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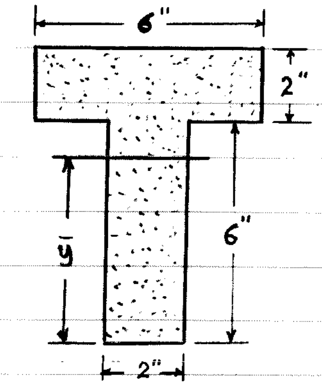
Examples (Bending Stresses)

Example 1:

Given:

The cross section & BMD shown for a beam



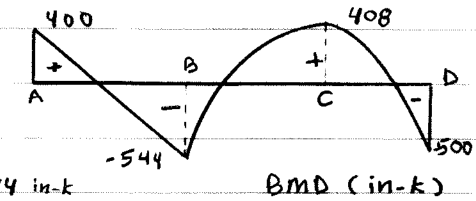
Req. d.:

The values & locations of the maximum positive and negative normal stresses

Sol.n.:

① Find the centroid : $\bar{y}\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} A_i \bar{y}_i$

$\bar{y} = [12(3) + 12(7)] / (12+12) = 5 \text{ in}$



② Calculate I : $I = \sum_{i=1}^{n} (\bar{I}_i + A_i d_i^2)$

$I = [\frac{1}{12}(2)6^3 + 12(2)^2] + [\frac{1}{12}(6)2^3 + 12(2)^2] = 136 \text{ in}^4$

③ Determine max. pos. & neg. M : $M^+_{max} = 408$ in-k , $M^-_{max} = 544$ in-k

④ Locate max y^+ & y^- : $y^+_{max} = 3"$ (top) ; $y^-_{max} = 5$ in (bottom)

⑤ Calculate max negative & positive stresses

$$\sigma = -\frac{My}{I}$$

$$\sigma^T_{max_C} = -\frac{408(-5)}{136} = 15 \text{ ksi} \quad (T) \quad \text{at the bottom of point C}$$

$$\sigma^T_{max_B} = -\frac{-544(3)}{136} = 12 \text{ ksi} \quad (T) \quad \text{at the top of point B}$$

$15 > 12 \Rightarrow$ $\boxed{\sigma^T_{max} = 15 \text{ ksi (T) at the bottom of point C}}$

$$\sigma^C_{max_B} = -\frac{-544(-5)}{136} = -20 \text{ ksi} \quad (C) \quad \text{at the bottom of point B}$$

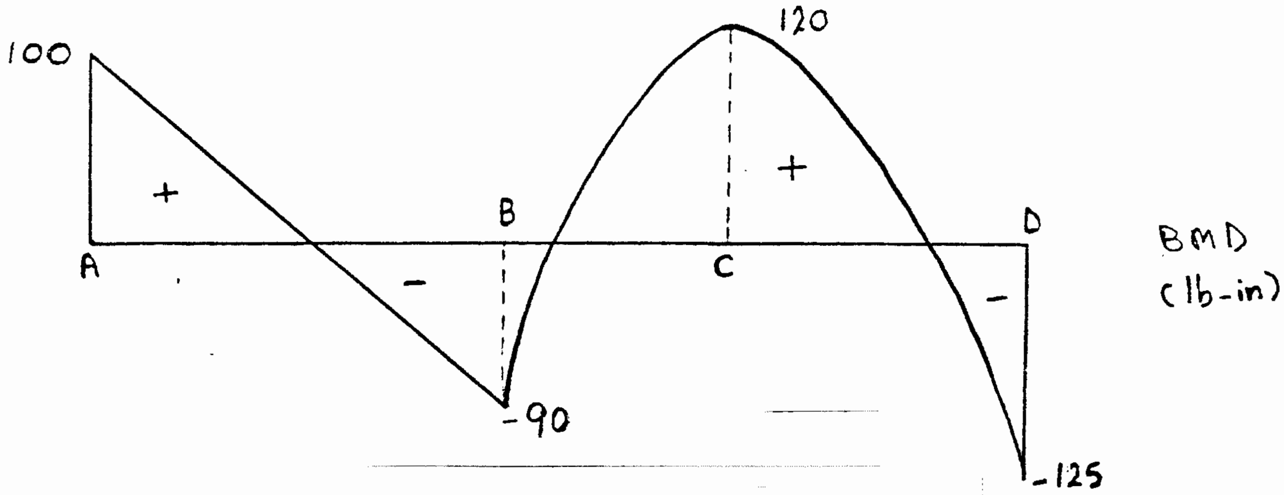
Clearly, the other option ($M = +408$ and $y = +3$) does not control (i.e., it will not give a bigger value for σ) because both M and y are smaller.

$\Rightarrow$ $\boxed{\sigma^C_{max} = 20 \text{ ksi (C) at the bottom of point B}}$

Important note: You need to check two points, one is the point of max. pos. M and the other is point of max. neg. M. For both points check the top and bottom of the cross section for max. ten. & comp. σ.

Example 2 :

The bending moment diagram (BMD) and the cross section of a beam are shown below. Determine the values and locations of the maximum positive and negative normal stresses.



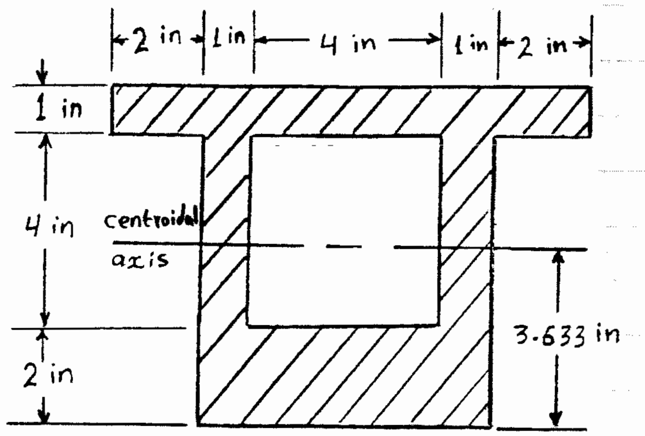
Soln. :

The steps in Example 1 will be followed

$\bar{y} = 3.633$ in

$$I = \frac{10(1)^3}{12} + 10(1)(\overbrace{7-3.633-0.5}^{3.367})^2 + 2\left[\frac{1(4)^3}{12} + 1(4)(4-3.633)^2\right] + \frac{6(2)^3}{12} + 6(2)(1-3.633)^2$$

$= 182.$ in^4



$M^+_{max} = 120$ in-lb @ C

$M^-_{max} = -125$ in-lb @ D

$\sigma = -My/I$

Point C :

$$\sigma^T_{max} = \frac{-120(-3.633)}{182} = 2.40 \text{ psi (T) at bottom}$$

$$\sigma^C_{max} = -\frac{120(3.367)}{182} = 2.22 \text{ psi (C) at top} \quad \text{(note - sign)}$$

$\Downarrow$ C

Point D :

$$\sigma^T_{max} = -\frac{-125(3.367)}{182} = 2.31 \text{ psi (T) at top}$$

$$\sigma^C_{max} = -\frac{-125(-3.633)}{182} = 2.50 \text{ psi (C) at bottom} \quad \text{(note - sign)}$$

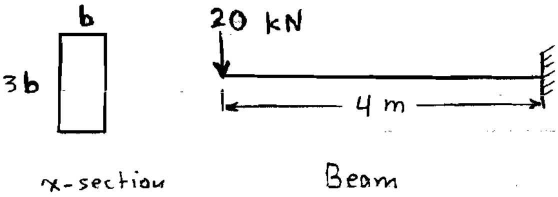
$\Rightarrow$ $\boxed{\sigma^T_{max} = 2.40 \text{ psi @ bottom of C}}$; $\boxed{\sigma^C_{max} = 2.50 \text{ psi @ bottom of D}}$

Example 3 :

Given :

The beam with the section shown

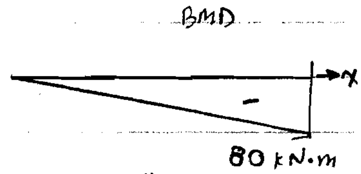
$\sigma_{max} = 30$ MPa (C & T)



Req'd. :

The minimum value of b (cross-sectional dimension)

Sol'n. :

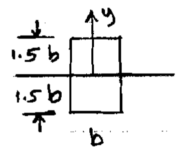


$$\sigma_{max} = \frac{M_{max} y_{max}}{I}$$

$M_{max} = 20(4) = 80$ kN.m (at the fixed end)

$y_{max} = 1.5\,b$

$I = \frac{1}{12} bh^3 = \frac{1}{12} b(3b)^3 = \frac{9}{4} b^4$



$\Rightarrow\ 30(10)^6 \equiv 80(10)^3(1.5b)/\frac{9}{4}b^4$

$\Rightarrow\ b^3 = 0.0017778\ m^3 \ \Rightarrow$

$\Rightarrow$ $\boxed{b = 0.121\ m = 121\ mm}$ the section is 121 by 363 mm

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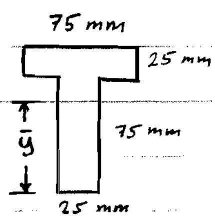
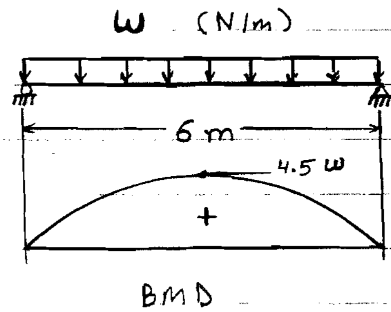
Example 4 :

Given:

The beam & section shown

$\sigma^T_{max} = 40$ MPa ; $\sigma^C_{max} = -25$ MPa





Req'd. :

a) The max. value of w

b) The force in the flange @ point of M_{max}

c) The moment of the force in the flange of (b) about N.A.

d) Show that $M_{from\,\sigma} = M_{from\,BMD}$ & $N_{total} = 0$ (pure bending)

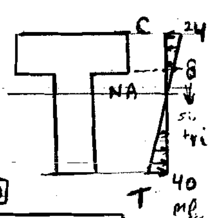
Sol'n. :

$\bar{y} = 62.5$ mm $= 0.0625$ m ; $I_{NA} = 3.32(10)^{-6}\ m^4$; $M_{max} = (\frac{wL}{2})(\frac{L}{2})(\frac{1}{2}) = 4.5\,w$

a) $\sigma = -My/I \Rightarrow \sigma^T_{max} = 40(10)^6 \equiv -4.5w(-0.0625)/3.32(10)^{-6} \Rightarrow w_{max} = 472.2$ N/m

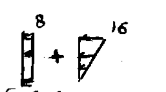
$\sigma^C_{max} = -25(10)^6 \equiv -4.5w\underbrace{(0.1-0.0625)}_{0.0375}/3.32(10)^{-6} \Rightarrow w_{max} = 491.9$ N/m

$\Rightarrow$ $\boxed{w_{max} = 472.2\ N/m}$



b) $\sigma_{bottom} = +40$ MPa , $\sigma_{top} = -\frac{4.5(472.2)0.0375}{3.32(10)^{-6}} = -24$ MPa (C)

$F_{flange} = \sigma_{av} A = -\frac{24+8}{2}(0.075 \times 0.025) = -0.03$ MN $= -30$ kN (C)



c) $M_{NA(F_{flange})} = 8(10)^6(0.075 \times 0.025)(0.0125 + 0.0125) + \frac{16(10)^6}{2}(0.075 \times 0.025)[0.0125 + \frac{2}{3}(0.025)]$

$\Rightarrow$ $\boxed{M_{Flan.} = 812.5\ N.m}$

d) $\Sigma N = -0.03 - \frac{8}{2}(0.0125 \times 0.025) + \frac{40}{2}(0.0625 \times 0.025) = 0$ ok

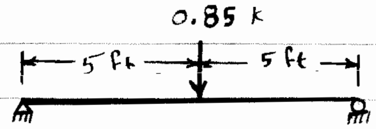
$M_{C} = 812.5 + 0.0125(10)^6(\frac{2}{3}0.0125) + 0.03125(10)^6(\frac{2}{3}0.0625) = 2125$ N.m $= M_{BMD}$ ok

Example 5 :

Given :

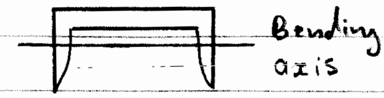
The beam shown with steel channel section

$\sigma_{max} = 18$ ksi $(\pm)$



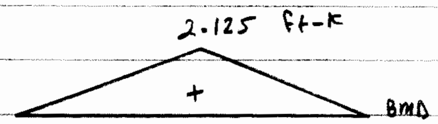
Req.d. :

The section with least weight



Sol,n. :

From the BMD shown, $M_{max} = 2.125$ ft-k.



The properties of several structural steel shapes are given in an appendix in the textbook.

$$\sigma_{max} = \frac{Mc}{I}$$ (do not worry about the sign as we have $\pm$ 18 ksi)

$$18 = \frac{2.125\ (12\ \text{in})\ c}{I} \Rightarrow \frac{I}{c} = 1.42\ \text{in}^3 = S = \text{Section modulus}$$

[Explanation of designation in the appendix: examples, W 18 X 60, S 24 X 100, C 15 X 50. The first letter is for the shape (W = wide-flange, S = standard, C = channel, L = leg ... etc). The following number is the nominal depth in inches (or mm), and the last number is the weight (or mass) in pound (or kg) per foot (or m) (length). Thus, C 15 X 50 means channel section 15 in deep by 50 lb/in. The weight is the measurment of the cost.]

From the table in the book, we need to select the section with S closest to 1.42 in^3 and with least weight. We may have more than one option. Note that in this example, bending is about the y axis, thus, we must use S_y.

options : ① C 12 X 20.7 ($S_y = 1.73$ in^3 ≥ 1.42 ok)

② C 10 X 25 ($S_y = 1.48$ in^3 ≥ 1.42 ok)

Note that option ② has S_y closest to 1.42 ; however, option ① is cheaper (less weight as $20.7 < 25$) and better (bigger S_y as $1.73 > 1.48$) $\Rightarrow$ $\boxed{\text{Select C 12 X 20.7}}$