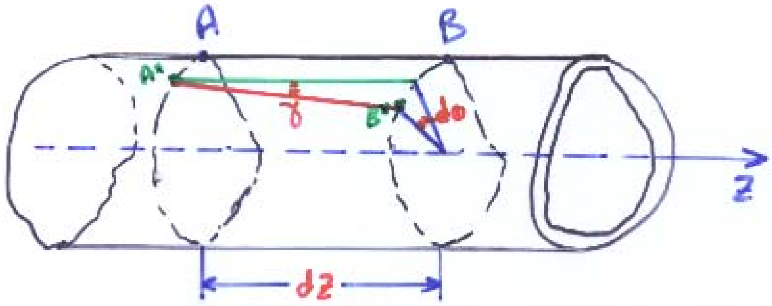
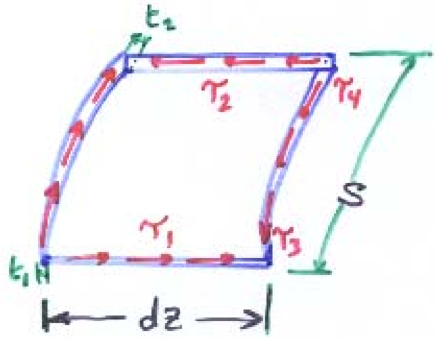
Elastic Twisting of Thin-Walled Closed Sections

* The assumptions made in circular sections are still valid. $\Leftarrow$ Closed

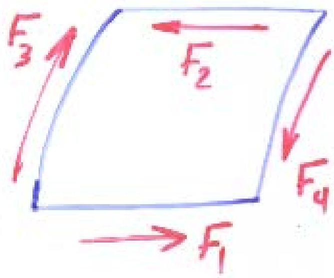


* The thickness may vary on a cross section, but it is constant along any line parallel to the z-axis.



* Shearing strain and, thus, Shearing stress is assumed to be distributed uniformly over the thickness (longitudinal or circumferential) $\Leftarrow$ Thin-Walled

$dz \Rightarrow$ τ_1 & τ_2 are the same at any point on the longitudinal planes they act



$S \Rightarrow$ τ_3 & τ_4 could be different

FBD

$F = (\text{stress})(\text{Area}) \Rightarrow$

$F_1 = \tau_1 t_1 dz$

$F_2 = \tau_2 t_2 dz$

From the FBD, $F_1 = F_2 \Rightarrow \tau_1 t_1 dz = \tau_2 t_2 dz$

$\Rightarrow \tau_1 t_1 = \tau_2 t_2 = \text{constant}$

As proved earlier, $\tau_1 = \tau_3$ and $\tau_2 = \tau_4$ } at the corner $\perp$ to each other

$\Rightarrow \tau_3 t_1 = \tau_4 t_2 = \text{constant} \Rightarrow$

τt is constant at any point on the cross section.

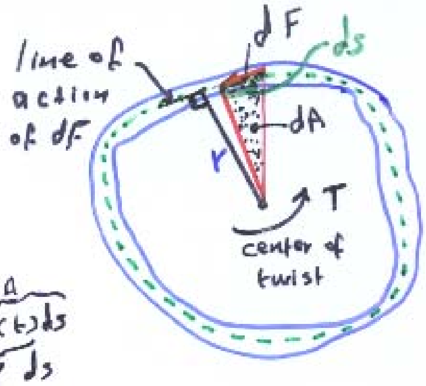
$\tau t = q$ = shear flow $\Leftarrow$ units $= \frac{\text{force}}{\text{length}}$ {N/m or lb/in}

Now, we need to find q :

$$dT = (dF)r$$

$$\int dT = \int (dF)r$$

$$\Rightarrow \quad T = \int_{A_m \leftarrow \{\text{mean perimeter}} (q\,ds)\,r \qquad \Leftarrow \quad dF = \tau A = \tau(t)ds = q\,ds$$



$$dA = \tfrac{1}{2} r\,ds$$

$$\Rightarrow \quad T = \int q\,(r\,ds) = \int q\,\tfrac{1}{2}(r\,ds)2 = \int 2q\,dA$$

Note that q is not a function of dA [[constant on the section as proved above]]

$$\Rightarrow \quad T = 2q \int_{A_m} dA$$

$\int_{A_m} dA = A_m =$ the area contained within the mean perimeter
« not material area! »

$$\Rightarrow \quad T = 2q A_m \quad \Rightarrow \quad \boxed{q = \frac{T}{2A_m}}$$

$$\Rightarrow \quad \boxed{\tau = \frac{q}{t} = \frac{T}{2tA_m}}$$

To find the angle of twist Φ, the energy (work) principles are used. $\Rightarrow$

Internal Work ((Energy)) = External Work

$\Rightarrow$ The following formula is derived (no need for the prove):

$$\boxed{\frac{d\Phi}{dz} = \frac{T}{4A_m^2 G}\int_{S_m} \frac{ds}{t}}$$

For x-sections made of a series of rectangular parts for each of which $t =$ constant, $\int \frac{ds}{t}$ is replaced by $\sum_{i=1}^{n} (s_i/t_i)$ $\Longrightarrow$

$$\boxed{\frac{d\Phi}{dz} = \frac{T}{4A_m^2 G}\sum_{i=1}^{n} \frac{S_i}{t_i}}$$

$n =$ number of parts

$\Phi = \left(\frac{d\Phi}{dz}\right)L$