CE 203-3 [072]

H.W # 10 - Solution

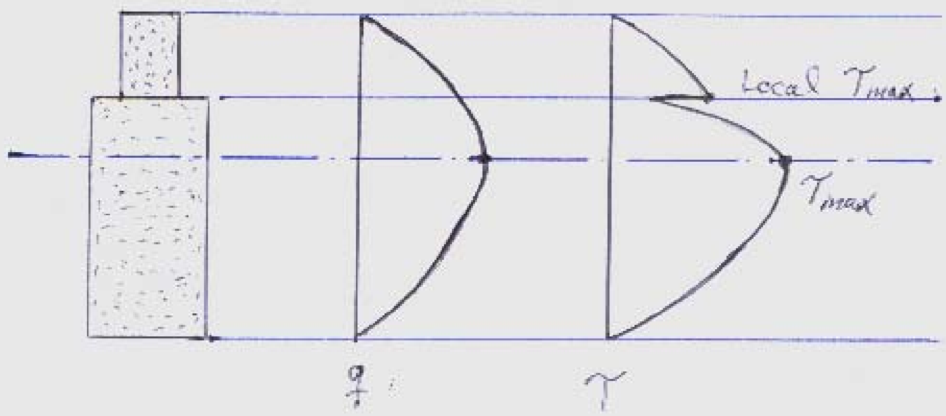
problem 1:-

Given :- The beam cross sections shown in Fig.

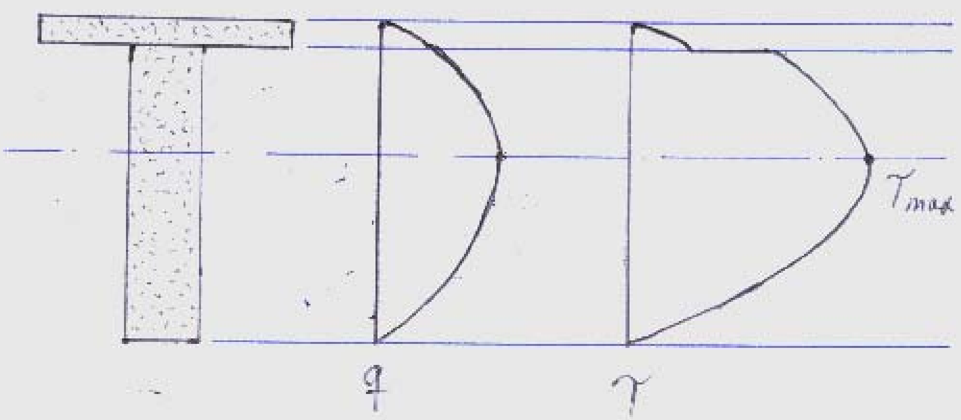
Req :- Sketch the shear flow and shear stress distributions for each of the beam cross sections.

Solution:-

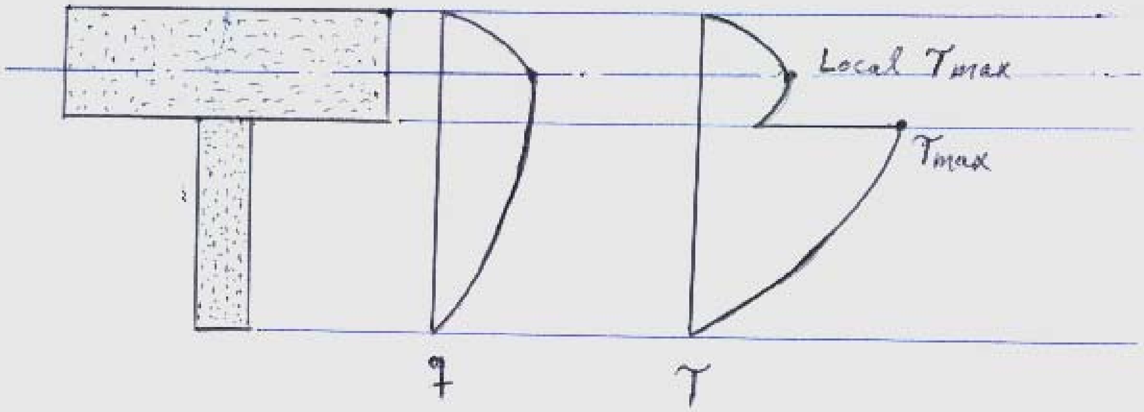
a)



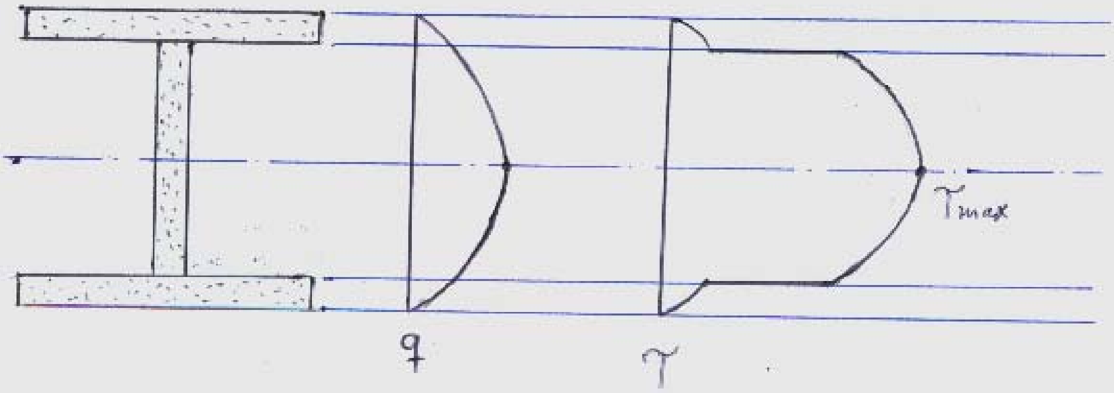
b)



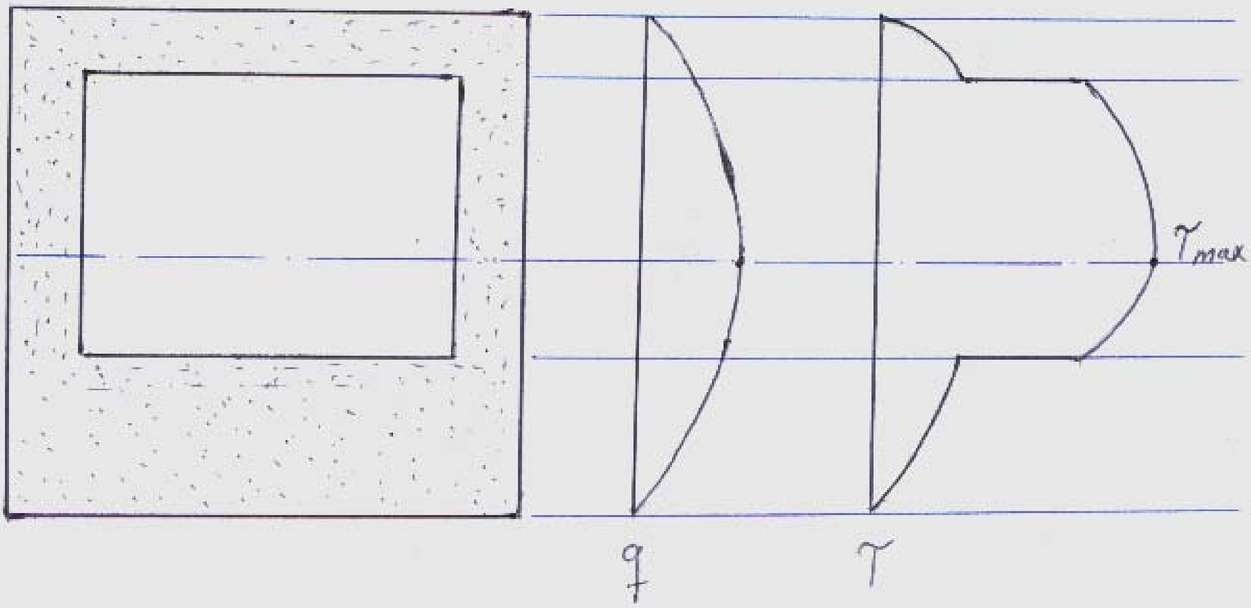
c)



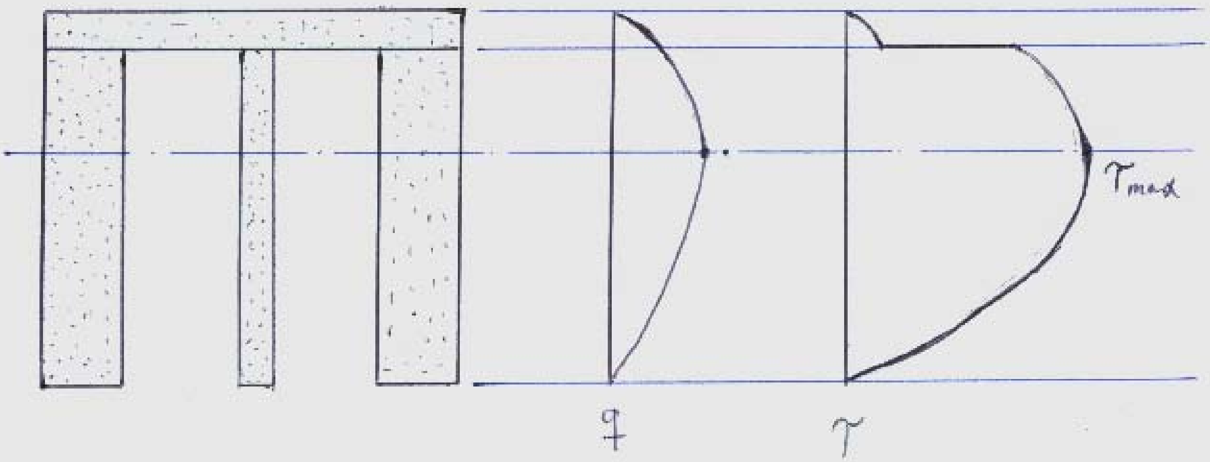
d)



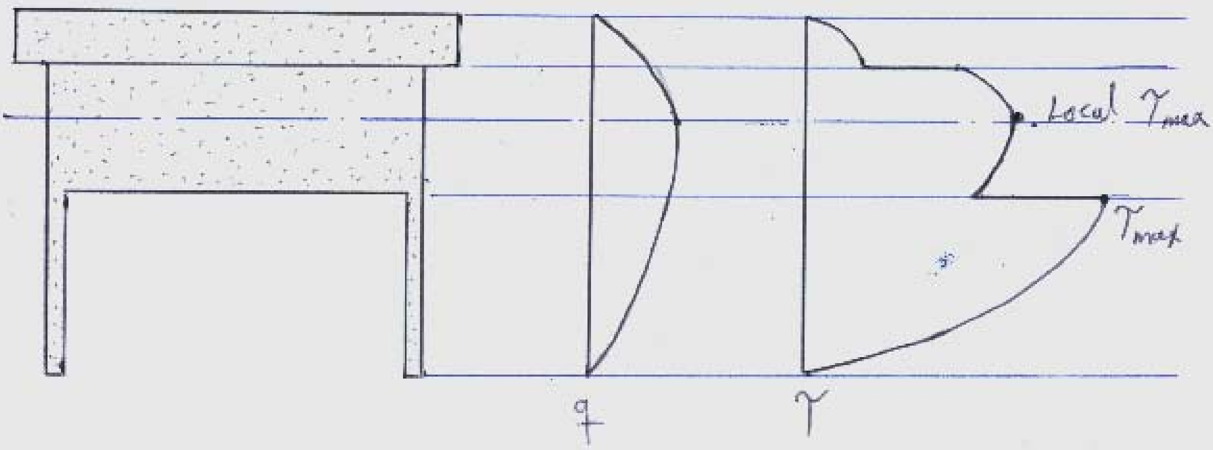
e)



f)



g)



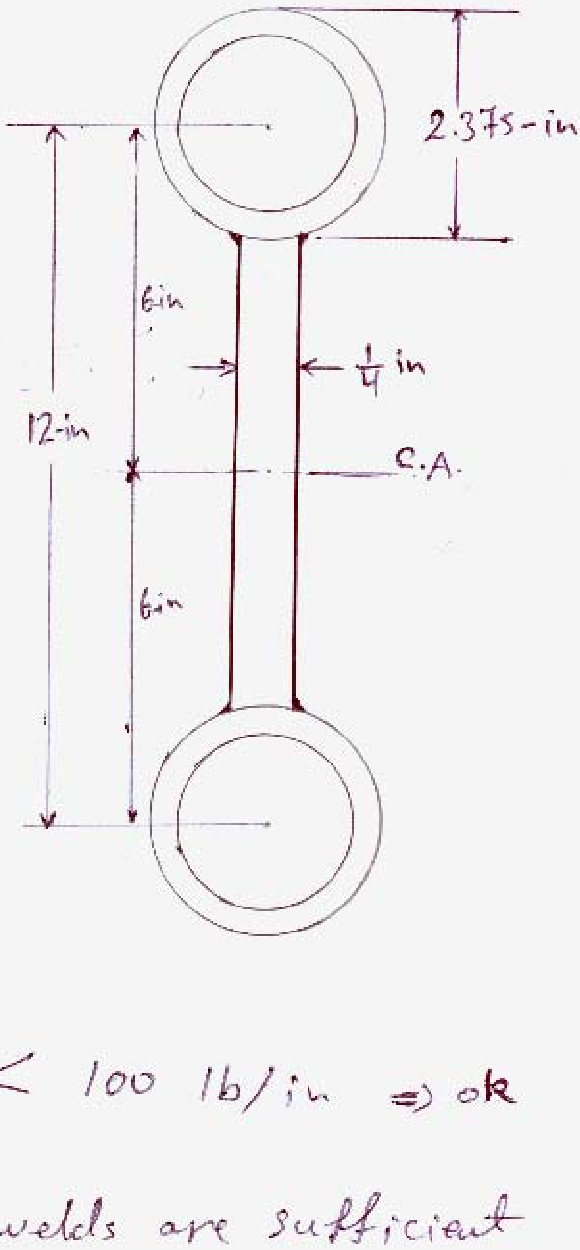
Problem 2:-

Given :- For the steel pipe
$A = 1.07 \text{ in}^2$, $I = 0.666 \text{ in}^4$.
$V = 2000$ lb , four fillet welds.

Req :- whether $\frac{1}{8}$-in fillet welds with a capacity of (100 lb/in) each will be sufficient.

Solution:-

For the composite section,



$$I_{C.A.} = 2\left[0.666 + 1.07(6)^2\right] + \left[\tfrac{1}{12}(0.25)(12-2.375)^2 + 0\right]$$

(why?!)

$I_{C.A.} = 96.952 \text{ in}^4$

shear at welds:-

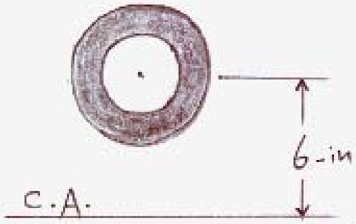
$Q = A\bar{y}$
$= (1.07)(6) = 6.42\text{-in}^3$

$q = \dfrac{VQ}{I}$

$\underbrace{2q}_{\text{(two weld)}} = \dfrac{(2000)(6.42)}{96.952}$

$\therefore q = 66.22$ lb/in < 100 lb/in $\Rightarrow$ ok

$\therefore$ $\boxed{\text{Yes}}$, the $\frac{1}{8}$-in fillet welds are sufficient (adequate).

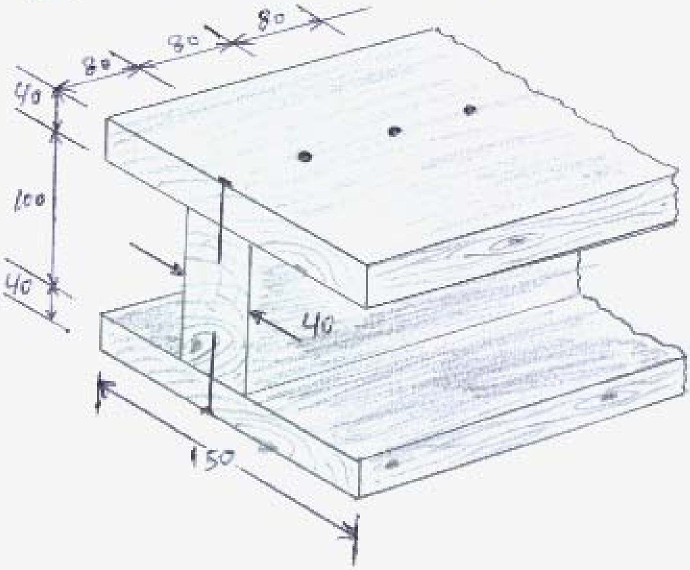


for shear at welds

Problem 3:-

Given :- A beam has the cross section shown in Fig.

Req :- a) The allowable shear force if $R_n = 500\,N$.
b) The required strength of the glue if the nails are replaced by glue, and if $V = 1200\,N$.



Dimensions in (mm)

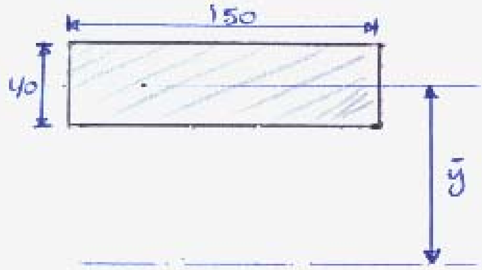
Solution:-

a) $q_{max} = \dfrac{V_{max}\,Q}{I}$

$\therefore V_{max} = \dfrac{q_{max}\,I}{Q}$

$Q = A\bar{y}$; $\bar{y} = \left(\frac{100}{2}\right) + 20$

$\bar{y} = 70\ mm$



$Q = (150 \times 40)(70) = 42 \times 10^4\ mm^3$
$= 42 \times 10^{-5}\ m^3$

$I = \left[\frac{1}{12}(150)(180)^3\right] - \left[\frac{1}{12}(150-40)(100)^3\right]$

$I = 6.3733 \times 10^7\ mm^4$
$= 6.3733 \times 10^{-5}\ m^4$

$R_n = q_{max} \cdot S \Rightarrow q_{max} = \dfrac{R_n}{S} = \dfrac{500}{80 \times 10^{-3}} = 6250\ N/m$

$q = \frac{VQ}{I} \Rightarrow V_{max} = \dfrac{(6250)(6.3733 \times 10^{-5})}{42 \times 10^{-5}} \Rightarrow \boxed{V_{max} = 948.4\ N}$

b) $\tau_{glue} = \dfrac{VQ}{Ib} = \dfrac{(1200)(42 \times 10^{-5})}{(6.3733 \times 10^{-5})(40 \times 10^{-3})} = 0.198 \times 10^6\ Pa$

$\boxed{\tau_{glue} = 0.198\ MPa}$ ⇐ This is the required shear strength of the glue.

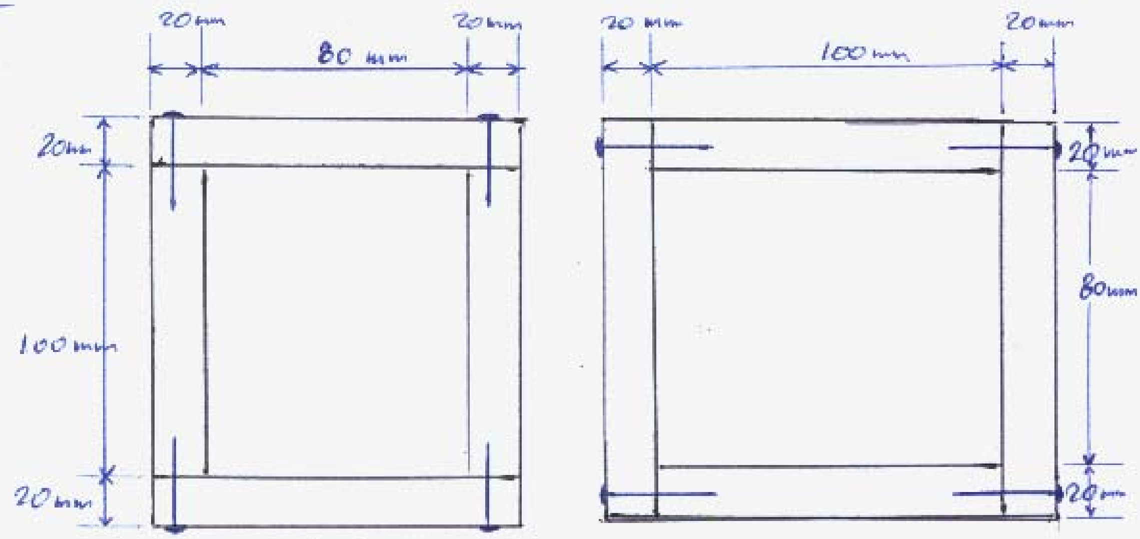
problem 4 :-

Given :- A "box beam" has the cross section shown. Note that (b) is similar to (a) except it is rotated (90°).

If $V = 10\,KN$, $M = 50\,KN.m$ $R_{n/nail} = 5\,KN$

Req :- The max. normal stress for each scheme
The max. permissible spacing of the nails for each scheme
which one is more efficient in bending and which one is better in shear? Explain.

Solution:-



(a) (b)

Scheme a)

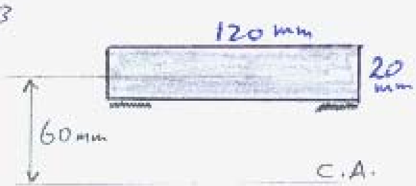
$$I = \left[\frac{1}{12}(120)(140)^3\right] - \left[\frac{1}{12}(80)(100)^3\right] = 2.077 \times 10^{-5}\ m^4$$

$$y_{max} = 70\,mm$$

$$\sigma_{max} = \pm \frac{M\,y_{max}}{I} = \frac{(50 \times 10^3)(70) \times 10^{-3}}{2.077 \times 10^{-5}} = 168.5 \times 10^6\ pa$$

$\boxed{\sigma_{max} = 168.5\ Mpa}$ ("T" and "C").

$$Q = A\bar{y} = (120)(20)(60) = 1.44 \times 10^{+5}\ mm^3$$



$$Q = 1.44 \times 10^{-4}\ m^3$$

$$q = \frac{QV}{I} = \frac{(10 \times 10^3)(1.44 \times 10^{-4})}{2.077 \times 10^{-5}} = 69330\ N/m$$

$R_n = 2 \times 5 = 10 \text{ KN}$ (2 rows of nails)

$R_n = q \cdot S \Rightarrow S_{max} = \frac{R_n}{q}$

$S_{max} = \frac{10 \times 10^3}{69330} = 0.144 \text{ m}$

$\boxed{S_{max} = 144 \text{ mm}}$

Schema b)

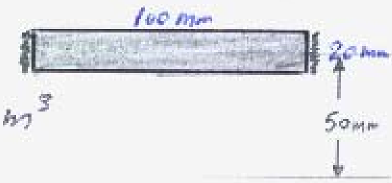
$I = \left[\frac{1}{12}(140)(120)^3\right] - \left[\frac{1}{12}(100)(80)^3\right] = 1.589 \times 10^7 \text{ mm}^4$

$I = 1.589 \times 10^{-5} \text{ m}^4$

$y_{max} = 60 \text{ mm}$

$\sigma_{max} = \pm \frac{M y_{max}}{I} = \frac{(50 \times 10^3)(60) \times 10^{-3}}{1.589 \times 10^{-5}} = 188.8 \times 10^6 \text{ pa}$

$\boxed{\sigma_{max} = 188.8 \text{ Mpa}}$ ("T" or "C")



$Q = (100)(20)(50) = 10^5 \text{ mm}^3 = 10^{-4} \text{ m}^3$

$q = \frac{VQ}{I} = \frac{(10 \times 10^3) \times 10^{-4}}{1.589 \times 10^{-5}} = 62930 \text{ N/m}$

$S_{max} = \frac{R_n}{q} = \frac{10 \times 10^3}{62930} = 0.159 \text{ m}$

$\therefore \boxed{S_{max} = 159 \text{ mm}}$

$\therefore$ scheme (a) is more efficient in bending because $\sigma_{max}^{(a)} = 168.5 \text{ Mpa} < \sigma_{max}^{(b)} = 188.8 \text{ Mpa}$.

$\therefore$ scheme (b) is better in shear because $S_{max}^{(b)} = 159 \text{ mm} > S_{max}^{(a)} = 144 \text{ mm}$

Problem 5 :-

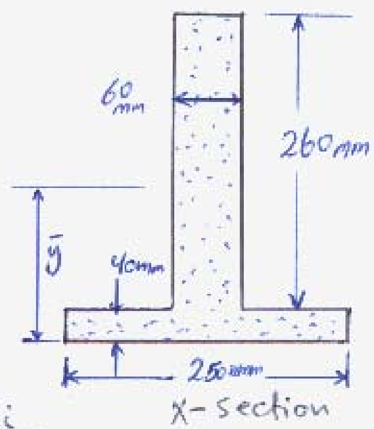
Given :- The beam shown in the fig. with its cross-section.

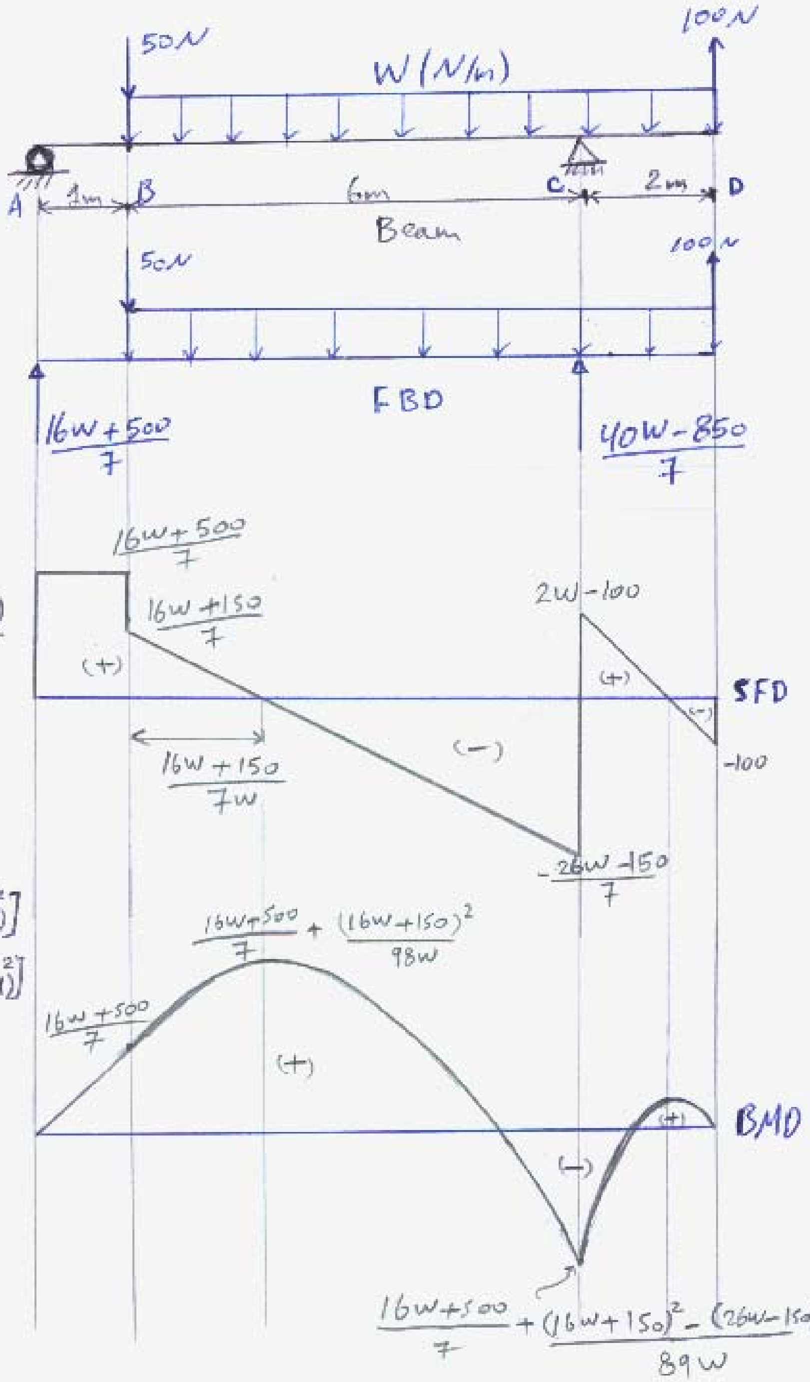
The allowable stresses are :-

$$\begin{cases} \sigma_t = 150 \text{ Mpa}; \\ \sigma_c = 120 \text{ Mpa}; \\ \tau = 80 \text{ Mpa}. \end{cases}$$

Req :- The max. Load (W) which can be applied.

Solution :-





$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i}$$

$$\bar{y} = \frac{(250\times40)(20) + (60\times260)(40+\frac{260}{2})}{(250\times40)+(60\times260)}$$

$$\bar{y} = 111.4 \text{ mm}$$

$$I = \Sigma(\bar{I}_i + A_i d_i^2)$$

$$I = [\tfrac{1}{12}(250)(40)^3 + (250\times40)(111.4-20)^2] + [\tfrac{1}{12}(60)(260)^3 + (60\times260)(170-111.4)^2]$$

$$I = 226.3\times10^{6} \text{ mm}^4$$

$$I = 226.3\times10^{-6} \text{ m}^4$$

First consider shear strength requirement.

$$|V_{max}| = \frac{26w - 150}{7} \quad \text{.... "assume"}$$

$$\tau_{max} = \frac{V\,Q_{max}}{I\,b}$$

$$80 \times 10^6 = \frac{(26w - 150)\left[111.4 \times 60 \times (111.4/2) \times 10^{-9}\right]}{7 \times (226.3 \times 10^{-6})(60 \times 10^{-3})}$$

$$\therefore\ w = 785 \times 10^3\ N/m$$

$$w = 785\ KN/m$$

* check V_{max}

$$\frac{26w - 150}{7} = 2894.3\ KN$$

$$\frac{16w + 150}{7} = 1815.7\ KN < 2894.3\ KN \Rightarrow \text{ok}$$

* Now consider tensile strength

$$M_{max} = \frac{16w + 500}{7} + \frac{(16w + 150)^2}{98w}$$

$$M_{max} = \frac{240w^2 + 5900w + 11250}{49w} \quad ; \quad \sigma_{max} = \frac{M_{max}\,y_{max}}{I}$$

$$\therefore\ 150 \times 10^6 = \frac{240w^2 + 5900w + 11250}{49w} \times \frac{(111.4 \times 10^{-3})}{226.3 \times 10^{-6}}$$

$$240w^2 - 14.93 \times 10^6 w + 11250 = 0$$

$$\therefore\ w = 62.2 \times 10^3\ N/m \quad \text{or} \quad w = 754 \times 10^{-6}\ N/m \approx 0$$

"This is not an option"

i.e. $\underline{\underline{w = 62.2\ KN/m}}$

* Now consider compressive strength

$$\sigma^{c}_{max} = \left| \frac{M_{max}\, y_{max}}{I} \right|$$

$$\left.\begin{aligned} M_{max} &= \frac{16w+500}{7} + \frac{(16w+150)^2}{98w} \\ M_{max} &= \frac{240w^2+5900w+11250}{49w} \end{aligned}\right\} \text{as above}$$

$$120\times10^6 = \frac{240w^2+5900w+11250}{49w} * \frac{(260+40-111.4)\times10^{-3}}{226.3\times10^{-6}}$$

$$240w^2 - 7.0495\times10^6 w + 11250 = 0$$

$\therefore$ $w = 29.4\times10^3$ N/m or $w = 1.60\times10^{-3}$ N/m ≈ 0

"This is not an option"

i.e. $\underline{\underline{w = 29.4 \text{ KN/m}}}$

$\therefore$ $\boxed{w_{max} = 29.4 \text{ KN/m}}$

$$\sigma_{T,c} = -\frac{My}{I}$$

* Check if $M^{(+)}$ controls both (T & c)

take $M^{(-)}_{max} = \frac{16w + 500}{7} + \frac{(16w+150)^2 - (26w-150)^2}{89w}$

$\therefore M^{(-)} = 67271.43 - 138600$

$M^{(-)} = -71328.57$ N.m

$$\sigma_T = -\frac{My}{I} = -\frac{(-71328.57)(260+40-111.4)\times 10^{-3}}{226.3\times 10^{-6}}$$

$\sigma_T = 59.45$ Mpa < 150 Mpa --- ok.

$$\sigma_c = -\frac{My}{I} = \frac{-(-71328.57)(-111.4)\times 10^{-3}}{226.3\times 10^{-6}}$$

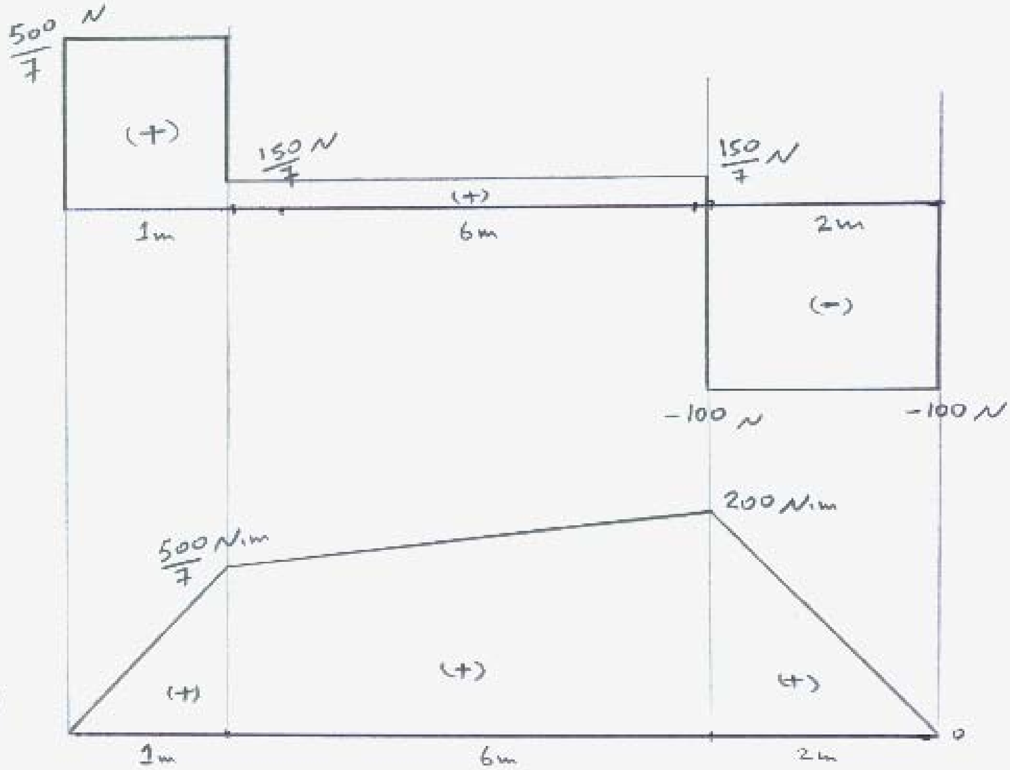
$\sigma_c = 35.113$ Mpa < 120 Mpa ---- ok

$\therefore M^{(+)}$ controls.

if $w = 0$

SFD

BMD



check V_{max}

$V_{max} = 100\,N < 2894.7 \Rightarrow$ ok.

$$\tau = \frac{VQ}{Ib} = \frac{(2650 - 150)(111.4 \times 60 \times (111.4/2) \times 10^{-9}}{7 \times (226.3 \times 10^{-6})(60 \times 10^{-3})}$$

$$\tau = 587.6\,\frac{N}{m^2} < 80 \times 10^{6}\,\frac{N}{m^2}$$

$$M_{max} = 200\ N.m$$

$$\sigma^{T}_{max} = \frac{200(0.1114)}{226.3 \times 10^{-6}} = 98.453\ Kpa < 150\ Mpa.$$

$$\sigma^{c}_{max} = \frac{200\ (0.1886)}{226.3 \times 10^{-6}} = 166.68\ Kpa < 120\ Mpa.$$