Material Behavior

σ : Stress
ε : Strain
↳ Epsilon

Standard tension test

$$\sigma_{ave} = \frac{N}{A_o} \quad \Leftarrow \text{Engineering stress}$$

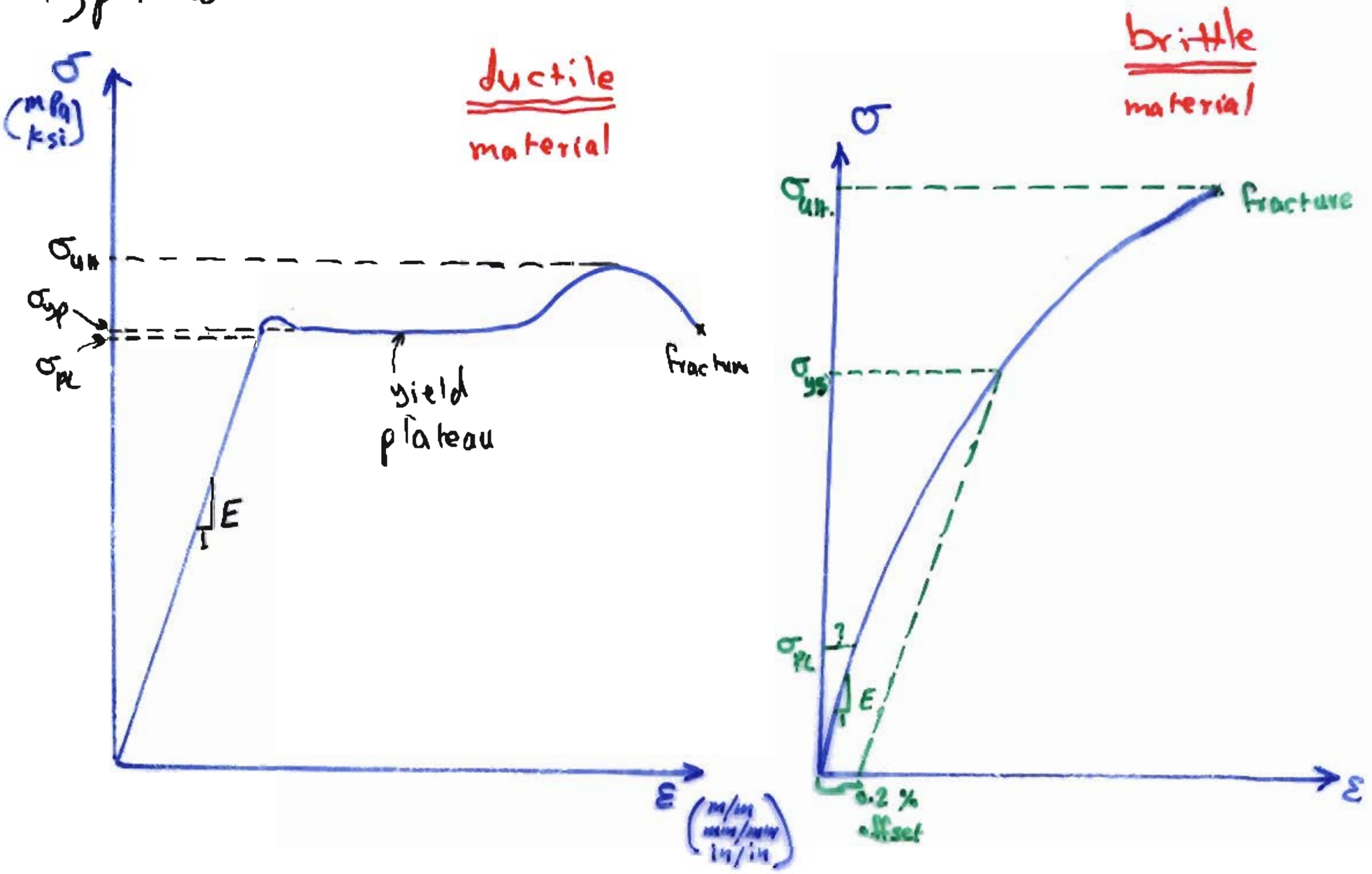
A_o = original area

$$\varepsilon = \frac{l - l_o}{l_o} \quad \Leftarrow \text{Engineering strain}$$

l = length at anytime during test

l_o = original length

Typical $\sigma - \varepsilon$ Curve



σ_{PL} = Proportional limit : the end of linear behavior

⊛ σ_{yp} = yield point stress : constant σ with increasing ε ((ductile materials))

σ_{ys} = yield strength : stress at offset (usually 0.2 %) ((for brittle materials with no σ_{yp}))

⊛ σ_{ult} = ultimate strength : the maximum value for σ in σ-ε curve

σ_{EL} = Elastic Limit : the end of elastic action (linear or nonlinear)

≠ σ_{PL} « technically »

≈ σ_{PL} for many materials

⊛ E = Elastic modulus = Young's modulus = modulus of elasticity

: the slope of the initial elastic portion.

It is a measurement of the material stiffness.

« Big" number $\doteq \frac{\sigma}{\varepsilon}$ ⇐ stress units

$E = \frac{\sigma}{\varepsilon} \Rightarrow$

$$\boxed{\sigma = E\varepsilon}$$

⇐ Hooke's Law

« linear elastic materials »

$E \approx 200$ GPa } steel
$\approx 30\,000$ ksi

≈ 70 GPa } Aluminum
≈ 10 ksi

≈ 20 GPa } Concrete
$\approx 3\,000$ ksi

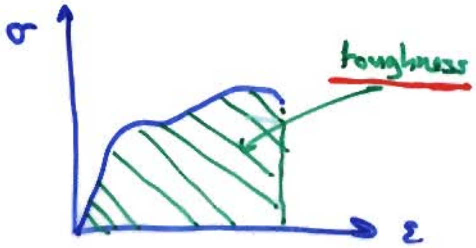
M_R = modulus of resilience : area under $\sigma - \varepsilon$ curve up to σ_{PL}

- units of energy per unit volume

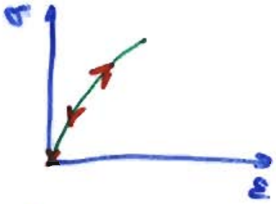
= a measurement of the capacity of the material to absorb energy w/o permanent deformation

Toughess = total area under $\sigma - \varepsilon$ curve

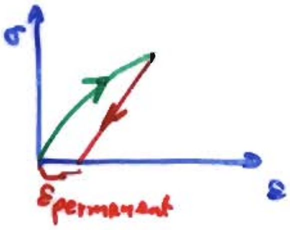
= a measurement of the material to undergo large permanent deformation prior to fracture
«energy / volume»



Elasticity : full recovery of ε after loading/unloading

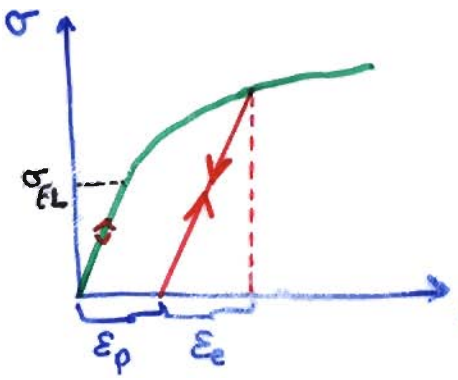


Plasticity : permanent «plastic» strain (deformation) after loading / unloading



Strain hardening : After plastic deformation and unloading, reloading.

σ_{EL} increases and, of course, modulus of resilience, but toughness decreases.
⇒ Strain-hardening



Ductile behavior : The toughness of the material is "large" } Steel/Aluminum
«can how "big" strain before rupture»
= f (temp., rate, type of loading, ...)

Brittle behavior : "opposite" of ductile behavior } concrete & glass

$$\% \text{ reduction of area} = \frac{A_0 - A_f}{A_0} \times 100 \Leftarrow \begin{array}{l} A_0 = \text{original area} \\ A_f = \text{final area} \end{array}$$

$$\% \text{ elongation} = \frac{l_f - l_0}{l_0} \times 100 \leftarrow \begin{array}{l} l_0 = \text{original length} \\ l_f = \text{final length} \end{array}$$

These two values (% reduction in area and elongation) are also used for the description of ductility as the toughness.

Isotropy: Isotropic materials if its elastic properties are the same in all directions as steel and aluminum not woo

Homogeneity: Homogeneous material if its elastic properties are the same throughout the body.

Safety Factor – Allowable Stress :

$$\text{Safety Factor} = \frac{\sigma_{strength}}{\sigma_{allowable}}$$

$$\sigma_{allowable} = \frac{\sigma_{strength}}{\text{Safety Factor}}$$

Why safety factor ?!

Uncertainty in :

- material properties ⇇ ? : - - -
- loading → how much
  - → number
  - → type
  - → location
  - ⋮
  - etc.
- dimensions
- ⋮
- etc.

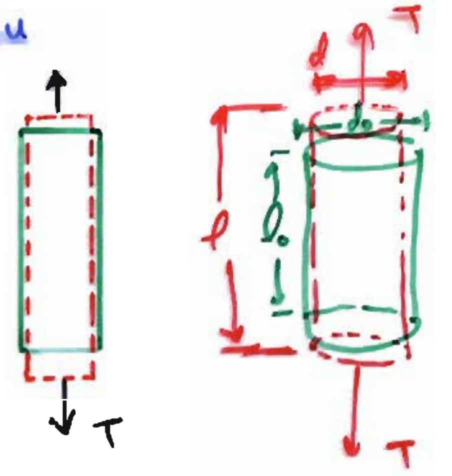
How large SF ?!

It depends on:
- uncertainties ....
- usage of structure (importance... ...)
- type of failure
- stress used in design ... etc.

Poissons Ratio (ν) ⇐ nu

$$\varepsilon_{axial} = \varepsilon_a = \frac{l - l_0}{l_0}$$

$$\varepsilon_{lateral} = \varepsilon_l = \frac{d - d_0}{d_0}$$



$$\nu = \left| \frac{\varepsilon_l}{\varepsilon_a} \right|$$

$0 \leq \nu \leq 0.5$ ⇐ range (theory)

$0.2 \leq \nu \leq 0.35$ ⇐ range for most engineering materials

* ε w/o σ

* σ w/o ε