General States of Stress and Strain

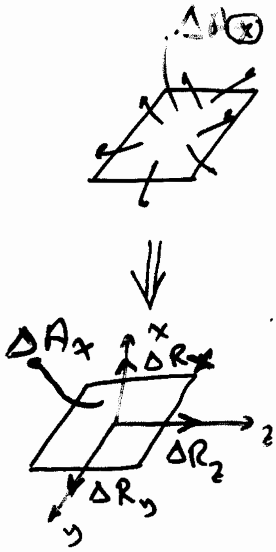
Stresses

The forces per unit area (stresses) are:

$$\frac{\Delta R_x}{\Delta A_x}$$

$$\frac{\Delta R_y}{\Delta A_x}$$

$$\frac{\Delta R_z}{\Delta A_x}$$



Let's now take the limit as

$\Delta A_x \to 0 \Rightarrow$

$$\lim_{\Delta A_x \to 0} \frac{\Delta R_x}{\Delta A_x} = \frac{dR_x}{dA_x} = \sigma_{x\,x}$$

↳ means direction in $\underline{\underline{x}}$-axis

↳ area $\perp$ to $\underline{x}$-axis

((Since x is repeated in $\underline{\underline{\sigma_{xx}}}$, we can drop one $x \Rightarrow \boxed{\sigma_x}$ ← normal

$$\lim_{\Delta A_x \to 0} \frac{\Delta R_y}{\Delta A_x} = \frac{dR_y}{dA_x} = \sigma_{xy} \ \underline{\underline{\text{or}}} \ \boxed{\tau_{xy}} \leftarrow \text{shear}$$

$$\lim_{\Delta A_x \to 0} \frac{\Delta R_z}{\Delta A_x} = \frac{dR_z}{dA_x} = \sigma_{xz} \ \underline{\underline{\text{or}}} \ \boxed{\tau_{xz}}$$

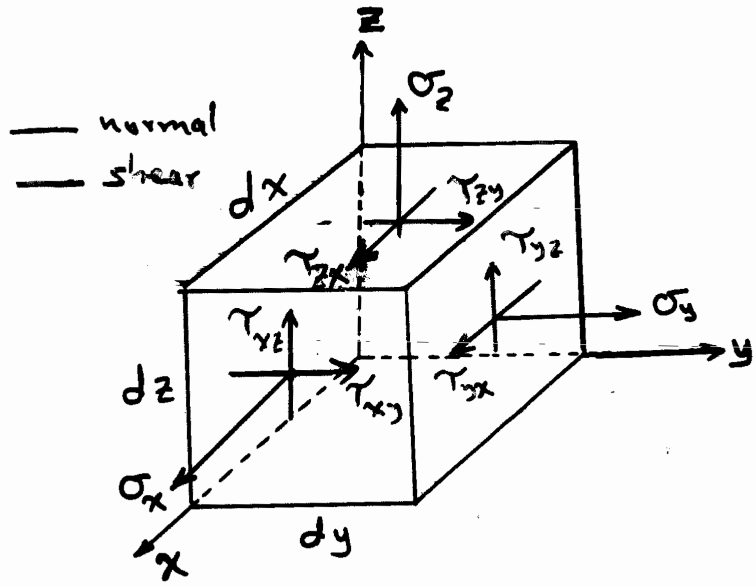
Similarly for $\Delta A_y \Rightarrow \underline{\underline{\tau_{yx}}}, \ \underline{\underline{\sigma_{y}}}, \ \underline{\underline{\tau_{yz}}}$

Similarly for $\Delta A_z \Rightarrow \underline{\underline{\tau_{zx}}}, \ \underline{\underline{\tau_{zy}}}; \ \underline{\underline{\sigma_{z}}}$

Now, let's consider 3-D differential volume element :

There are 9 stress components on the positive faces

((Note that the stresses on the negative faces are equal & opposite))



* 3 normal stresses :

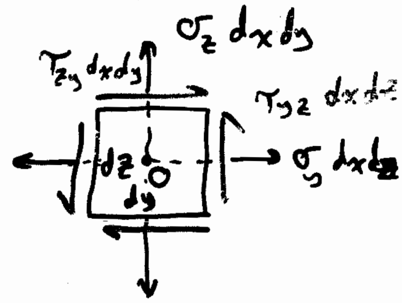
$$\sigma_x \ , \ \sigma_y \ , \ \sigma_z$$

* 6 shearing stresses :

$$\tau_{xy} \ , \ \tau_{xz} \ , \ \tau_{yx} \ , \ \tau_{yz} \ , \ \tau_{zx} \ , \ \tau_{zy}$$

Now, we need to find some relations among shearing stresses (if there are any).

Consider the differential area shown ⇒



$\circlearrowleft^{+} \ \Sigma M_o = 0 \Rightarrow$

$$2\left[\tau_{yz}\, dx\, dz\left(\frac{dy}{2}\right)\right] - 2\left[\tau_{zy}\, dx\, dy\left(\frac{dz}{2}\right)\right] = 0$$

$$\Rightarrow \quad \tau_{yz} = \tau_{zy}$$

Similarly,

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{xz} = \tau_{zx}$$

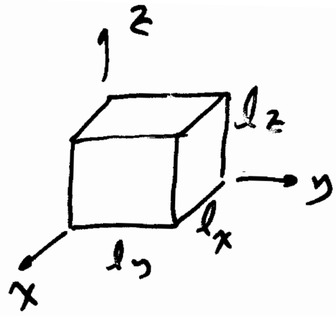
⇒ There are only <u>6</u> <u>independent</u> stress components at any point :

$$3 \text{ normal} : \quad \sigma_x \ , \ \sigma_y \ , \ \sigma_z$$

$$3 \text{ shear} : \quad \tau_{xy} \ , \ \tau_{xz} \ , \ \tau_{yz}$$

Strains:

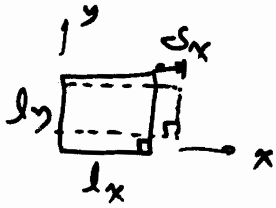
General strain state:



3-D

Consider **normal** strains:

Take x as an example



2-D

$$\varepsilon_x = \frac{\delta_x}{l_x}$$

Similarly,

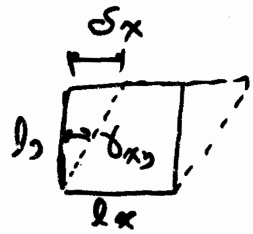
$$\varepsilon_y = \frac{\delta_y}{l_y}$$

$$\varepsilon_z = \frac{\delta_z}{l_z}$$

Normal strains are a measure of volumetric alteration

Note that the 90° angles **remain** 90°

Now, consider **shearing** strains:

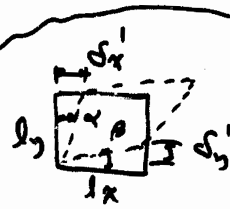


$$\gamma_{xy} = \frac{\delta_x}{l_y}$$

Similarly,

$$\gamma_{xz} = \frac{\delta_x}{l_z}$$

$$\gamma_{yz} = \frac{\delta_y}{l_z}$$



or

optional $\Leftarrow$ $\gamma_{xy} = \underbrace{\frac{\delta_x'}{l_y}}_{\alpha} + \underbrace{\frac{\delta_y'}{l_x}}_{\beta}$; $\gamma_{xz} = \frac{\delta_x'}{l_z} + \frac{\delta_z'}{l_x}$; $\gamma_{yz} = \frac{\delta_y'}{l_z} + \frac{\delta_z'}{l_y}$

Summary $\begin{cases} \text{3 normal strains: } \varepsilon_x, \varepsilon_y, \varepsilon_z \\ \text{3 shearing strains: } \gamma_{xy}, \gamma_{xz}, \gamma_{yz} \end{cases}$

* Normal strains change lengths: $+\Delta l$ gives ⊕ ⇒ T (C ⇒ ⊖)

* Shearing strains change the 90° angles: $<90° = ⊕$; $>90° = ⊖$