#12

Torsion: Solid Noncircular Sections

(Rectangular Sections)

Theory:

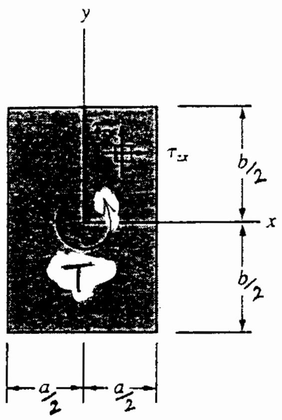


Figure 5–32

Rectangular cross section. Analytical solutions have been obtained for the torsion of rectangular, elastic members of sides a and b as shown in Figure 5–32. The components of shearing stress are given by formulas in terms of infinite series. The mathematical treatment is beyond the scope of this book. The analysis shows that shearing stresses at the corners are zero. Note that the corners of a rectangular cross section are its most remote points. Accordingly, the stress is zero at the most remote point of the rectangular cross section. Recall that the stress was a maximum at the most remote point for circular cross sections. This observation emphasizes how different the stress distributions are for these two cross sections. The maximum shearing stress for a rectangular cross section occurs at the midpoint of the long side. It can be calculated analytically and the result expressed in terms of an infinite series.

The maximum shearing stress and the unit angle of twist are frequently expressed in the forms

$$\left.\begin{aligned} \tau_{max} &= \frac{T}{\alpha b a^2} \\ \frac{d\phi}{dz} &= \frac{T}{\beta b a^3 G} \end{aligned}\right\} \qquad \Phi = \frac{TL}{\beta b a^3 G} \tag{5–60}$$

where a is the length of the short side of the rectangle and b is the length of the long side. The coefficients α and β are determined so that, when they are substituted into Eq. (5–60), τ_{max} and $d\phi/dz$ coincide with values τ_{max} and $d\phi/dz$ calculated from the more advanced analyses. Numerical values for α and β for various values of aspect ratio b/a are listed in Table 5–1. For thin rectangles—that is, for b much greater than a—$\alpha = \beta = 1/3$.

TABLE 5–1 Torsion Constants for Rectangular Bars

b/a	1.00	1.50	1.75	2.00	2.50	3.00	4	6	8	10	∞
α	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.298	0.307	0.312	0.333
β	0.141	0.196	0.214	0.229	0.249	0.263	0.281	0.298	0.307	0.312	0.333

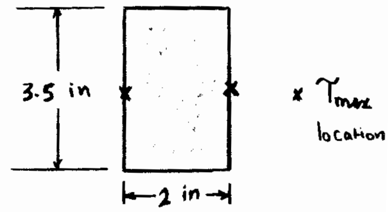
Torsion : Solid Noncircular Sections

(Rectangular Sections)

Example :

Given :

An aluminum shaft ($G = 4000$ ksi)
with the rectangular cross section shown
$T = 6$ in-k



Req.d :

a) The value and location of the maximum shearing stress
b) The unit angle of twist

Soln. :

a) $\tau_{max} = \dfrac{T}{\alpha\, b\, a^2}$ @ the middle of the long leg

$\dfrac{b}{a} = \dfrac{3.5}{2} = 1.75$ ((note that $b \geq a$ ⇒ You must choose **b** as the bigger dimension))

⇒ From the table attached,

$\alpha = 0.239$
$\beta = 0.214$

} Use interpolation if the value of $\frac{b}{a}$ is not "exactly" in the table.

$$\Rightarrow \tau_{max} = \frac{6}{0.2385\,(3.5)(2)^2}$$

$$\Rightarrow \tau_{max} = 1.797 \text{ ksi}$$ @ the middle of the long leg ((as shown))

b) $$\frac{d\Phi}{dz} = \frac{T}{\beta\, b\, a^3 G}$$

$$= \frac{6}{0.214\,(3.5)(2)^3(4000)}$$

$$\Rightarrow \frac{d\Phi}{dz} = 2.503(10)^{-4} \text{ rad/in}$$