Shear Force Diagram (SFD)

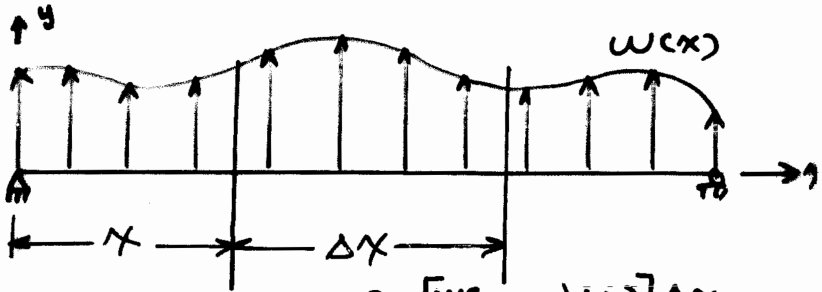
Bending Moment Diagram (BMD)

* You have already studied one method for drawing the SFD & BMD in Statics. The <u>FBD's</u> and <u>equations</u> were used. Review !!

* Now, a new method called summation (or integration or area) method will be developed. No need for FBDs and equations. It is a very quick and convenient method.

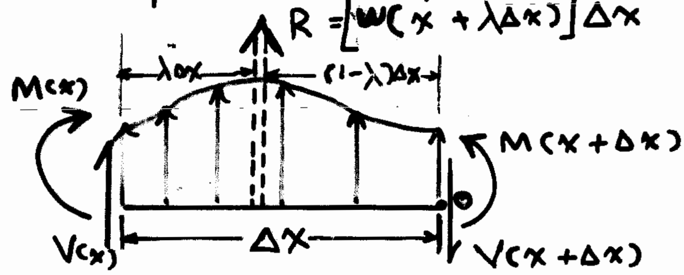
SFD & BMD by Summation Method :

In the FBD, the distributed load is replaced by an equivalent concentrated load (Resultant):



$$R = [w(x + \lambda \Delta x)]\,\Delta x$$

$$0 \le \lambda \le 1$$



FBD

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$$V(x) + [w(x+\lambda\Delta x)]\Delta x - V(x+\Delta x) = 0$$

Divide by Δx, rearrange, and let $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{V(x+\Delta x) - V(x)}{\Delta x} = \lim_{\Delta x \to 0} w(x + \lambda \Delta x)$$

$$\Rightarrow \boxed{\frac{dV}{dx} = w(x)}$$

$\circlearrowleft + \Sigma M_o = 0 \Rightarrow$

$$-M(x) + M(x+\Delta x) - V(x)\Delta x - [w(x+\lambda\Delta x)](\Delta x)(1-\lambda)(\Delta x) = 0$$

Divide by Δx, rearrange, and let $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{M(x+\Delta x) - M(x)}{\Delta x} = \lim_{\Delta x \to 0} V(x) + \lim_{\Delta x \to 0} [w(x+\lambda\Delta x)](1-\lambda)\Delta x$$

$$\Rightarrow \boxed{\frac{dM}{dx} = V(x)}$$

Interpret the equations above geometrically (slope & area):

By integrating the two eqs. above on the interval i and $i+1$:

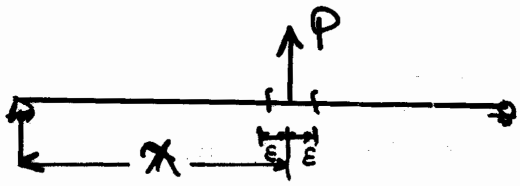
$\underline{\Delta V} = V_{i+1} - V_i = \underline{\text{area}}$ under the $\underline{\text{load}}$ diagram between x_i and x_{i+1}

$\underline{\Delta M} = M_{i+1} - M_i = \underline{''}$ " " $\underline{\text{shear}}$ " " " " "

* Note that the load must be continuous.

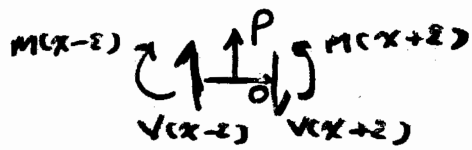
* If the load w is of order x^n, then V is of order x^{n+1} and M is of order x^{n+2}.

* Concentrated Loads:



$+\uparrow \Sigma F_y = 0 \Rightarrow$

$P + V(x-\varepsilon) - V(x+\varepsilon) = 0$



$\Rightarrow \lim_{\varepsilon \to 0} \left[V(x+\varepsilon) - V(x-\varepsilon) \right] = P$

$$\Rightarrow \boxed{V(x+) - V(x-) = \Delta V = P}$$

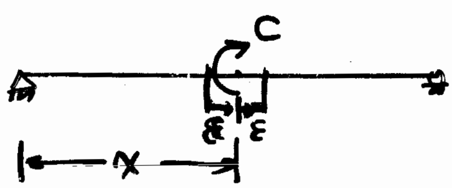
$\circlearrowleft\!+ \ \Sigma M_o = 0 \Rightarrow$

$-M(x-\varepsilon) + M(x+\varepsilon) - V(x-\varepsilon)(2\varepsilon) - P\varepsilon = 0$

$\lim_{\varepsilon \to 0} \left[M(x+\varepsilon) - M(x-\varepsilon) \right] = \lim_{\varepsilon \to 0} \left[V(x-\varepsilon)(2\varepsilon) + P\varepsilon \right]$

$$\Rightarrow \boxed{M(x+) - M(x-) = \Delta M = 0}$$

* Concentrated Couple:



We do as we did above $\Rightarrow$

$\Sigma F_y = 0$; $\Sigma M = 0$ $\Rightarrow$

$$\boxed{V(x+) - V(x-) = \Delta V = 0}$$

$$\boxed{M(x+) - M(x-) = \Delta M = C}$$ $\Leftarrow$ note $\oplus$ C