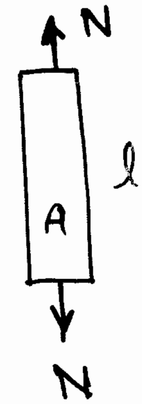
Material Behavior

σ : Stress
ε : Strain
↳ Epsilon

Standard tension test



$$\sigma_{ave} = \frac{N}{A_0} \quad \Leftarrow \text{Engineering stress}$$

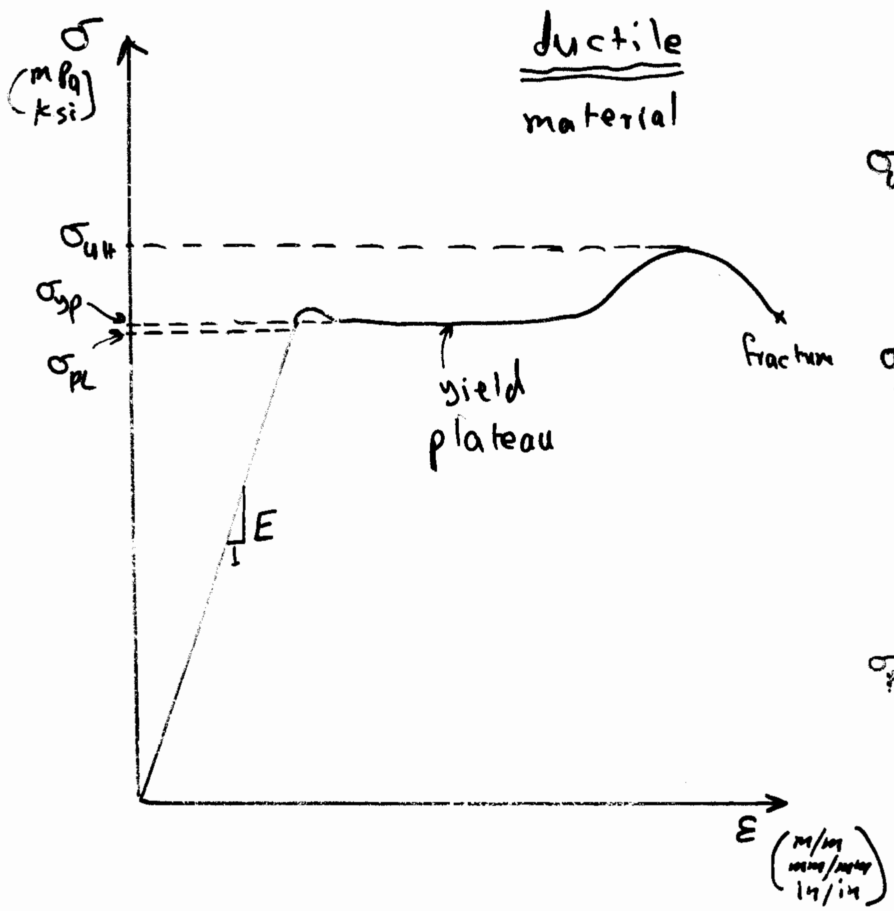
A_0 = <u>original</u> area

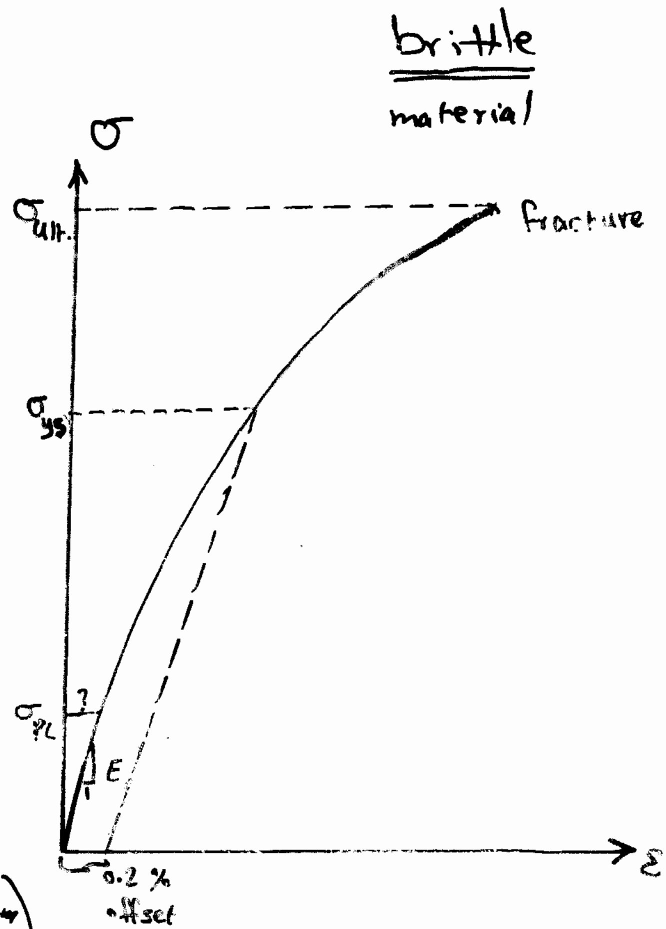
$$\varepsilon = \frac{l - l_0}{l_0} \quad \Leftarrow \text{Engineering strain}$$

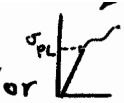
l = length at anytime during test

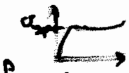
l_0 = <u>original</u> length

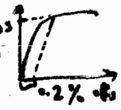
Typical $\sigma - \varepsilon$ Curve





σ_{PL} = Proportional limit : the end of linear behavior 

⊛ σ_{yp} = yield point stress : constant σ with increasing ε ((ductile materials 

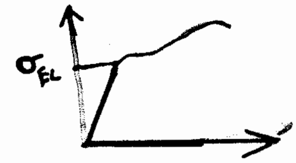
σ_{ys} = yield strength : stress at offset (usually 0.2 %) ((for brittle materials with no σ_{yp})) 

⊛ σ_{ult} = ultimate strength : the maximum value for σ in σ-ε curve 

σ_{EL} = Elastic Limit : the end of elastic action (linear or nonlinear

$\neq \sigma_{PL}$ « technically »

$\approx \sigma_{PL}$ for many materials

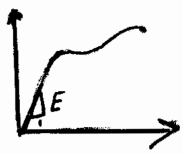


⊛ E = Elastic modulus = Young's modulus = modulus of elasticity

: the slope of the initial elastic portion.

It is a measurement of the material stiffness

« Big" number $\div \frac{\sigma}{\varepsilon}$ ⇐ stress units



$$E = \frac{\sigma}{\varepsilon} \Rightarrow$$

$$\boxed{\sigma = E\varepsilon}$$

⇐ Hooke's Law

« linear elastic materials »

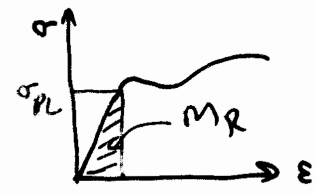
$E \approx 200$ GPa, $= 30\,000$ ksi } steel

≈ 70 GPa, ≈ 10 ksi } Aluminum

≈ 20 GPa, $\approx 3\,000$ ksi } Concrete

M_R = modulus of resilience : **area** under $\sigma-\varepsilon$ curve up to σ_{PL}

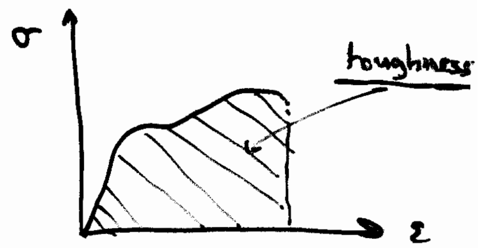
* units of energy per unit volume



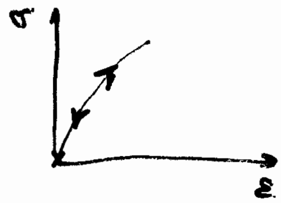
= a measurement of the capacity of the material to absorb energy w/o permenent deformation

Toughess = **total area** under $\sigma-\varepsilon$ curve

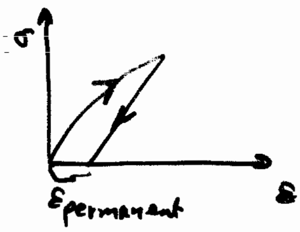
= a measurement of the material to undergo large permanent deformation prior to fracture
((energy / volume))



Elasticity : full recovery of ε after loading/unloading

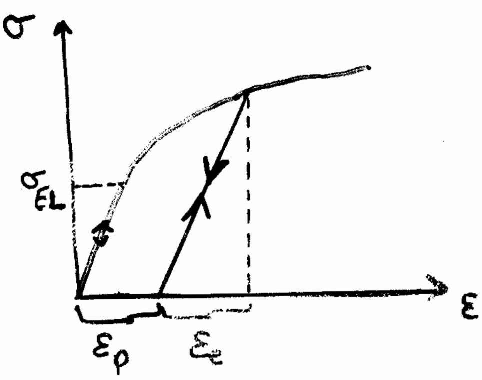


Plasticity : permanent « plastic » strain (deformation) after loading / unloading



Strain hardening : After plastic deformation and unloading, reloading.

σ_{EL} increases and, of course, modulus of resilience, but toughness decreases.
⇒ Strain-hardening



Ductile behavior : The toughness of the material is "large" } Steel/Aluminum
<< can have "big" strain before rupture >>
= f (temp., rate, type of loading, ...)

Brittle behavior : "opposite" of ductile behavior } concrete & glass

$$\% \text{ reduction of area} = \frac{A_o - A_f}{A_o} \times 100 \quad \Leftarrow \quad A_o = \text{original area}, \; A_f = \text{final area}$$

$$\% \text{ elongation} = \frac{l_f - l_o}{l_o} \times 100 \quad \Leftarrow \quad l_o = \text{original length}, \; l_f = \text{final length}$$

These two values (% reduction in area and elongation) are also used for the description of ductility as the toughness.

Isotropy: Isotropic materials if its elastic properties are the same in all directions as steel and aluminum (not wood)

Homogeneity: Homogeneous material if its elastic properties are the same throughout the body.

Safety Factor – Allowable Stress:

$$\text{Safety Factor} = \frac{\sigma_{strength}}{\sigma_{allowable}}$$

$$\sigma_{allowable} = \frac{\sigma_{strength}}{\text{Safety Factor}}$$

Why safety factor ?!

<u>Uncertainty</u> in:

* material properties ⇉ ⋮ - - -
* loading → how much, number, type, location, ⋮ etc.
* dimensions

⋮ etc.

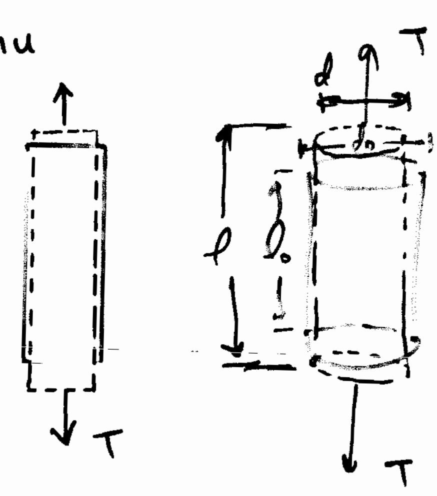
How large SF ?!

It depends on:
* uncertainties - - - -
* usage of structure (importance.... ...)
* type of failure
* stress used in design .... etc.

Poisson's Ratio (ν) ⇐ nu

$$\varepsilon_{axial} = \varepsilon_a = \frac{l - l_0}{l_0}$$

$$\varepsilon_{lateral} = \varepsilon_l = \frac{d - d_0}{d_0}$$



$$\nu = \left| \frac{\varepsilon_l}{\varepsilon_a} \right|$$

$0 \leq \nu \leq 0.5$ ⇐ range (theory)

$0.2 \leq \nu \leq 0.35$ ⇐ range for most <u>engineering materials</u>

* ε w/o σ

* σ w/o ε

⋮