#13

Torsion of Sections Composed of Narrow Rectangles

Theory & Examples

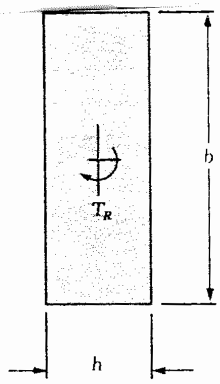


Figure 4-34

For the long, narrow rectangular section of height b and base h shown in Figure 4-34, the following approximations are obtained from the exact analysis

$$\left.\begin{aligned} J &= \tfrac{1}{3}bh^3 \\ \tau_{max} &= \frac{T\ h}{J} \end{aligned}\right\} \qquad (4\text{-}61)$$

Here b is the long side of the narrow rectangle, and h is the short side. The maximum shearing stress occurs at the midpoint of the long side of the rectangle.

The value of J for sections composed of several long, narrow rectangles is approximately

$$J = \sum_{i=1}^{n} J_i, \qquad (4\text{-}62)$$

where J_i is the value of J for ith rectangular segment. This formula can be used to calculate the value of J for angles, channels, and wide-flange and other sections that are made up of narrow rectangles.

The corresponding approximate angle of twist is

$$\frac{d\varphi}{dz} = \frac{T}{JG} \quad ; \quad \Phi = \frac{TL}{JG} \qquad (4\text{-}63)$$

EXAMPLE 1:

Determine the magnitude and location of the maximum shearing stress in each leg of the aluminum angle section shown in Figure 4-35 when a 5-ft length is subjected to twisting couples of 3000 in.-lb at its ends. Determine the relative rotation of the ends of this torsion member.

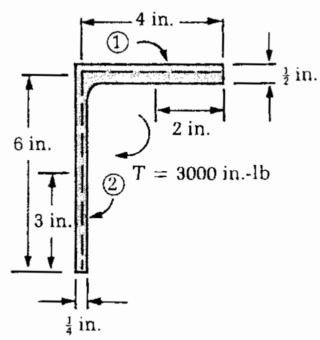


Figure 4-35

SOLUTION

This cross section consists of two narrow rectangles. An approximate value of polar moment of inertia is obtained from Eqs. (4-61) and (4-62). Accordingly,

$$J = J_1 + J_2 = \tfrac{1}{3}(4)(0.5)^3 + \tfrac{1}{3}(6)(0.25)^3 = 0.1979 \text{ in}^4 \qquad \text{(a)}$$

The magnitude of the maximum shearing stress in each leg is given by the second formula in Eqs. (4-61). Thus

$$\left.\begin{aligned} \tau_{max}^{(1)} &= \frac{3000}{0.1979}(0.5) = 7580 \text{ psi} \\ \tau_{max}^{(2)} &= \frac{3000}{0.1979}(0.25) = 3790 \text{ psi} \end{aligned}\right\} \qquad \text{(b)}$$

Note that $\frac{4}{\frac{1}{2}} = 8 < 10$

but $\approx$ OK!

These stresses occur at the midlengths of the horizontal and vertical legs of the section.

The relative rotation of the ends of the bar is obtained through the use of Eq. (4-63). Accordingly,

$$\varphi = \frac{3000(60)}{0.1979(4 \times 10^6)} = 0.227 \text{ rad} \qquad \text{(c)}$$

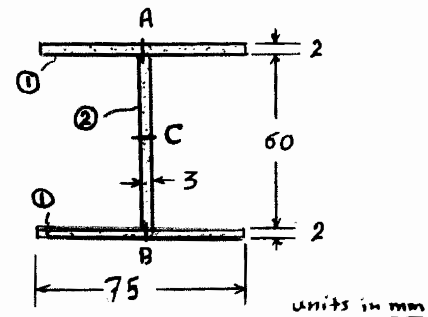
Example 2 :

Given :

A shaft with the cross section shown

$T = 1880$ N.mm

$G = 80$ GPa



Req.d.:

a) the value and location of the maximum shearing stress

b) the unit angle of twist

Soln. :

First, check if "narrow" rectangles :

$$\underbrace{\frac{b}{a} = \frac{b}{h}}_{\text{same}} = \frac{75}{2} = 37.5 > 10$$

$$\frac{b}{h} = \frac{60}{3} = 20 > 10$$

$\Rightarrow$ ok $\Rightarrow$ Use narrow rectangular sections theory

a)

$$\tau_{max} = \frac{Th}{J}$$

$$J = \sum_{i=1}^{3} J_i = \sum_{i=1}^{n} \left(\tfrac{1}{3} bh^3\right)_i = \tfrac{1}{3}\left[\overset{\text{two legs}}{2}(75)(2)^3 + 60(3)^3\right] = 940 \text{ mm}^4$$

* Very important : h is always the smaller dimension.

$$\tau_{max}^{①} = \frac{Th_①}{J} = \frac{1880(2)}{940} = 4 \text{ N/mm}^2 = 4 \text{ MPa}$$

$$\tau_{max}^{②} = \frac{Th_②}{J} = \frac{1880(3)}{940} = 6 \text{ N/mm}^2 = 6 \text{ MPa}$$

$\Rightarrow$ $\tau_{max} = 6$ MPa @ C (middle of the thickest leg)

* important : τ_{max} is always at the middle of the thickest leg for this type of section (narrow rectangular), because T and J are common for all legs.

b)

$$\frac{d\phi}{dz} = \frac{T}{JG}$$

$$= \frac{1880(10)^{-3}}{940(10^{-3})^4\, 80(10)^9} \Rightarrow \frac{d\phi}{dz} = 0.025 \text{ rad/m}$$