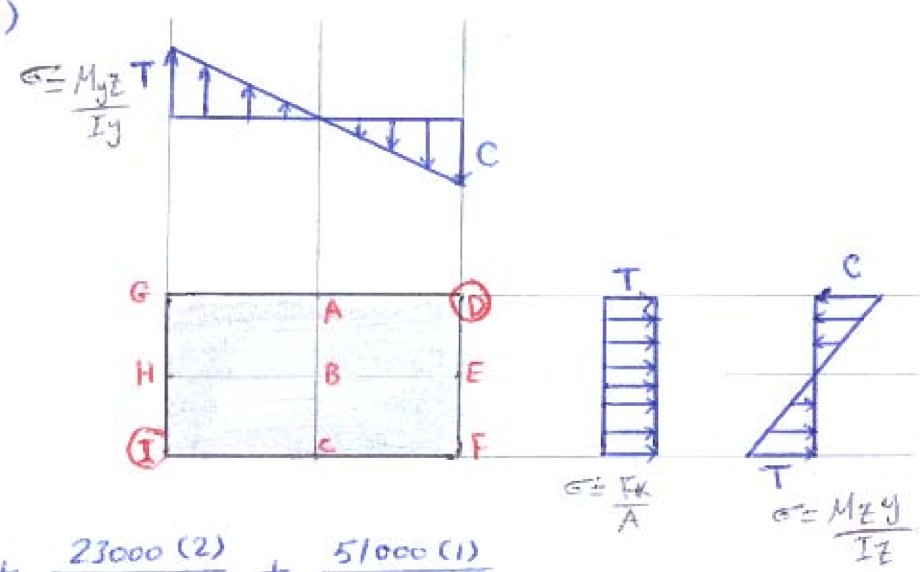
$$I_y = \frac{1}{12}(2)(4)^3 = 10.667 \text{ in}^4$$

$$I_z = \frac{1}{12}(4)(2)^3 = 2.6667 \text{ in}^4$$

$$\sigma = \mp \frac{N}{A} \mp \frac{M_y z}{I_y} \mp \frac{M_z y}{I_z}$$

clearly σ^T_{max} in @ point (I)

σ^c_{max} in @ point (D)



$$\sigma^T_{max} = \frac{1000}{8} + \frac{23000\,(2)}{10.667} + \frac{51000\,(1)}{2.6667}$$

$$= 125 + 4313 + 19123$$

$$\boxed{\sigma^T_{max} = 23{,}561 \text{ psi} \quad @\ I}$$

$$\sigma^c_{max} = 125 - 4313 - 19123$$

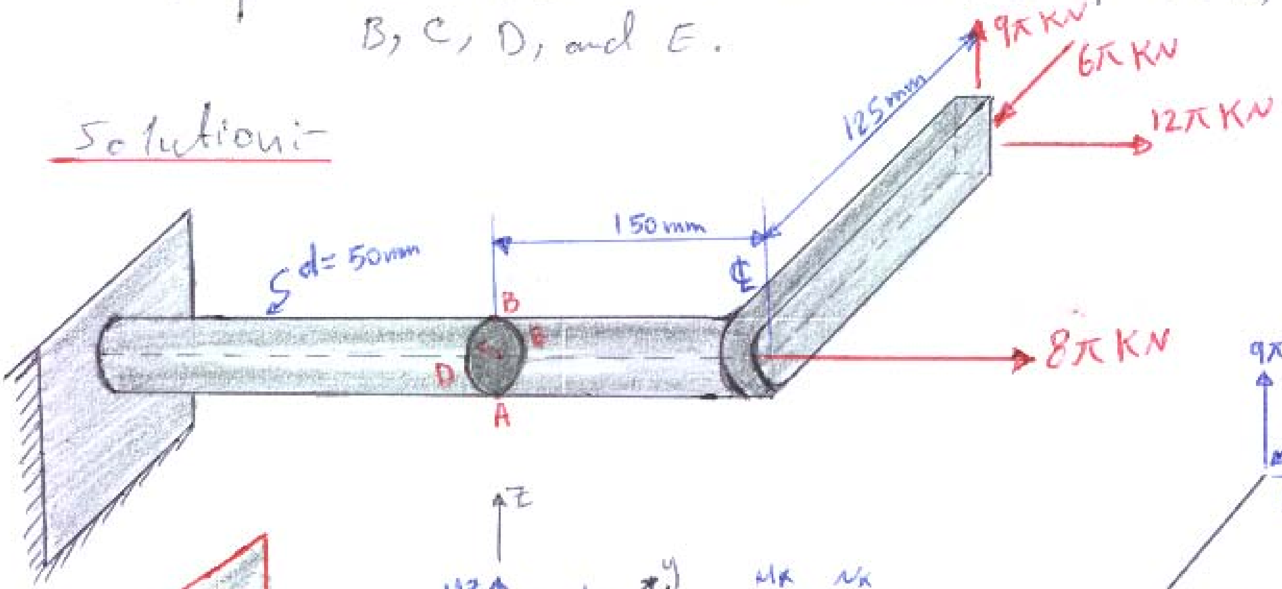
$$\boxed{\sigma^c_{max} = 23{,}311 \text{ psi} \quad @\ D}$$

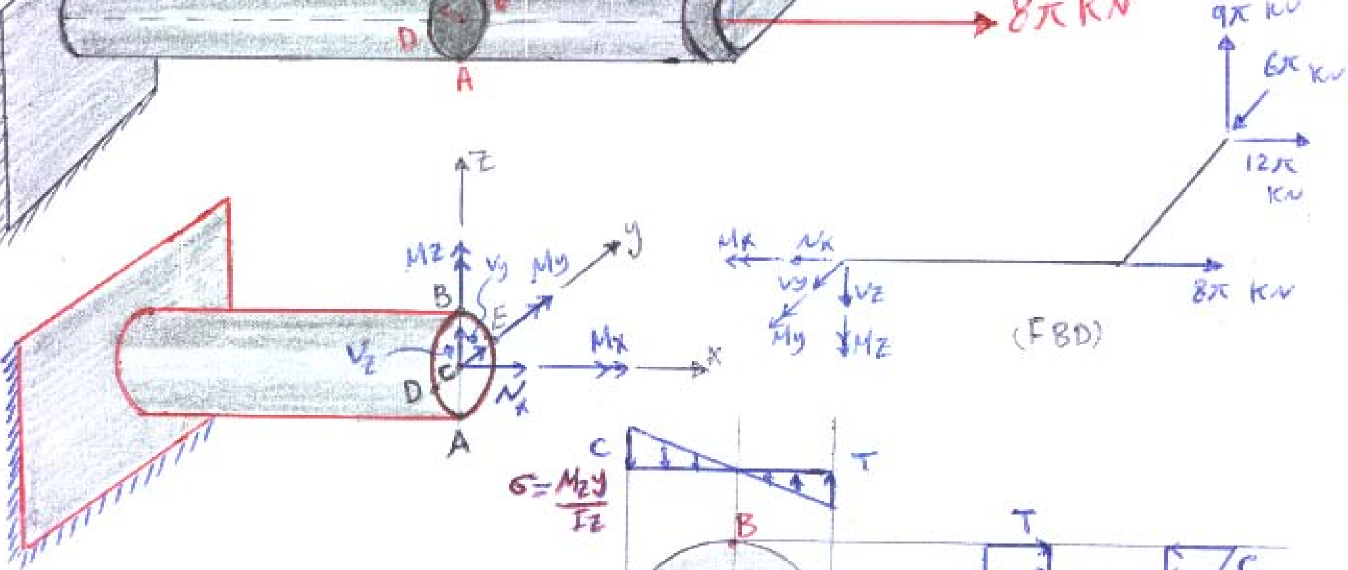
Problem 5 :-

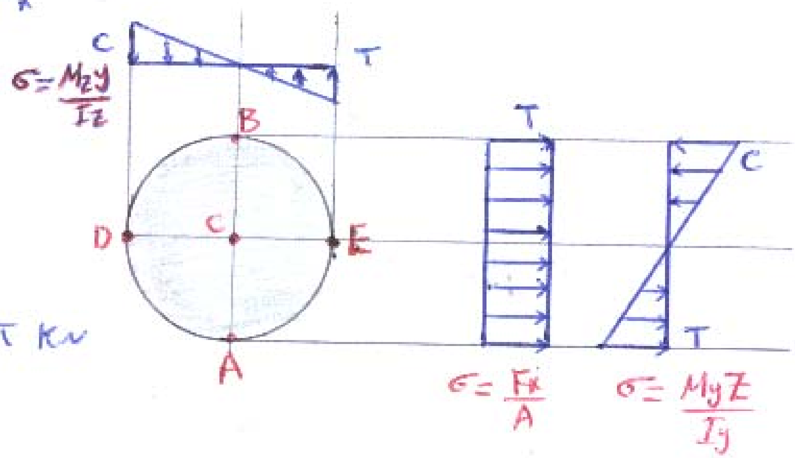
Given :- In the member shown in Fig. points (D) and (E) are at the end of horizontal diameter.

Req :- The normal and shear stresses at points A, B, C, D, and E.

Solution:-







From (FBD):-

$N_x = 8\pi + 12\pi = 20\pi$ KN

$V_y = -6\pi$ KN

$V_z = 9\pi$ KN

$M_x = 9\pi \times 10^3 \times 125 \times 10^{-3} = 1.125\pi \times 10^3$ N.m $= T$ (torque)

$M_y = -9\pi \times 10^3 \times 150 \times 10^{-3} = -1.35\pi \times 10^3$ N.m

$M_z = -12\pi \times 10^3 \times 125 \times 10^{-3} - 6\pi \times 10^3 \times 150 \times 10^{-3} = -2.4\pi \times 10^3$ N.m

$$I_x = I_y = \frac{\pi d^4}{64} = \frac{\pi(50)^4}{64} = 9.766\pi \times 10^4 \ mm^4$$

$$= 9.766\pi \times 10^{-8} \ m^4$$

$$J = \frac{\pi d^4}{32} = \frac{\pi(50)^4}{32} = 1.953\pi \times 10^5 \ mm^4$$

$$= 1.953\pi \times 10^{-7} \ m^4$$

$$A = \pi r^2 = \pi(25)^2 = 6.25\pi \times 10^{12} \ mm^2$$

$$= 6.25 \times 10^{-4} \ m^2$$

$$\sigma = (\pm)\frac{N}{A} (\pm) \frac{M_y z}{I_y} (\pm) \frac{M_z y}{I_z}$$

(y, z) coordinate of points A, B, C, D and E are

$$\binom{y}{z} \rightarrow A\binom{0}{-25}, \ B\binom{0}{25}, \ C\binom{0}{0}, \ D\binom{-25}{0}, \ E\binom{25}{0}$$

$$\sigma A = \frac{20\pi \times 10^3}{6.25\pi \times 10^{-4}} + \frac{-1.35\pi \times 10^3 (-25 \times 10^{-3})}{9.766\pi \times 10^{-8}} - \frac{-2.4\pi \times 10^3 \times 0}{9.766\pi \times 10^{-8}}$$

$$= 32 + 345.59 + 0 \Rightarrow \boxed{\sigma_A = 377.59 \ Mpa \ (T)}$$

$$\sigma B = \frac{20\pi \times 10^3}{6.25\pi \times 10^{-4}} + \frac{-1.35\pi \times 10^3 (25 \times 10^{-3})}{9.766\pi \times 10^{-8}} - \frac{-2.4\pi \times 10^3 \times 0}{9.766\pi \times 10^{-8}}$$

$$= 32 - 345.59 + 0 \Rightarrow \boxed{\sigma_B = -313.59 \ Mpa \ (C)}$$

$$\sigma_C = \frac{20\pi \times 10^3}{6.25\pi \times 10^{-4}} + \frac{-1.35\pi \times 10^3 (0)}{9.766\pi \times 10^{-8}} - \frac{-2.4\pi \times 10^3 (0)}{9.766\pi \times 10^{-8}}$$

$$= 32 - 0 + 0 \Rightarrow \boxed{\sigma_C = 32 \ Mpa \ (T)}$$

$$\sigma_D = \frac{20\pi \times 10^3}{6.25\pi \times 10^{-4}} + \frac{-1.35\pi \times 10^3 (0)}{9.766\pi \times 10^{-8}} - \frac{-2.4\pi \times 10^3 (-25 \times 10^{-3})}{9.766\pi \times 10^{-8}}$$

$$= 32 - 0 - 614.38 \Rightarrow \boxed{\sigma_D = -582.38 \ Mpa \ (C)}$$

$$\sigma_E = \frac{20\pi \times 10^3}{6.25\pi \times 10^{-4}} + \frac{-1.35\pi \times 10^3 (0)}{9.766\pi \times 10^{-8}} - \frac{-2.4\pi \times 10^3 (25\times 10^{-3})}{9.766\pi \times 10^{-8}}$$

$$= 32 + 0 + 614.38 \Rightarrow \boxed{\sigma_E = 646.38 \text{ Mpa (T)}}$$

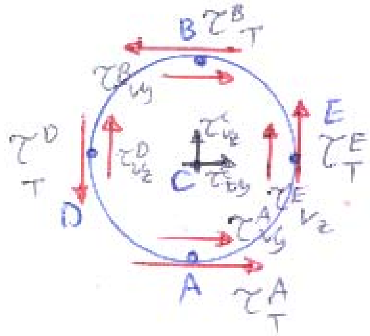
* Shear stresses at A, B, C, D, E

$$Q = A\bar{y} = \frac{\pi r^2}{2} \times \frac{4r}{3\pi} = \frac{2}{3} r^3$$

$$= \frac{2}{3}(25)^3 = 1.042 \times 10^4 \text{ mm}^3$$

$$= 1.042 \times 10^{-5} \text{ m}^3$$

[These directions are according to the sign convution in the first fig]



$$\tau^A_T = \tau^B_T = \tau^D_T = \tau^E_T = \frac{Tr}{J}$$

$$= \frac{1.125\pi \times 10^3 \times 25 \times 10^{-3}}{1.953\pi \times 10^{-7}} = 144 \text{ Mpa.}$$

$$\tau^A_{vy} = \tau^C_{vy} = \tau^B_{vy} = \frac{V_y Q}{I_y b} = \frac{-6\pi \times 10^3 \times 1.042 \times 10^{-5}}{9.766\pi \times 10^{-8} \times 50 \times 10^{-3}}$$

$$= -12.80 \text{ Mpa.}$$

$$\tau^D_{vz} = \tau^C_{vz} = \tau^E_{vz} = \frac{V_z Q}{I_z b} \quad \frac{9\pi \times 10^3 \times 1.042 \times 10^{-5}}{9.766\pi \times 10^{-8} \times 50 \times 10^{-3}}$$

$$= 19.20 \text{ Mpa.}$$

* Result shear stresses:-

$$\tau^A = \tau^A_T - \tau^A_{vy} = 144 - 12.80 \Rightarrow \tau_A = 131.2 \text{ Mpa } (\longrightarrow)$$

$$\tau^B = \tau^B_T + \tau^B_{vy} = 144 + 12.80 \Rightarrow \tau_B = 156.8 \text{ Mpa } (\longleftarrow)$$

$$\tau^D = \tau^D_T - \tau^D_{vz} = 144 - 19.2 \Rightarrow \tau^D = 124.8 \text{ Mpa } (\downarrow)$$

$$\tau^E = \tau^E_T + \tau^D_{vz} = 144 + 19.2 \Rightarrow \tau^E = 163.2 \text{ Mpa } (\uparrow)$$

$$\tau^C = \sqrt{(\tau^C_{vy})^2 + (\tau_{vz})^2} = \sqrt{(12.80)^2 + (19.20)^2} \Rightarrow \tau^C = 23.08 \text{ Mpa } \angle\theta$$

$$\theta = \tan^{-1}\frac{19.20}{12.80} \Rightarrow \theta = 56.31^\circ$$

Note that:
$\vec{\tau}^C = \vec{\tau}_{v_y} + \vec{\tau}_{v_z}$ (Vector)