#9

Examples

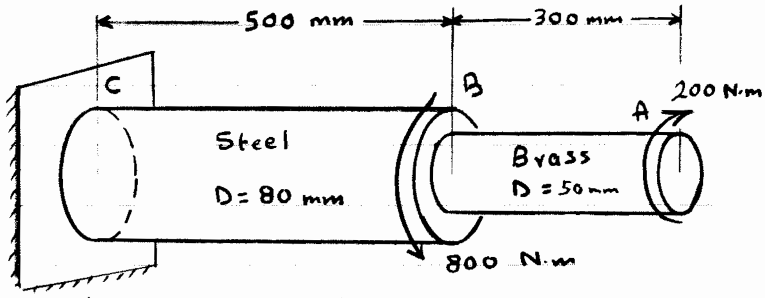
Torsion : Circular Shafts

Statically Determinate & Indeterminate

Example 1:

Given:

The shaft shown



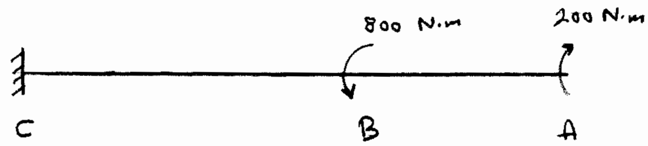
Req.d. :

a) The value and location of the maximum shearing stress

b) τ distribution at section of τ_{max}

c) The angle of twist of A relative to C ($\Phi_{A/C}$)

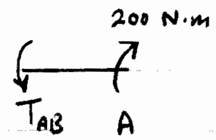
Soln. :



a) We can not know by inspection whether τ_{max} is in AB or BC. (Why?!)

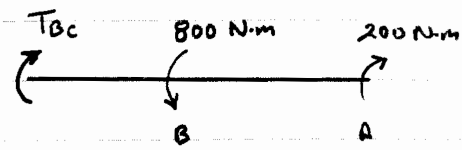
$\Rightarrow$ Calculate τ_{max} in both.

$T_{AB} = 200$ N.m



$$\tau = \frac{Tr}{J}$$

(imp. : T is the internal torque)



$T_{BC} = 800 - 200 = 600$ N.m

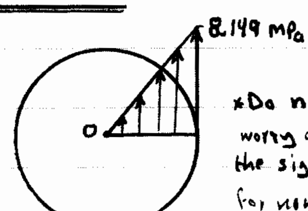
$$\tau_{max} = \frac{T r_{max}}{J} = \frac{T r_{out}}{J}$$

$$\Rightarrow \tau_{max}^{AB} = \frac{200(0.025)}{\frac{\pi}{2}(0.025)^4} = 8.149 (10)^6 \text{ Pa} = 8.149 \text{ MPa}$$

$$\tau_{max}^{BC} = \frac{600(0.04)}{\frac{\pi}{2}(0.04)^4} = 5.968 (10)^6 \text{ Pa} = 5.968 \text{ MPa}$$

$\Rightarrow$ $\tau_{max} = 8.149$ MPa @ the outside radius of portion AB

b) $\tau = \frac{Tr}{J} = \frac{200\, r}{\frac{\pi}{2}(0.025)^4} = 3.2595 (10)^8\, r$ $\Rightarrow$ distribution



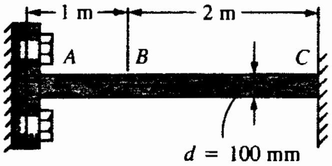
*Do not worry about the sign for now!!

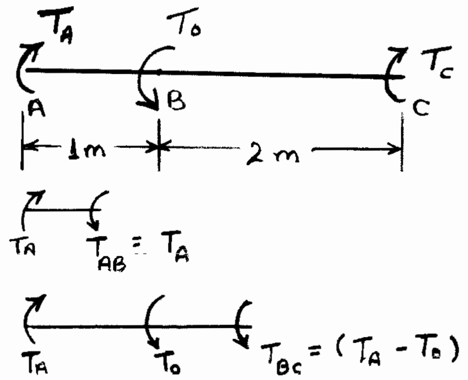
c) $\Phi_{A/C} = \Sigma \Phi = \Phi_{A/B} + \Phi_{B/C}$

$$= \left(\frac{TL}{JG}\right)_{A/B} + \left(\frac{TL}{JG}\right)_{B/C} = \frac{-200(0.3)}{\frac{\pi}{2}(0.025)^4\, 39(10)^9} + \frac{+600(0.5)}{\frac{\pi}{2}(0.04)^4\, 84(10)^9}$$

$$= -0.002573 + 0.0008881 = -0.001685 \text{ rad} \Rightarrow \Phi_{A/C} = 0.09655^\circ$$

An aluminum (G = 28 GPa) shaft is rigidly supported at its right end as shown in Figure below. The left end of the shaft is connected to a rigid support via a flange plate. The flange plate allows the shaft to rotate 0.010 rad before the bolts provide rigid support. Determine the maximum torque that can be applied at point B if the shearing stress is not to exceed 50 MPa.





Given & Reqd.:

As above

Sol'n.:

First, check if $\phi_{A/C}$ controls (i.e., $\phi_{A/C}$ reaches $\phi_{A/C\,max} = \pm 0.010$ before τ reaches 50 MPa $\Rightarrow$ the problem becomes stat. indet.).

$$J = \frac{\pi}{2}(0.05)^4 = 9.818(10)^{-6}\ m^4$$

$$\phi = \frac{TL}{JG} \Rightarrow T = \frac{0.01}{2}(9.818)(10)^{-6}(28)(10)^6 = 1.375\ kN.m$$

Check, now, τ: $\tau = \frac{Tr}{J} \Rightarrow \tau_{max} = \frac{1.375\,(0.05)}{9.818(10)^{-6}(10)^3} = 7.0\ MPa < 50\ MPa$

Since τ did not reach τ_{max}, more T can be applied $\Rightarrow$ The problem becomes stat indet. « There are other ways for checking. Think !! »

① Equilibrium: $T_o - T_A - T_C = 0$

② Geometric Compatibility: $\phi_{A/C} = -0.01$ rad «note direction»

③ Material properties (Equations): $d\phi/dz = T/JG$

$$\phi_{A/C} = \frac{T_A(1)}{GJ} + \frac{(T_A - T_o)2}{GJ} \equiv -0.01 \Rightarrow 3T_A - 2T_o = -0.01\,GJ$$

Assume τ_{AB} Controls (Why ?!) $\Rightarrow$

$$\tau_{AB} = T_A\,c/J \Rightarrow 50(10)^3 \equiv T_A(0.05)/9.818(10)^{-6} \Rightarrow T_A = 9.818\ kN.m$$

$$\Rightarrow T_o = \frac{1}{2}\left[0.01(28)(10)^6(9.818)(10)^{-6} + 3(9.818)\right] = 16.1\ kN.m$$

Now, check τ_{BC}: $T_C = T_o - T_A = 6.283\ kN.m$

$\Rightarrow \tau_C = 32.0\ MPa < 50\ MPa$ ok $\Rightarrow$ $\boxed{T_o = 16.1\ kN.m}$

⊛ Solve the problem by other method(s) ! $\Rightarrow$ Compare.